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Theoretical study of rock mass investigation efficiency

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This report concerns a study which was conducted for SKB. The conclusions and viewpoints presented in the report are those of the authors and do not necessarily coincide with those of the client.

Abstract

The study concerns a mathematical modelling of a fractured rock mass and its investigations by use of theoretical boreholes and rock surfaces, with the purpose of analysing the efficiency (precision) of such investigations and determine the amount of investigations necessary to obtain reliable estimations of the structural-geological parameters of the studied rock mass. The study is not about estimating suitable sample sizes to be used in site investigations, The purpose of the study is to analyse the amount of information necessary for deriving estimates of the geological parameters studied, within defined confidence intervals and confidence levels. In other words, how the confidence in models of the rock mass (considering a selected number of parameters) will change with amount of information collected form boreholes and surfaces.

The study is limited to a selected number of geometrical structural-geological parameters:

- Fracture orientation: mean direction and dispersion (Fisher Kappa and SR1).
- Different measures of fracture density (*P10, P21* and *P32*).
- Fracture trace-length and strike distributions as seen on horizontal windows.

A numerical Discrete Fracture Network (DFN) was used for representation of a fractured rock mass. The DFN-model was primarily based on the properties of an actual fracture network investigated at the Äspö Hard Rock Laboratory. The rock mass studied (DFN-model) contained three different fracture sets with different orientations and fracture densities. The rock unit studied was statistically homogeneous. The study includes a limited sensitivity analysis of the properties of the DFN-model.

The study is a theoretical and computer-based comparison between samples of fracture properties of a theoretical rock unit and the known true properties of the same unit. The samples are derived from numerically generated boreholes and surfaces that intersect the DFN-network. Two different boreholes are analysed; a vertical borehole and a borehole that is inclined 45 degrees. Borehole lengths are varied between 20 and 1000 metres. Circular horizontal rock surfaces are also analysed, the radii of these surfaces were varied between 4 and 150 metres. The results of the study are based on both parametrical and non-parametrical statistical tests (parametrical tests for Fisher spherical distributions).

The detailed results of the study are given as calculated borehole lengths and radii of rock surfaces (sample sizes), necessary for estimating structural-geological parameters of each fracture set, for a given confidence interval and a given confidence level. The sensitivity analysis, demonstrates and discuses how sample size varies with the properties of the DFN-model (fracture density [*P32*] and fracture radius distribution.) In addition the results of the study includes discussions of (i) the optimal orientation of a borehole, (ii) the exchangeability of samples from several shorter boreholes and smaller surfaces contra samples from fewer but larger boreholes and surfaces, and (iii) the applicability of parametrical tests in relation to sampling bias.

Different methods for calculation of the structural-geological parameters from samples taken in boreholes and on surfaces are discussed and analysed in the study, e.g. for fracture orientation the eigenvalues and resultant vector methods (with inclusion of Terzaghi-correction). For the trace-length and strike distributions, moments and shape of distributions have been analysed (with inclusion of curve fitting procedures).

Sammanfattning

Denna studie är en matematisk modellstudie av en sprickig bergmassa och dess undersökning med hjälp av observationer i teoretiska borrhål och kartering av teoretiska bergytor (hällar), med syftet att analysera sådana undersökningars effektivitet och precision, och bestämma den undersökningsmängd som är nödvändig för att erhålla pålitliga uppskattningar av bergmassans strukturgeologiska egenskaper (parametrar). Det är inte syftet med denna studie att uppskatta en lämplig stickprovsstorlek att användas vid platsundersökningar. Studien syftar istället till att analysera den informationsmängd som är nödvändig för att erhålla uppskattningar av de studerade geologiska parametrarna med bestämda konfidensinterval och konfidensnivåer. Alltså en analys av hur konfidens i modeller av bergmassan (för vissa utvalda parameter) förändras med mängden information som erhålls från borrhål och hällar.

Studien är begränsad till ett utvalt antal geometriska strukturgeologiska parametrar.

- Sprickorientering: medelriktning och dispersion (Fisher Kappa och SR1).
- Olika mått på sprickdensitet (*P10*, *P21* och *P32*)
- Sprickspårlängd och sprickspårriktning.

Den studerade bergmassan (sprickigt berg) representerades av numeriska DFN-modeller (nätverk av diskreta sprickor). DFN-modellerna baserades huvudsakligen på egenskaperna hos ett verkligt spricksystem som har undersökt vid Äspö berglaboratorium (HRL). Den analyserade bergmassan (DFN-modellen) innehåller tre olika sprickset, med olika orientering och värden på sprickdensitet. Den studerade bergenheten var statistiskt homogen. Studien inkluderar en begränsad sensitivitetsanalys av DFN-modellens egenskaper.

Studien är en teoretisk och datorbaserad jämförelse mellan egenskaper som uppvisas av stickprov från en bergenhet och bergenhetens kända egenskaper. Stickprov erhölls från numeriskt genererade borrhål och bergytor, som genomkorsar DFN-modellen. Två olika borrhål analyserades; ett vertikalt borrhål och ett borrhål som vinklades 45 grader. Borrhålens längd varierades mellan 20 m och 1000 m. Cirkulära horisontella bergytor analyserades också, radien på dessa ytor varierades mellan 4 m och 150 m. Studiens resultat baserades på både parametriska och icke-parametriska statistiska tester (parametriska tester mot sfäriska Fisherfördelningar).

Studiens detaljerade resultat är beräknade borrhålslängder och radier på hällar (stickprovsstorlek), nödvändiga för uppskattning av spricksetens strukturgeologiska parametrar, vid givna konfidensinterval och konfidensnivåer. Sensitivitetsanalysen demonstrerar och diskuterar hur stickprovsstorlek varierar med DFN-modellens egenskaper (sprickdensitet [*P32*] och sprickradiusfördelning). Dessutom inkluderar studien en diskussion om (i) mest fördelaktig orientering för ett borrhål, och (ii) utbytbarheten av stickprov från flera korta borrhål och små hällar kontra stickprov från få men långa borrhål och stora hällar, och (iii) parametriska testers tillämpbarhet i relation till systematiska avvikelser i stickprovsundersökningarna.

Olika metoder för att beräkna stukturgeologiska parametrar från stickprov tagna i borrhål och hällar diskuteras och analyseras i studien; för sprickorientering egenvärdesmetoden och resultantvektorsmetoden (med Terzaghikorrektion); för sprickspårsfördelningar analyserades moment och form på fördelningar (med bl.a. kurvpassnings mot lognormal fördelningar).

Contents

Executive summary

Introduction

SKB will conduct site investigations for selecting a suitable place to locate the deep repository for nuclear waste. Rock units are to be investigated by use of deep boreholes and mapping of rock outcrops (other methods will also be used). Based on analysis of observations collected from boreholes and rock outcrops (and other investigations), site descriptive models of the rock mass are established. These models describe the geological parameters of the rock mass. Many geological parameters are heterogeneous and vary spatially (e.g. fracture density, hydraulic conductivity etc), therefore the confidence in the established models depends on the number and size of boreholes and rock outcrops used for investigating the rock mass and for establishing the site descriptive models. However, due to practical and economical limitations the number of possible boreholes etc is limited. Considering site investigations and the rock mass analysed in this study (a selected rock unit with specific properties), the results of this study will indicate lengths of boreholes and sizes of rock surfaces, necessary for deriving estimates of the selected and analysed structural geological parameters, within defined confidence intervals and confidence levels.

However, it is important to note that this study is not about estimating the necessary sample sizes to be used in site investigations. The necessary amount of information that needs to be collected at a site investigation is best calculated based on statistical analysis at different stages of sampling (preliminary and confirmatory sampling) and in combination with safety analysis calculations (i.e. sensitivity analyses of such calculations). Theoretically, the necessary sample sizes and acceptable uncertainties in estimation of the true properties (parameters) of a rock mass depend on the properties of the investigated site and the results of safety analyses calculations. Large uncertainties could be accepted for parameters with little importance in the safety analysis, or for remote rock volumes that carries small importance in the safety analysis; while parameters and rock volumes that the safety analysis calculations has identified as being important for the performance of the investigated site, such parameters and volumes needs to be investigated in more detail to produce reliable estimates with a small amount of uncertainty.

Purpose

This study is a mathematical modelling of a fractured rock mass and its investigations by use of theoretical boreholes and rock surfaces, with the purpose of analysing the efficiency and precision of such investigations. The general purpose of this study is to investigate how knowledge of selected geological parameters depend on information collected from boreholes and rock surfaces and how this information varies with length and inclination of boreholes, as well as on size of rock surfaces. In other words, how the confidence in the models of the rock mass (considering a selected number of parameters) will change with amount of information collected form boreholes and rock surfaces.

This study is limited to a selected number of geometrical parameters of a fracture system. Considering the site investigation program, /Stråhle, 2001/ defines such parameters. In this study the following geological parameters are investigated:

(i) Fracture orientation.

(ii) Fracture density (frequency)

(iii) Fracture trace length

In this study fracture orientation is analysed considering mean directions and dispersions of the different fracture sets. Fracture density (frequency) is analysed considering different density parameters (*P10*, *P21* and *P32*). Fracture trace-length and fracture strike distributions (based on direction of fracture traces) are analysed considering distribution characteristics.

In general, the method of the study is to numerically generate a fracture network and numerically analyse it, by use of theoretical boreholes and surfaces. A comparison between the known true properties of the network (the parameters) and the derived properties (the samples) will reveal the deviation between the true properties and the derived properties, and the size of deviation will indicate how the knowledge will vary with the amount of investigation.

Terminology

Some of the terms used in this study are explained in the next section.

Methodology – general

This study is a theoretical and computer-based comparison between (i) samples of fracture properties of a theoretical rock mass (a fracture network) as revealed by observations in simulated boreholes and on simulated rock surfaces; and (ii) the known true properties (parameters) of the theoretical rock mass. Discrete fracture networks (DFN-models) represent the rock mass, and the computer program Eblafrac generated the DFN-models. In this study the properties of the fracture network of the rock mass are known, and these networks constitute the ”reality” studied.

Thus, the numerically generated fracture network is the studied population. The boreholes studied are theoretical lines that cut through the fracture network. The fractures that intersects the borehole (the observed fractures) form a sample of the fracture population. The rock surfaces studied are theoretical planes that cut through the fracture network. The fractures that intersect the plane (the observed fracture traces) form a sample of the fracture population. The properties of the samples are estimates of the properties of the population.

Properties of the studied fracture network – DFN model

The studied fracture network represents the rock mass at the Prototype Repository at the Äspö Hard Rock laboratory. The fracture network model, used in this study, is the DFN 2 model presented in /Hermanson et al, 1999/. The main objective of the DFN 2 modelling was to establish a discrete fracture network model, representing the rock mass at the Prototype Repository, which could be used for simulation of groundwater flow. Hence, the model was not intended for rock mechanical purposes. The DFN 2 model underestimates the total number of fractures in the rock mass at the Prototype Repository, as small fractures with minor or negligible hydraulic importance is not included in the model. To what degree the DFN 2 model represents the actual properties at the Prototype Repository are not analysed in this study.

The fracture network studied consists of three fracture sets. Set 1 and Set 2 have a sub-vertical orientation and Set 3 is sub-horizontal. The largest dispersion in fracture orientation (deviations about the mean direction) takes place within Set 1. For the other two fracture sets, the dispersion is much less and about the same. On the average, the largest fractures occur within Set 2, the smallest fractures are within Set 1. The fracture density, given as fracture area per unit volume (*P32*), varies between the fracture sets; Set 2 has the largest *P32*-value and Set 1 the smallest *P32*-value. A summary of the properties of the fracture network is given in Table 2-1 through Table 2-3 (page 31). The fractures are defined as circular planar discs with varying values of radii.

Properties of the studied boreholes and rock surfaces

We have studied two different boreholes, a vertical and an inclined borehole; the orientation of the inclined borehole is 45 degrees from vertical. For both boreholes, the lengths (of the boreholes) were varied from 20 metres and up to 1000 metres.

We have also studied rock surfaces. The rock surfaces are analysed for fracture traces. A studied rock surface is called a window. All the analysed windows are horizontal; they correspond to horizontal rock outcrops. The geometrical shape of the windows studied is circular. The radius of the windows was varied from 4 metres and up to 150 metres.

For the boreholes and the windows, the number of realisations of the rock mass were varied between 500 and 1000. Hence, for every borehole length and rock surface area studied, a large number of different realisations of the fracture network were analysed. The large number of realisations is necessary to obtain reliable statistics.

Terzaghi correction

One-dimensional sampling is sampling along a straight line (a scanline). Such sampling of fracture orientation in a three-dimensional fracture system will introduce an orientation sampling bias. For compensation of this sampling bias /Terzaghi, 1965/ proposed the application of a geometrical correction factor, see Appendix B. In this study all fracture orientation data, derived from sampling the boreholes, are corrected for sampling bias by use of the Terzaghi correction. No Terzaghi correction was included when fracture densities (*P10*, *P21* and *P32*) were estimated. In this study fracture data gathered from surfaces (e.g. distribution of trace lengths) have not been corrected for orientation sampling bias.

Classification of observed fractures into fracture sets

In this study each fracture was marked with its proper set identity since this is known at the generation of the fracture. In a real situation, different methods and algorithms for identifying and delimiting sets will be necessary to ensure objective set identifications. Different methods for identification of fracture sets will produce different results. The reason why we have used the known true fracture set identity and not applied a fracture set identification algorithm is because we do not want the efficiency of the fracture set identification algorithm to influence the result of the study.

Aspects of the applied statistical tests

From a statistical point of view, the unknown properties of the rock mass are the properties of a population studied; we will call these properties the true properties. Samples will produce estimates of the true properties (estimates of the population); these estimates are called the sample properties. In general the sample properties deviate somewhat from the true properties. In reality when observing fractures in boreholes and on outcrops, and when predicting properties of the rock mass based on these observations, it is impossible to exactly calculate how much the sample properties deviate from those of the population, as the properties of the population are unknown. Nevertheless, considering the purpose of a real investigation there are probably some demands on accuracy, which correspond to an acceptable deviation in estimated properties. Decisions and conclusions are founded on the sample properties, hence large deviations between the sample properties and the true properties are not acceptable, but small deviations are acceptable as such deviations are of no practical importance.

In this study, the properties of the rock mass are known, hence (in this study) it is possible to calculate the deviation between sample properties and the true properties. Primarily this study concerns tests in which the calculated deviation between sample properties and true properties is compared to different selected acceptable deviations of the test variable studied (first category of tests). The acceptable deviations are called the test criterions. However, this study also includes tests that do not directly correspond to a selected acceptable deviation, but to a given level of confidence in estimating the true properties (second category of tests). The difference between these two types of tests should be noted. The purpose of the first category of tests is to determine when the size of the sample is large enough to produce an acceptable estimate of the true properties (e.g deviation <= 15 degrees), with a certain probability (e.g. >= 90%). The purpose of the second category of tests is to demonstrate the probability for a given hypothesis of the properties of the population, to be rejected or accepted, at a certain selected level of confidence (e.g. 99%).

For the first category of tests, the selected acceptable deviation is constant for all sizes of sample; in the second category of tests, the selected level of confidence is constant for all sizes of sample. The first category of tests are carried out as non-parametric tests, hence we make no assumptions regarding the statistical distributions of the properties of the studied fracture network or regarding systematic bias in the sampling procedure. The first category of tests could be considered as calculation of the sample size that is necessary to reach a confidence level, considering a given confidence interval. The confidence interval corresponds to the above-discussed acceptable deviation (test criterions). The sample size corresponds to a length of borehole or size of area. The second category of tests are carried out as parametric tests, for which we assume that the orientation of the fractures of the studied network are according to Fisher distributions and that no sampling bias takes place.

When performing statistical tests, it is common that different sample sizes are selected beforehand, and for such an analysis a point estimate of an unknown parameter refers to different fixed sizes of sample. That is however not the case in this study. In this study the number of observed fractures (i) along a studied borehole or (ii) on a studied surface, gives the sample size. Hence, for unknown boreholes or areas, the actual sample sizes are unknown, even if the lengths of the boreholes or sizes of areas are known, and the sample sizes are revealed when the samples are taken. The point estimates of this study refer not directly to different fixed sizes of sample, but to different fixed lengths of boreholes or sizes of area. On the average, the sample size increases with length of borehole and size of area. However, as the sample size will vary somewhat for a given borehole length or size of area, this variation will be a source of uncertainty.

When studying the results of the tests it is important to remember that we are analysing a large number of samples that produce estimates of the true properties of the population. Hence, the statistical tests are applied to distributions of estimates corresponding to different lengths of borehole or areas of rock surfaces.

Applicability and limitations of the presented results

Results and conclusions given in this study are only directly applicable to the fracture networks studied; however, rock masses with similar fracture networks will produce similar results. Nevertheless, great care should be taken when generalising results and conclusions given in this study. It is important to note the following:

- Considering the studied parameters of the rock mass, the results correspond to a rock unit having statistically homogeneous properties. When analysing real data from field investigations, the applicability of this assumption needs to be statistically evaluated.

- The rock unit studied is of a certain size and is assigned statistically homogeneous properties. Sample sizes have been calculated—sample sizes that are necessary to reach a certain confidence level when predicting the properties of the rock unit. When applying the results of this study to an actual rock unit it is not a prerequisite that the actual rock unit must be of the same size and form as the unit used in this study when the necessary sample sizes were calculated. However, the actual rock unit needs to be larger than the calculated necessary sample size, and it should carry statistically properties that are close to homogeneous within the volume considered. For example, it is a result of this study that for a certain rock mass the mean direction of a certain fracture set could be estimated (within a certain acceptable deviation) using a vertical borehole with a length of 20 m. Such a result is applicable to a rock unit that is larger than 20 m and carries statistically homogeneous properties within that scale.

- The fracture network studied does not contain any spatial correlation of the fractures. For a fracture network that has such a correlation, the necessary length of boreholes and size of rock outcrops, for producing an estimate with a certain confidence, is larger than for the network of this study.

- The effects of different methods for identification of fracture sets are not included in this study.

- The fracture orientations observed in boreholes were corrected for sampling bias by use of Terzaghi correction; such a correction is essential and should always be included when analysing fracture orientation data from boreholes.

- This study is a theoretical study, all data form the boreholes and rock-surfaces are numerically collected from a numerical fracture-network. No measurement errors occur in this study and all data is collected with the same high precision and quality.

Below are a few important observations that should be considered when generalising the results and conclusions given in this study (more details are given in the sensitivity analysis presented in Chapter 9 [page 179])

- The results depend on the properties of the fracture network, i.e. fracture orientation and fracture density (intensity) and fracture size. Generally, for a fracture network with a higher fracture density than that of the network studied, the lengths of boreholes and sizes of rock outcrops, necessary for deriving an estimate within a certain confidence interval and at a certain confidence level, is less than for the network studied. It follows that for a rock mass with a lower fracture density, the necessary length of boreholes and size of rock outcrops is larger than for the network of this study. Also the type of distribution of orientations within a fracture set (e.g. Fisher distribution) will influence the necessary lengths and areas.

- The dispersion of the orientations of the fractures of a fracture set will influence the length of a borehole and the size of a rock outcrop (window), necessary for deriving an estimate with a certain confidence. In general, when analysing a fracture set with a large dispersion, the necessary length of borehole and size of rock outcrop is larger than for a fracture set with a smaller dispersion (everything else being equal).

- When analysing a fracture set with a sampling structure, i.e. a borehole (a scan-line) or a rock-outcrop (a window). The length or size of the sampling structure, necessary for deriving an estimate with a certain confidence, depends on the orientation of the sampling structure in relation to the mean orientation of the fracture set studied. In general the most favourable orientation of a sampling structure is an orientation parallel to the mean direction (defined by trend and plunge) of the fracture set studied, i.e. on the average the fracture planes should be at right angles to the structure. For boreholes, the use of Terzaghi-correction will compensate for the systematic bias caused by sampling a three-dimensional fracture system with a one-dimensional scan-line. Therefore, a borehole that is approximately at right angle to the mean direction (defined by trend and plunge) of the fracture set (i.e. on the average the fracture planes are along the borehole) can be used for sampling. The Terzaghi-correction is not perfect and for very small confidence intervals (acceptable deviations), the remaining bias may come to dominate the derived estimates.

- Consider estimations based on observations in boreholes. Everything else being equal, the necessary length of borehole for producing an estimate with a certain confidence level (for a given confidence interval) is linearly proportional to the fracture density of the population studied (*P32, P21* or the *P10*-value).

- For a rock mass with a given fracture density, the mean and the variance of the fracture radius distributions (fractures defined as circular planar discs), will not influence the number of fractures that intersects a borehole. Hence, on the average a small number of large fractures will produce the same number of fracture observations in a borehole as a large number of small fractures, presuming that the fracture density of the rock mass is the same (*P32* is constant). It follows that for estimations based on observations in boreholes, the necessary length of borehole for producing an estimate with a certain confidence level (for a given confidence interval), is independent on mean and variance of the fracture radius distributions, presuming that the fracture density of the rock mass is the same (*P32* is constant).

- Consider estimations based on observations on surfaces. Everything else being equal, the necessary size of rock-outcrop (window) for producing an estimate with a certain confidence level is not linearly proportional to the fracture density of the population studied (the *P32*-value or the *P21*-value). Estimation of the trace-length distributions is difficult, the necessary size of window for producing an estimate with a certain confidence level depends on (i) the orientations of window studied in relation to that of the fracture set studied, (ii) the size of the window studied in relation to the properties of the fracture-radius distribution that created the fracture traces, as well as on (iii) the fracture density (the *P32*-value) and the dispersion of the fracture set studied. It follows that it is difficult to make any general conclusions regarding the necessary window size for estimating the properties of a trace-length distribution (with a certain confidence). For estimation of the mean of a strike distribution (derived from the directions of fracture traces), the necessary window-radius for deriving an estimate with a certain confidence level is related to the fracture density (*P32*-value) in a non-linear way. However, for fracture networks that are equal, except for the *P32*-value, this relationship can be analytically estimated.

Summary of detailed results

Below we will present some detailed results; a more complete summary of results is given last in the main report, Figure 9-1 (page 216) through Figure 9-4 (page 219). The results given below correspond to a confidence level of 90 percent. The confidence interval (acceptable deviation) considering fracture set mean direction is defined as a deviation of plus/minus 15 degrees from the true value of the population (deviation as an acute angle between two vectors, see Section 4.1 [page47]). The confidence interval (acceptable deviation) for the values of: fracture set dispersion, fracture densities (*P10, P21* and *P32*) as well as the moments of trace-length distributions and strike distributions (from direction of fracture traces), is defined as a range of plus/minus 15 percent of the true values of the population (centred on the true values). The length of borehole or radius of studied window corresponds to a sample size—the size that is necessary to reach the confidence level. The results given below are only examples of results that can be deduced from the figures of the main report.

Considering the fracture network studied (Table 3-1 through Table 2-3 [page 31]) and the results (summarised in Figure 9-1 [page 216] through Figure 9-4 [page 219], we conclude the following.

- For estimates of the mean directions of the fracture sets, the necessary borehole lengths are as follows: Considering a vertical borehole: 140 m (Set 1), 50 m (Set 2) and 20 m (Set 3). Considering an inclined borehole: 90 m (Set 1), 35 m (Set 2) and 35 m (Set 3).
- For estimates of the dispersion of the fracture orientations of the fracture sets, the necessary borehole lengths could be large, e.g. 400–1100 m, if the dispersion is very large and the value of *P32* (fracture density) is small, as for Set 1. For fracture sets 2 and 3, the necessary borehole lengths are between 100 and 500 metres, dependent on direction of borehole and dispersion parameter studied.
- For estimates of the *P10* (fracture frequency) of the fracture sets, the necessary borehole lengths are as follows: Considering a vertical borehole: 400 m (Set 1), 300 m (Set 2) and 150 m (Set 3). Considering an inclined borehole: 350 m (Set 1), 150 m (Set 2) and 210 m (Set 3).
- For estimates of the *P21*-values (trace length per area) of the different fracture sets, the necessary radius of a horizontal circular window is as follows: 24 m (Set 1), 22 m (Set 2) and 40 m (Set 3).
- For a direct estimate of the *P32* (fracture density) of the fracture sets, from borehole data, the necessary borehole lengths are very different depending on direction of the different fracture sets and the *P32*-values of the fracture sets. Considering a vertical borehole: 850 m (Set 1), 650 m (Set 2) and 150 m (Set 3). Considering an inclined borehole: 480 m (Set 1), 320 m (Set 2) and 380 m (Set 3).
- For indirect estimates of the *P32* of the fracture sets, from borehole and surface data by use of the *P10* or the *P21* parameters, the values of borehole lengths or surface radii are the same as for the estimations of the *P10* or the *P21* parameters. However, the convergence criteria of the trial and error procedure, necessary for such an estimation of *P32*, will reduce the confidence level of such estimations (although that reduction could be small).
- For estimates of the moments of the trace-length distributions of the fracture sets, the necessary radius of a horizontal circular window is as follows: Considering mean of distribution: 32 m (Set 1), 45 m (Set 2) and 52 m (Set 3). Considering standard deviation of distribution: 52 m (Set 1), 12 m (Set 2) and 70 m (Set 3).
- For estimates of the shapes of the trace-length distributions of the fracture sets (by use of non-parametrical goodness-of-fit tests), the necessary radius of a horizontal circular window is as follows: 13 m (Set 1), 38 m (Set 2) and 32 m (Set 3).
- For estimates of the mean of the strike distributions of the fracture sets (calculated from direction of fracture traces), the necessary radius of a horizontal circular window is as follows: 35 m (Set 1), 18 m (Set 2) and 60 m (Set 3).
- For estimates of the shape of the strike distributions of the fracture sets (calculated from direction of fracture traces) a non-parametrical goodness-of-fit test was used, the necessary radius of a horizontal circular window is as follows: 13 m (Set 1), 11 m (Set 2) and 24 m (Set 3).

When comparing the results for the different fracture sets, it is demonstrated that Set 1 is the fracture set most difficult to analyse, because this set has a large dispersion and the smallest value of *P32* (fracture density) of the three sets studied.

When comparing the results of a specific fracture set considering different borehole orientations, the variation in results is in line with the variation in number of fractures observed in boreholes with different orientations.

Considering the orientation of the fractures of a fracture set, it is more difficult to estimate the dispersion of the fracture orientations than the mean of the fracture orientations.

Considering fracture set 3 and horizontal windows, the large radius necessary for good estimates of the parameters of Set 3 is caused by the sub-horizontal orientation of Set 3, because the fractures of a sub-horizontal fracture set only rarely intersects a sub-horizontal surface. A fracture set with such an orientation is not well analysed by use of sub-horizontal surfaces, unless a correction for sampling bias is applied and in this study such a correction was not used when the surface data were analysed. (Correction for orientation sampling bias was only applied to borehole data.)

Estimation of the trace-length distributions is difficult, as such estimations (among other things) depend on the size of the window studied in relation to the properties of the fracture radius distribution that created the fracture traces. Therefore the results for different fracture sets could be very different, for the same size of window.

On parametric tests and calculated confidence intervals

Parametric statistical tests were carried out regarding mean direction and dispersion of the three fracture sets of the population, considering observations of fracture orientation in theoretical boreholes (see Sections 3.5 [page 57] and 4.4 [page 87]). As the population (the fracture network) is created by use of Fisher distributions, the tests were based on the assumption that samples were drawn from (represent) Fisher distributions.

The tested hypothesis was that the mean direction and the dispersion of the population, as estimated by the samples, are equal to the known true properties of the population. We know that this is a correct hypothesis; but due to sampling bias, remaining in the samples after application of Terzaghi correction, the hypothesis will not necessarily be confirmed by the samples.

The results of the tests demonstrate a larger amount of rejected samples, than the amount prescribed by the confidence level of the tests. The following conclusion can be made: If we assume that (i) samples are drawn from perfect Fisher distributions and that (ii) the systematic sampling bias is fully corrected by use of Terzaghi correction; we may derive confidence intervals, based on parametrical statistical analysis, that are to small and which do not reflect the actual uncertainties. This is especially the case if the sample size is large (a sample that contains a large number of fracture observations) as the confidence intervals, derived through parametric statistical analyses, are small for such samples.

On optimal orientation of a borehole

Based on observations in theoretical boreholes, we have estimated fracture set orientation, mean direction and dispersion, as well as the fracture density parameters *P10* and *P32*. Two different boreholes have been used, a vertical and an inclined borehole. By comparing the efficiency of the point estimates, as produced by the two boreholes, we can make conclusions regarding the optimal orientation of a borehole.

Let us first consider the *P10*-parameter (fracture frequency in a borehole); it is a direction-dependent parameter and as such it is calculated without Terzaghi correction. The point estimate of the *P10* parameter relates to borehole length and not to number of fractures in a sample. However, the efficiency of the point estimate increases with number of fractures observed in a sample; hence for a given borehole length, the borehole that intersects most fractures will produce the most efficient point estimate as regards the *P10*-parameter. Considering the two borehole directions studied, the inclined borehole (45 deg.) produces on the average, when adding together all three fracture sets, the largest samples (number of fractures per metre of borehole), and consequently as regards *P10* the point estimate is most effective for the inclined borehole.

For all parameters analysed by use of boreholes, on the average the most efficient point estimate takes place for the borehole direction for which most fractures are intersected. Hence, in order to reach the largest efficiency when analysing a single fracture set, the borehole should not necessarily be an inclined borehole, but directed so that the mean direction (defined by trend and plunge) of the fracture set studied is parallel to the borehole (i.e. on the average the fracture planes are at right angles to the borehole), because on the average this is the borehole direction that produces the largest samples (for a given borehole length). Consequently, different borehole directions are optimal for different fracture sets.

The borehole length necessary for deriving acceptable estimates of all properties studied of all fracture sets studied is determined by the length necessary for deriving an acceptable estimate of the property and fracture set that is the most difficult to estimate. The properties that are easier to estimate will be derived within the borehole length necessary for the most difficult estimation. For example, if we want to estimate the mean orientation and dispersion (Kappa) of the three fracture sets studied, by use of a vertical borehole, the necessary length is 500 m (confidence level=90%; confidence interval =+/–10 degrees (orientation) and +/–15% (Kappa)). By use of an inclined borehole, the necessary length is 500m as well. For the vertical borehole the most difficult parameter to estimate is the dispersion of Set 1, consequently this is the parameter that determines the borehole length for the vertical borehole. For the inclined borehole the most difficult parameter to estimate is the dispersion of Set 3, and consequently this is the parameter that determines the borehole length for the inclined borehole. For both boreholes the necessary borehole length is 500m.

Even if the necessary length of borehole was the same for the two borehole orientations, as this length was determined by the most difficult estimation, the necessary lengths for estimating the other parameters were not the same. As a measure of the average efficiency of a borehole orientation we have calculated the average necessary length for estimating certain parameters in the same borehole (average necessary borehole length is defined by equation 10-1 [page 211]).

The results for the *P10* and *P32* parameters are given in Table 9-1 [page 212]. Considering fracture frequency *P10* and a vertical borehole, the necessary lengths are 400 m (Set 1), 300 m (Set 2) and 150 m (Set 3), producing an average necessary length of 283 m (confidence interval= +/–15% of true value and confidence level= 90%). For an inclined borehole the average necessary length is 236 m. The average necessary length of the inclined borehole is 84% of that of the vertical borehole. Considering fracture density *P32* (based on borehole data), the average necessary length of the inclined borehole (393 m) is 71% of that of the vertical borehole (550 m). Hence, the inclined borehole produces on the average the best estimates, especially for the *P32* parameter. On the other hand, if the acceptable deviation (confidence interval) is *not* set as very small and the available borehole lengths are large, the direction of the borehole is not very important, as acceptable estimates could be derived for any direction.

Estimates of fracture set orientation should, as little as possible, be dependent on the orientation of the investigation borehole. Therefore all orientation data from boreholes should be corrected by use of Terzaghi correction (see Appendix B). The Terzaghi correction will compensate for most of the systematic sampling bias. After application of Terzaghi correction, the sample sizes necessary for deriving an estimate with a certain confidence, should only be weakly dependent on the orientation of the borehole, however the necessary lengths will still be dependent on dispersion and fracture density; and as the Terzaghi correction is not perfect and some systematic bias will remain in the samples, it follows that some borehole orientations are better than other orientations. The number of fractures observed and the efficiency (completeness) of the Terzaghi correction depends on the acute angle between the borehole and the mean orientation of the fracture set studied. When considering the efficiency (completeness) of the Terzaghi correction, different directions of borehole are optimal for different fracture sets (as they occur in a rock unit). The remaining bias will have the least influence if the bias is distributed in a symmetric way around the predicted mean orientation, which is achieved for boreholes that are at right angles or parallel to the mean direction of the fracture set.

Hence, for best efficiency of the Terzaghi correction, the borehole should be directed in a way that the mean direction (trend and plunge) of the fracture set studied is parallel to the borehole (i.e. fracture planes at right angles to the borehole), as most fractures are intersected for this direction, and because the remaining bias will be symmetric for such a direction. A borehole direction that is at right angle to the mean direction (trend and plunge) of a fracture set (i.e. borehole direction along fracture planes) could (theoretically) be an efficient investigation borehole, assuming that it is has a large length. Because for very large lengths of such a borehole direction, the derived estimate will be close to the true value, as the remaining bias is symmetrically distributed for such a borehole direction.

For a borehole that is not parallel and not at right angles to the mean direction of the fracture set studied, and if the acceptable deviation (confidence interval) is set as very small, for such a situation the necessary borehole lengths could be infinite (especially for large values of the confidence level). Because the estimates might converge not towards the true value but towards a value that is slightly off the true value, due to the remaining sampling bias (see Figure 2-7 and Appendix B). (If the acceptable deviation (confidence interval) is set as very small, the estimate may converge towards a value outside of the confidence interval.)

The necessary average lengths, considering mean direction of fracture sets, are given in Table 9-2 [page 213]. The average necessary length of the inclined borehole (53 m) is 76% of that of the vertical borehole (70 m), for an acceptable deviation (confidence interval) of 15 degrees and a confidence level of 90%. For an acceptable deviation of 10 degrees, the average necessary length of the inclined borehole (113 m) is 85% of that of the vertical borehole (133 m). And finally, for an acceptable deviation of 5 degrees, the average necessary length of the inclined borehole is undefined. Because by use of an inclined (45 deg) borehole it is not possible to estimate the mean direction of Set 1 at such a small acceptable deviation (confidence interval) together with a confidence level of 90%. Hence, the inclined borehole is better than the vertical borehole, except if the confidence interval (acceptable deviation) and confidence level is set as very small, for such a situation the direction of the borehole has to be optimised for each fracture set. On the other hand, if the acceptable deviation (confidence interval) is not very small, the direction of the borehole is not very important, as acceptable estimates could be derived for any direction, and the difference in total lengths for different borehole directions is not very large.

The necessary average lengths, considering dispersion of a fracture sets, are given in Table 9-3 [page214]. Considering dispersion in fracture orientation, as represented by the SR1 dispersion parameter, the average necessary length of the inclined borehole (390 m) is 81% of that of the vertical borehole (483 m), for an acceptable deviation (confidence interval) of +/–15% of the true values and a confidence level of 90%. This is in line with the results for the mean direction (above). It should however be noted that the different necessary lengths for each individual fracture set, considering the SR1 parameter (see Section 4.2, page 64), are very large (e.g. vertical borehole, Set 1=1100 m, Set 2=250 m and Set 3=100 m). Considering dispersion in fracture orientation, as represented by the Kappa dispersion parameter (see Section 4.3, page 75), the average necessary length of the inclined borehole (427 m) is 114% of that of the vertical borehole (373 m). This is different from the results regarding mean direction, and it follows from the remaining sampling bias of the inclined borehole.

Thus, it is more difficult to predict dispersion than mean value (which is the way it should be, as dispersion is a measure of variance), it follows that the borehole direction is more important when estimating dispersion than when estimating mean direction of a fracture set.

The borehole direction is also more important when estimating *P32* than when estimating *P10*. In general, the necessary lengths of boreholes are larger when estimating *P32* than for estimation of *P10*. However, if the borehole direction and mean direction (trend and plunge) of the fracture set is parallel, the *P10*-value in the borehole is equal to the *P32*-value of the fracture set; this conclusion underlines the importance of borehole direction.

If the acceptable deviation (confidence interval) is not very small, and large borehole lengths are available, any borehole direction will do, but if the acceptable deviation (confidence interval) has to be very small and/or only short borehole lengths are available, for such a situation the borehole direction is important and needs to be optimised considering each fracture set. In general it is better to have three somewhat shorter boreholes, with different optimised directions, than one borehole with a large length.

On number of investigation boreholes and rock surfaces

In this study the analysed fracture network is statistically homogeneous, it follows that the results are only applicable to a rock unit with statistically homogeneous properties. Considering the use of boreholes for investigation of fracture sets orientation (mean direction and dispersion) and the *P10* fracture density parameter, the necessary size of samples for the estimation of the parameter does not have to come from a single borehole. If the rock mass has statistically homogeneous properties, the analysed sample can come from several different boreholes that together produce the necessary size of sample. For example, three boreholes of length 50 metres can together form a sample representing approximately the same size of sample as observations in a single borehole of length 150 metres (presuming that they all are in the same rock unit with statistically homogeneous properties). Hence, in practise when analysing a real rock mass, it is very important to know which observations belong to which rock unit, especially if several boreholes are used; that is however also a concern when analysing observations from a single borehole with a large length.

It is however a different situation when considering the mapping of fracture trace-length distributions on rock surfaces. There are several biases that come from sampling a three dimensional system with a two-dimensional surface of a given form (e.g. circular), this is discussed in Section 6.2 [page 131]; but regarding the topic of this section, the most important bias is the boundary truncation of the large fracture traces. This is stated in Section 6.3.1 [page136] in the following way “ The efficiency of a point estimate increases with sample size, however for the sampling of traces also the size of the studied window is important. The observations are made on windows that have a limited size, and the upper tail of the trace-length distribution (traces with a large length) can only be directly observed on windows of a size (radius) comparable to length of the large traces. Hence, for small windows there will be a systematic bias in the estimate of the trace-length distribution, due to boundary truncation, even if the sample size is large. (Small window sizes could be sufficient if it is possible to fit a mathematical distribution to the observed truncated trace-length distributions, even if such a curve fitting procedure will introduce uncertainty regarding the ability of such a distribution to represent the part of the true distribution that is unknown at small window sizes.) ”

It follows from the statement above that regarding the trace-length distribution it is not possible to replace observations on one large window with observations on several smaller windows, even if all windows are from the same rock unit with statistically homogeneous properties.

It is again a different situation when considering observations of fracture strike distributions, derived from directions of fracture traces, as observed on rock surfaces. As for the trace-length distribution there are several biases that come from sampling a three dimensional system with a two-dimensional surface of a given form (see Section 6.2 [page 131]). However, there is no systematic bias in the estimate of the strike distribution, due to boundary truncation of large fracture traces. Hence, when estimating the strike distribution it is possible to replace observations on one large window with observations on several smaller windows and thereby gather one large sample, presuming that all windows are from the same rock unit with statistically homogeneous properties.

Terminology

Below we will explain some of the terms used in this study. The terms are not given in an alphabetic order, but based on connecting topics.

POPULATION The collection of all individual units possessing some characteristics of interest. In this stydy the population is the fractures of the analysed fracture network.

PARAMETER A numerical characteristics of a population, which may be known or may require estimation.

SAMPLE A part of the population (or a subset of the units of the population). The sample is provided by some process or selection; with the object of investigating characteristics of the population. In this study samples are derived through observations in boreholes or on rock surfaces, the sample consists of fracture data (orientations etc).

MODAL VECTOR A vector can represent the mean direction of a fracture set. In this study the mean directions of the fracture sets of the population of fractures are called the modal vectors.

REPRESENTATIVE VECTOR In this study the orientation of a sample is calculated based on two different methods, which both produces vectors with the same orientation, but of different sizes: (i) the eigen values method and (ii) the resultant vector method (see Appendix A). In this study, the vector that is derived from the eigenvalues method is called ”the representative vector”

RESULTANT VECTOR In this study the orientation of a sample is calculated based on two different methods, which both produces vectors with the same orientation, but of different sizes: (i) the eigen values method and (ii) the resultant vector method (see Appendix A). In this study, the vector that is derived from the resultant vector method is called “the resultant vector”.

ACUTE ANGLE An acute angle is the smallest angle between two vectors. Generally in this study when we discuss an acute angle, we mean the smallest angle between the modal vectors of a fracture population and the representative vectors (or the resultant vector) of a sample taken from the population. When we discuss borehole directions, the acute angel is the smallest angle between the borehole and surrounding fractures.

KAPPA (FISHER KAPPA) DISPERSION PARAMETER The Fisher distribution /Fisher, 1953/ is characterised by a modal vector (mean direction) and a concentration parameter called kappa, the distribution has a rotational symmetry about the modal vector. The larger the value of kappa the more the distribution is concentrated towards the modal vector. Kappa is often called a dispersion parameter, but actually it is a concentration parameter, since the larger the value of kappa the more the concentrated the distribution.

SR1 AND SR2 DISPERSION PARAMETERS The mean direction of a group of fractures can be calculated based on the eigenvalues method, as proposed by /Mardia, 1972/; this method is discussed in Appendix A. The method will provide us with a representative vector. In addition the method will provide us with three eigenvalues (L1, L2 and L3), these three values provide direct information about the distribution of the group of fractures studied (the fracture cluster studied). Based on the eigenvalues, two different dispersion parameters are calculated, as proposed by /Woodcock, 1977/, these two parameters are called, SR1 and SR2, they are defined as follows: SR1= LN(L1/L2) and SR2= LN(L2/L3). The relation between these parameters can be used to quantify the shape of the cluster, e.g. concentric, girdle, etc.

EIGENVALUES METHOD A method for calculation of the mean direction of a sample of fractures, see Appendix A.

RESULTANT VECTOR METHOD A method for calculation of the mean direction of a sample of fractures, see Appendix A.

TERZAGHI CORRECTION When performing one-dimensional sampling along a line (e.g. a borehole) of a three-dimensional fracture system, there will be a systematic sampling bias. The correction of this sampling bias is called the Terzaghi correction /after Terzaghi, 1965/. See also Section 2.5 and Appendix B.

POINT ESTIMATE An estimate, based on observed data (samples), of the properties of the population (parameters) is called a point estimate. As the observed data varies from sample to sample, also the point estimate will vary.

POINT ESTIMATE, EFFICIENCY A point estimate is based on samples. As the observed data varies from sample to sample, also the point estimate will vary. However, as size of sample is increased, for an efficient point estimate: (i) the mean of different estimates should converge towards the parameter value; and (ii) the variance of different estimates should decrease. The efficiency of a point estimate is the progress, with size of sample, towards the parameter.

HYPOTHESIS TESTING Hypothesis testing can be carried out in many different ways, a classical approach is as follows. Hypothesis testing is a procedure in which we test if samples confirm certain assumed properties of the population. The first step is to establish a theory of the population; this theory is the **NULL HYPOTHESIS.** The next step is the test of samples, if a sample confirms the theory the sample is accepted, and otherwise the sample is rejected. The amount of accepted and rejected samples will provide us with information regarding the correctness and soundness of the null hypothesis

NULL HYPOTHESIS A selected hypothesis regarding the properties of samples and population.

PARAMETRIC TEST Statistical test which assumes that the analysed population is distributed according to a known probability distribution, e.g. the Fisher distribution.

NON-PARAMETRIC TEST Statistical test for which no assumptions are made regarding the probability distribution of the studied population.

CONFIDENCE INTERVAL Suppose θ is a parameter to be estimated. A confidence interval for θ is an interval of values computed from a sample, which includes the unknown value of θ with some specified probability. Some authors prefer the following definition: the confidence interval for a hypothesis test consists precisely of all those values for which the null hypothesis is not rejected at some specified degree of probability.

CONFIDENCE LEVEL The probability (e.g. 95% or 99%) that a confidence interval will cover the unknown parameter value.

1 Introduction and purpose

1.1 Introduction

SKB will conduct site investigations for selecting a suitable place to locate the deep repository for nuclear waste. Rock units are to be investigated by use of deep boreholes and mapping of rock outcrops (other methods will also be used). Based on analysis of observations collected from boreholes and rock outcrops (and other investigations), site descriptive models of the rock mass are established. These models describe the geological parameters of the rock mass. Many geological parameters are heterogeneous and vary spatially (e.g. fracture density, hydraulic conductivity etc), therefore the confidence in the established models depends on the number and size of boreholes and rock outcrops used for investigating the rock mass and for establishing the site descriptive models. However, due to practical and economical limitations the number of possible boreholes etc is limited. Considering site investigations and the rock mass analysed in this study (a selected rock unit with specific properties), the results of this study will indicate lengths of boreholes and sizes of rock surfaces, necessary for deriving estimates of the selected and analysed structural geological parameters, within defined confidence intervals and confidence levels.

However, it is important to note that this study is not about estimating the necessary sample sizes to be used in site investigations. The necessary amount of information that needs to be collected at a site investigation is best calculated based on statistical analysis at different stages of sampling (preliminary and confirmatory sampling) and in combination with safety analysis calculations (i.e. sensitivity analyses of such calculations). Theoretically, the necessary sample sizes and acceptable uncertainties in estimation of the true properties (parameters) of a rock mass depend on the properties of the investigated site and the results of safety analyses calculations. Large uncertainties could be accepted for parameters with little importance in the safety analysis, or for remote rock volumes that carries small importance in the safety analysis; while parameters and rock volumes that the safety analysis calculations has identified as being important for the performance of the investigated site, such parameters and volumes needs to be investigated in more detail to produce reliable estimates with a small amount of uncertainty.

1.2 Purpose

This study is a mathematical modelling of a fractured rock mass and its investigations by use of theoretical boreholes and rock surfaces, with the purpose of analysing the efficiency and precision of such investigations. The general purpose of this study is to investigate how knowledge of selected geological parameters depend on information collected from boreholes and rock surfaces and how this information varies with length and inclination of boreholes, as well as on size of rock surfaces. In other words, how the confidence in the models of the rock mass (considering a selected number of parameters) will change with amount of information collected form boreholes and rock surfaces.

This study is limited to a selected number of geometrical parameters of a fracture system. Considering the site investigation program, /Stråhle, 2001/ defines such parameters. In this study the following geological parameters are investigated:

1. Fracture orientation.
2. Fracture density (frequency)
3. Fracture trace length.

In this study fracture orientation is analysed with respect to mean directions and dispersions of the different fracture sets. Fracture density (frequency) is analysed with respect to different parameters (*P10*, *P21* and *P32*). Fracture trace-length and fracture strike distributions (based on fracture traces) are analysed with respect to distribution characteristics.

In general, the method of the study is to numerically generate a fracture network and numerically analyse it, by use of theoretical boreholes and surfaces. A comparison between the known true properties of the network (the parameters) and the derived properties (the samples) will reveal the deviation between the true properties and the derived properties, and the size of deviation will indicate how the knowledge will vary with the amount of investigation.

2 Methodology

2.1 General

This study is a theoretical and computer-based comparison between (i) samples of fracture properties of a theoretical rock mass (a fracture network) as revealed by observations in simulated boreholes and on simulated rock surfaces; and (ii) the known true properties (parameters) of the theoretical rock mass. Discrete fracture networks (DFN-models) represent the rock mass; the computer program Eblafrac generated the DFN-models, an example of a numerically generated fracture network is given in Figure 2-1. In this study the properties of the fracture network of the rock mass are known, and these networks constitute the ”reality” studied.

Thus, the population studied is numerically generated fracture networks, networks that represent fractures of a rock unit. The boreholes studied are theoretical lines that cut through the fracture network. The fractures that intersects the borehole (the observed fractures) form a sample of the fracture population. The rock surfaces studied are theoretical planes that cut through the fracture network. The fractures that intersect the plane (the observed fracture traces) form a sample of the fracture population. The properties of the samples are estimates of the properties of the population.

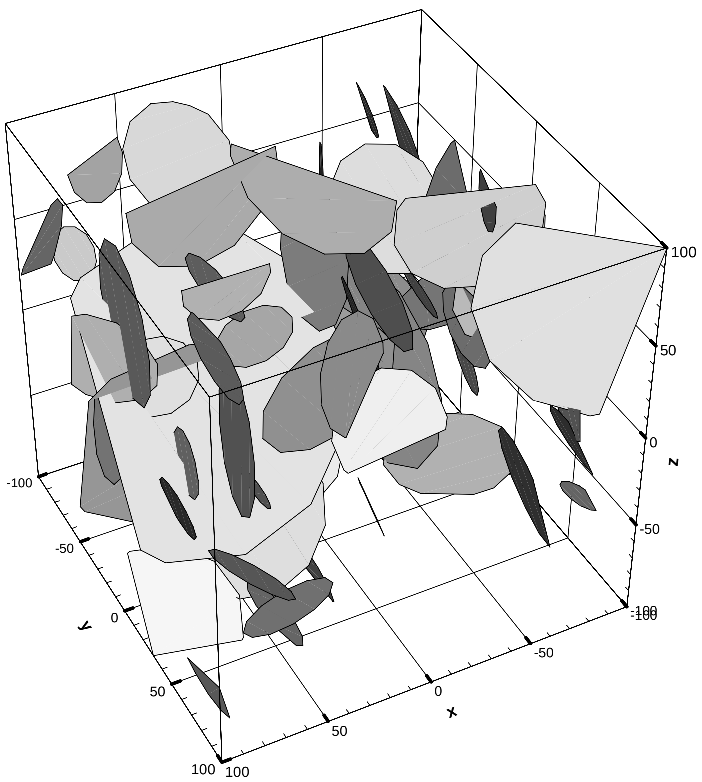


Figure 2-1. *Example of a numerically generated fracture network, the fractures have the shape of planar circular discs.*

2.2 Spherical data, co-ordinate system and projection

2.2.1 General

The fracture networks studied are numerically generated, and it follows that some simplifications have been introduced as regards the shape of the fractures of the networks, in comparison to the fracture network of an actual rock unit. In this study planar circular discs represent fractures, and when analysing the orientation of a fracture, a normal to the planar disc represents the fracture (a normal to a fracture-plane). The studied normal is a straight line in space with a certain orientation.

By spherical data we mean the orientation of a straight line in space. Hence, the data that we shall be dealing with are spherical data that represent fracture planes. (In addition, we will also analyse direction and length of the traces that the fracture-planes studied will create as they intersect a planar surface. Such lines on a planar surface are called fracture traces.)

Normals to fracture planes are lines in space, they have an orientation in space, but points in two directions; the lines are not vectors, but undirected lines called *axes.* There are many different ways of representing a three-dimensional unit vector or axis, because different methods have been developed by different scientific disciplines (e.g. Geology, Astronomy and Mathematics), but also for the purpose serving different needs within a discipline, e.g. Polar co-ordinates, Geographical co-ordinates, Geological co-ordinates (see below). The methods used in this study are briefly presented below.

2.2.2 Geological co-ordinates

The data that we shall be dealing with are lines that represent fracture planes. For the definition of the lines that represents the fracture planed we will use geological co-ordinates. Study a planar feature e.g. a fracture plane or a bedding plane:

In modern structural geology, the orientation of a planar feature is defined by its direction of dip and its angle of dip. The *dip direction* is the bearing of the line of maximum slope on the plane, in the direction of downward slope. Its value can vary between 0 and 360 degrees. The *dip angle* is the angle between the line of maximum slope and the horizontal. For some purposes, it is convenient to define a third parameter, called the strike of the plane, although dip direction and dip angle alone define the orientation of a plane unambiguously. The *strike* is the direction of a horizontal line on the planar feature and is thus, by definition, normal to the dip direction. The strike has two possible direction values, differing by 180 degrees. This ambiguity is generally treated by applying what has become known as the "right hand rule", i.e. the strike direction is the one towards which one faces when the plane slopes downwards towards one's right.

The orientation of a planar feature may also be given by a normal (or *pole*) to the plane studied. The normal has its base on the fracture plane and at the origin of a unit sphere. The pole is at the intersection of the normal with the lower or upper hemisphere of the unit sphere; in geology the lower hemisphere is normally preferred. By the concept of a normal and a pole, it is possible to define two orientation variables, called *trend* and *plunge* (or pole trend and pole plunge). The trend is the angle between North and the vertical projection of the normal onto a horizontal plane, in the direction of plunge. The plunge is the angle between the normal and the horizontal surface.

2.2.3 Spherical projection

A projection of spherical data is a representation of spherical data in the plane. As for the spherical co-ordinate systems, there are several different commonly used spherical projections based on different requirements. Geologist that analyses structural geological data commonly use an equal area projection, called Lambert or Schmidt projection and a equal angle projection called Stereographic projection, (or Wulff projection). In this study we have used the Stereographic projection (equal angle). For such a projection great and small circles projects as circular areas. Hence, a contour plot of a unimodal data set, which exhibits circular contours when projected onto a Stereographic net, indicates that the data are isotropic about their mean direction.

2.3 Properties of the studied fracture network – DFN model

The studied fracture network represents the rock mass at the Prototype Repository at the Äspö Hard Rock laboratory. The fracture network model, used in this study, is the DFN 2 model presented in /Hermanson et al, 1999/. The main objective of the DFN 2 modelling was to establish a discrete fracture network model, representing the rock mass at the Prototype Repository, which could be used for simulation of groundwater flow. Hence, the model was not intended for rock mechanical purposes. The DFN 2 model underestimates the total number of fractures in the rock mass at the Prototype Repository, as small fractures with minor or negligible hydraulic importance is not included in the model. To what degree the DFN 2 model represents the actual properties at the Prototype Repository are not analysed in this study.

The fracture network studied consists of three fracture sets. Set 1 and Set 2 have a sub-vertical orientation and Set 3 is sub-horizontal. The largest dispersion in fracture orientation (deviations about the mean direction) takes place within Set 1. For the other two fracture sets, the dispersion is much less and about the same. On the average, the largest fractures occur within Set 2, the smallest fractures are within Set 1. The fracture density, given as fracture area per unit volume (P32), varies between the fracture sets; Set 2 has the largest P32 value and Set 1 the smallest P32 value. The tables below give a summary of the properties of the fracture network (Table 2-1, Table 2-2 and Table 2-3). It is important to note the difference between the *P32*-value of a fracture set and the number of fractures of a certain set than on the average takes place in a volume of a given size. Considering the DFN-network studied, a modelled domain of cylindrical shape with height 1000 metres and radius 150 metres, will contain approximately 3.5 millions of fractures. Set 1 contains 25% of the fracture area and 70% of the number of fractures. Set 2 contains 47% of the fracture area and 15% of the number of fractures. Set 3 contains 28% of the fracture area and 15% of the number of fractures.

As previously stated, this study is a theoretical comparison between (i) the sample properties of a fracture network, and (ii) the true properties (parameters) of the fracture network. It is important that the DFN-models created by the Eblafrac computer code honour the theoretical properties that we have assigned to the DFN-models. To ensure this, the size of the modelled fracture networks is large (modelled domain), so that boundary effects will only have a minimal influence on the properties of the network.

For the analysis of the vertical and inclined boreholes, the modelled domain is of a cylindrical shape, the main axis (height) of the cylinder has a length of 1000 metres and the radius of the cylinder is about 200 metres (base case). The borehole is located at the centre of the cylinder, along the main axis of the cylinder. For the analysis of horizontal rock surfaces, the modelled domain is also of a cylindrical shape, the height of the cylinder is about 450 metres and the radius of the cylinder is of about 400 metres. The plane studied is horizontal and located at the centre of the cylinder.

The fracture network inside the cylinders contains several millions of fractures. For deriving reliable statistics, the fracture network inside the cylinder was generated between 500 and 1000 times, thereby creating the same number of independent realisations of the fracture networks surrounding the borehole or the rock surface. It follows that each studied scenario of this study (e.g. vertical borehole, inclined borehole, sensitivity-cases etc) involves the generation and analyses of billions of fractures.

Considering the models used for analyses of boreholes, the models were analysed for 50 different lengths of borehole between 20 and 1000 metres. Considering the models used for analyses of rock surfaces, the models were analysed for 34 different circular horizontal surfaces with radii between 2 and 150 metres.

The properties of the fracture networks studied were supervised during the generation of the networks; thereby we checked that the networks honoured the theoretical properties assigned to the networks. As the modelled domain is large and contains a large number of fractures, the properties of the fracture networks is expected to be close to the theoretical properties assigned to the networks. Analyses of the generated fracture networks (inside the cylinders studied) demonstrated the following. The Fisher kappa values (a measure of dispersion in fracture orientation see Section 2.8) of the fracture networks were very close to the theoretical values assigned to the networks. The deviations between the theoretical values of kappa and the kappa values of the modelled fracture networks were always less then 0.2 percent of the theoretical values. The deviations in mean orientation of the different fracture sets were less than 0.1 degrees. Similar very small deviations were observed for the other properties studied. Hence, the fracture networks studied have properties in accordance to the theoretical properties assigned to them. Examples of the orientation of the fractures of the studied population are demonstrated in Figure 2-2 below (stereographic projection is discussed in Section 2.2.3). An example of the fracture traces that the fracture network creates on a circular horizontal rock surface is given in Figure 2-5.

As previously stated, the DFN-model used in this study (as the base case) is the DFN 2 model presented in /Hermanson et al, 1999/. The DFN 2 model underestimates the total number of fractures in the rock mass at the Prototype Repository, as small fractures with minor or negligible hydraulic importance is not included in the model. We have therefore established an alternative DFN-model, which includes a larger number of small fractures, but has the same value of fracture density (*P32*-value) as the DFN 2 model. For this alternative DFN-model, the results considering the necessary sample sizes for reliable estimation of parameters studied are presented in Chapter 8 (Limited Sensitivity Analysis).

Table 2-1. Size of fractures.

Set No.	**1**	**2**	**3**
Fracture shape	Planar discs	Planar discs	Planar discs
Distribution	TLogNormal (1)	TLogNormal (1)	TLogNormal (1)
Mean radius [m] (2)	2	8	5
Mean of LN(radius) (3)	0.346574	2.049129	1.36209
Stdv radius [m] (4)	2	2	4
Stdv of LN(radius) [m] (5)	0.832555	0.246221	0.703346
Termination % (6)	0	0	0
Lower bound [m] (7)	0.0025	0.0025	0.0025
Upper bound [m] (7)	10000	10000	10000

(1) A Log-Normal distribution that is truncated at lower and upper bounds.
(2) Mean of distribution.
(3) Mean of the natural logarithms of the values of radius
(4) Standard deviation of distribution.
(5) Standard deviation of the natural logarithms of the values of radius
(6) Amount of fractures that terminate at other fractures.
(7) Upper and lower boundaries for the truncated Log-Normal distribution

Table 2-2. Orientation of fractures.

Set No.	**1**	**2**	**3**
Distribution	Fisher (1)	Fisher (1)	Fisher (1)
Dispersion (κ)	4.84	8.35	8.33
Pole trend (degrees)	129.0	37.0	290.6
Pole plunge (degrees)	6.3	5.8	84

(1) Spherical distributions, like the Fisher distribution, are discussed in Section 2.8

Table 2-3. Fracture density (P32).

Set No.	**1**	**2**	**3**
P32 = Fracture area per unit volume	0.85	1.59	0.97

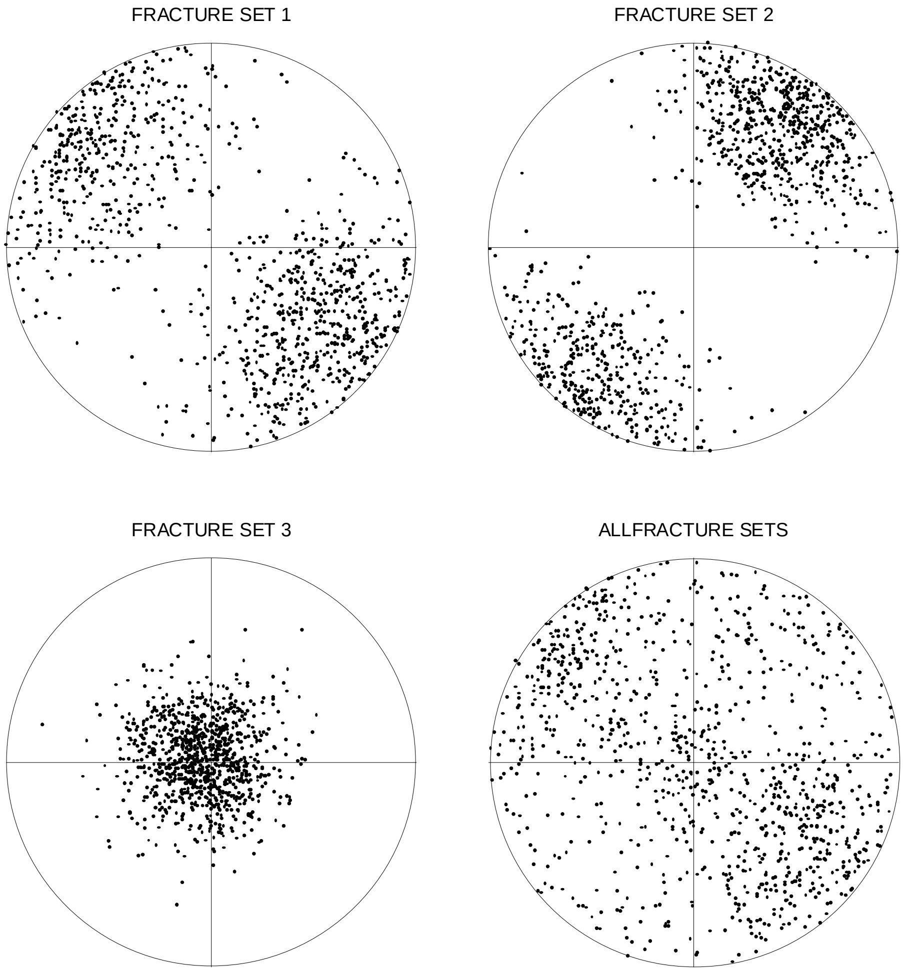


Figure 2-2. *Lower hemisphere equal angle stereoplots of fracture pole orientation and distribution. The plotted fracture poles represents one realisation of the **fracture population** studied (the fracture network). The figures are based on a stereographic projection, see Section 2.2.3. Each of the four figures includes 1000 fracture poles. Considering the figure that contains all fracture sets (lower right) the number of fracture poles per set corresponds to the P32 values of the different sets and the fracture size distribution. On the average:*
Set 1 contains 25% of the fracture area and 70% of the number of fractures.
Set 2 contains 47% of the fracture area and 15% of the number of fractures.
Set 2 contains 28% of the fracture area and 15% of the number of fractures.

The orientations of the fractures, as observed in a vertical and an inclined borehole, are demonstrated below in Figure 2-3 and Figure 2-4, respectively. Note that as the figures gives the fractures observed in boreholes, the distribution of fractures with an orientation close to parallel to the boreholes are not well represented, as such fractures rarely intersects the boreholes. This is an example of a systematic bias that will take place when sampling a three-dimensional fracture system, with a one-dimensional boreholes. In this study, previously to analysing the data, the bias is corrected by use of Terzaghi correction, see Section 2.5 and Appendix B. (It should be pointed out that the Terzaghi correction is not a perfect correction that will remove all the sampling bias, but it is an efficient method for minimising the above discussed sampling bias).

When comparing Figure 2-2, Figure 2-3 and Figure 2-4, one should note that they are based on different realisations of the fracture network.

When studying the fracture poles given in the figures above and below, it is perhaps of interest to know how many fractures that on the average are observed in the analysed boreholes (the efficiency of the point estimates of the parameters studied depend on the number of observed fractures). The number of fractures per metre in a borehole is called the *P10* value, see Section 5.1. The efficiency of the estimation of *P10* values is analysed in this study. The *P10* values depend on orientation of borehole. Two boreholes with different orientation are used in this study, a vertical and an inclined borehole.

The average *P10* values for the different boreholes are given below, considering boreholes of 1000 m length. No correction for sampling bias (Terzaghi correction) is applied when calculating the *P10* values.

Vertical borehole:	Set 1: Mean= 0.292 fractures per metre. Set 2: Mean= 0.434 fractures per metre. Set 3: Mean= 0.848 fractures per metre.
Inclined borehole: (Trend=90deg, Plunge=45deg)	Set 1: Mean= 0.456 fractures per metre. Set 2: Mean= 0.732 fractures per metre. Set 3: Mean= 0.551 fractures per metre

On a two-dimensional structure (a surface), the fractures of the DFN-model will create two-dimensional lines, such lines are called fracture traces. An example of fracture traces (created by the DFN-model) as seen on a circular horizontal rock surface is given in Figure 2-5.

The orientations of the fractures, as observed on a circular horizontal surface, are demonstrated below in

Figure 2-6. Note that as the figure gives the orientation of fractures observed on a horizontal surface, the distribution of fractures with an orientation close to parallel to the surface (sub-horizontal fractures) are not well represented, as such fractures only rarely intersects the surface.

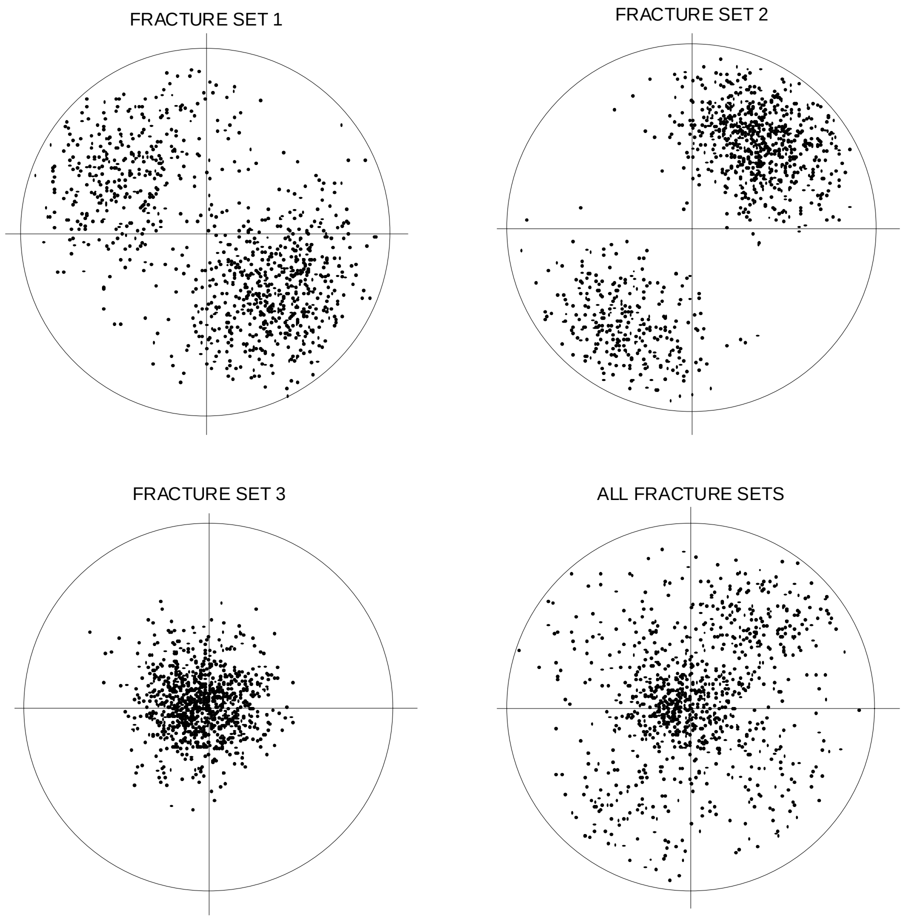


Figure 2-3. *Lower hemisphere equal angle stereoplots of fracture pole orientation and distribution, as seen in a* ***vertical borehole****. The plotted fracture poles represents one realisation of the fracture population studied (the fracture network). The figures are based on a stereographic projection, see Section 2.2.3. Each of the four figures includes 1000 fracture poles.*
Considering the figure that contains all fracture sets (lower right) the number of fracture poles per set corresponds to the P10 values of the different sets. On the average, as seen in the borehole:
Set 1 contains 18% of the number of fractures.
Set 2 contains 28% of the number of fractures.
Set 3 contains 54% of the number of fractures.
Note that this data is not corrected for sampling bias (Terzaghi correction is not included in this figure).

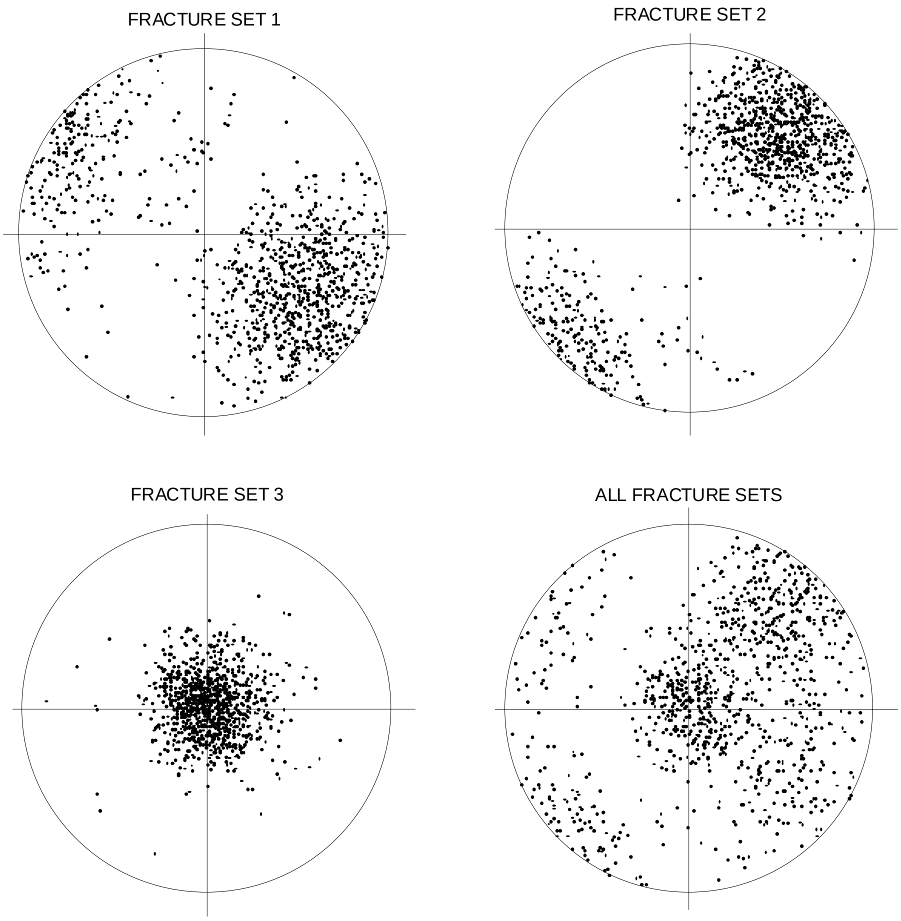


Figure 2-4. *Lower hemisphere equal angle stereoplots of fracture pole orientation and distribution, as seen in an inclined borehole (Trend= 90 deg. Plunge= 45 deg.). The plotted fracture poles represents one realisation of the fracture population studied (the fracture network). The figures are based on a stereographic projection, see Section 2.2.3. Each of the four figures includes 1000 fracture poles.*
Considering the figure that contains all fracture sets (lower right) the number of fracture poles per set corresponds to the P10 values of the different sets. On the average, as seen in the borehole:
Set 1 contains 26% of the number of fractures.
Set 2 contains 42% of the number of fractures.
Set 3 contains 32% of the number of fractures.
Note that this data is not corrected for sampling bias (Terzaghi correction is not included in this figure).

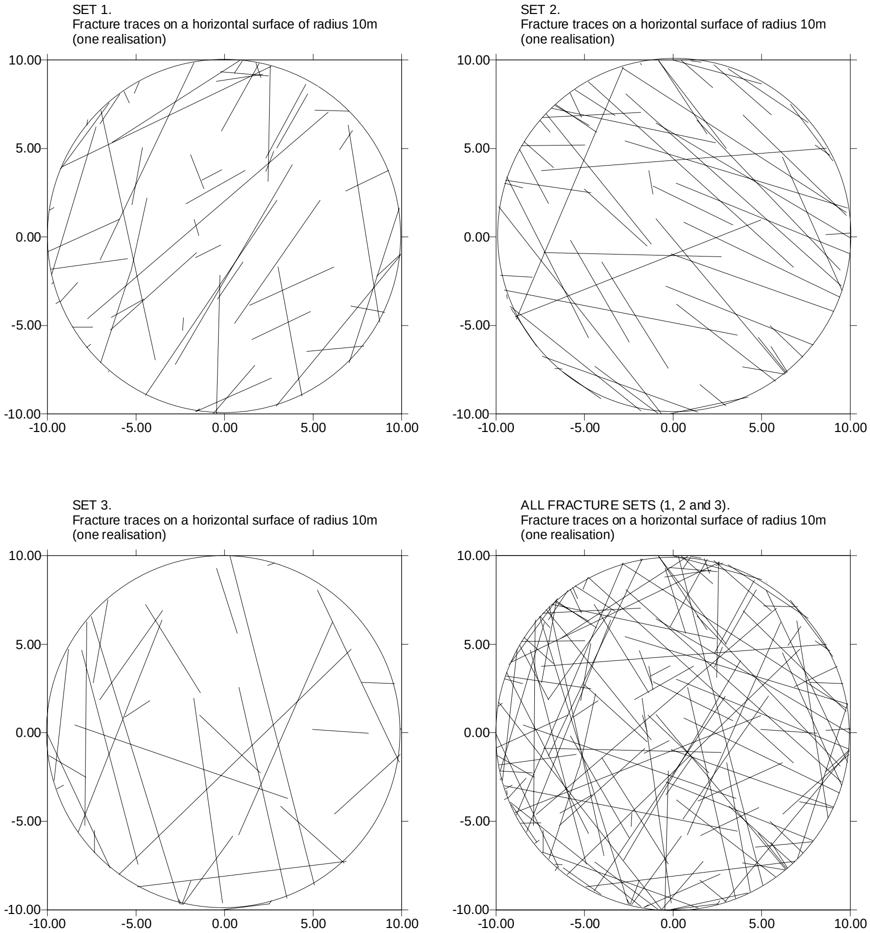


Figure 2-5. *Fracture traces on a circular horizontal surface with radius 10m. The plotted fracture traces represents one realisation of the fracture population studied (the fracture network). The traces were numerically generated. Each of the four figures includes a different number of traces, dependent on the orientation and density of the fracture set studied. The length of the traces divided by the surface area is the P21 parameter. For very large horizontal surfaces, the P21 values are as follows: Set1 P21= 0.77, Set 2 P21= 1.51, Set 3 P21= 0.41; hence 29% of the trace-lengths belongs to Set 1, and 56% belongs to Set 2, and 15% belongs to Set 3. Considering the number of traces on a very large horizontal surface, on the average 45% belongs to Set1, 43% belongs to Set 2 and 12% belongs to Set 3.*

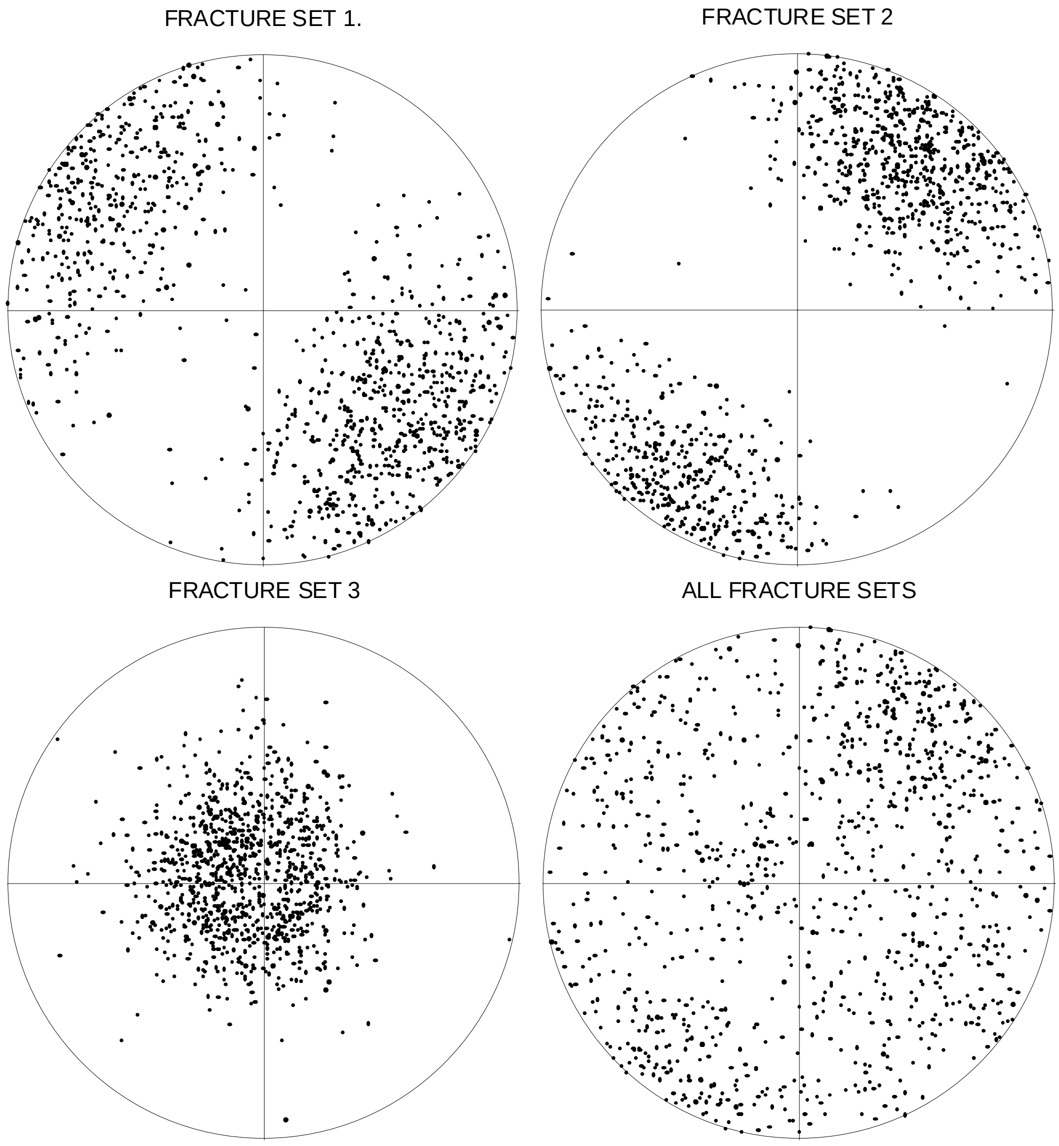


Figure 2-6. *Lower hemisphere equal angle stereoplots of fracture pole orientation and distribution, as seen on a circular* ***horizontal surface****. The plotted fracture poles represents one realisation of the fracture population studied (the fracture network). The figures are based on a stereographic projection, see Section 2.2.3 Each of the four figures includes 1000 fracture poles.*
Considering the figure that contains all fracture sets (lower right) the number of fracture poles per set corresponds to the average number of fracture traces on a circular area with radius 150 metres. On the average, as seen on the surface:
Set 1 contains 35% of the number of fractures.
Set 2 contains 51% of the number of fractures.
Set 3 contains 14% of the number of fractures
Note that this data is not corrected for sampling bias (Terzaghi correction is not included in this figure).

2.4 Properties of the studied boreholes and rock surfaces

The purpose of this study is to simulate the sampling of fractures, or fracture traces, as observed in a borehole or on a rock surface. We have studied two different boreholes, as regards orientation of boreholes. The first borehole is a vertical borehole. The second borehole is an inclined borehole. Using the same method of orientation as used for the normals to the fracture-planes (see Section 2.2.2); the orientation of the inclined borehole is trend=90 degrees and plunge=45 degrees (hence, the borehole is inclined 45 degrees from horizontal). For both boreholes, the lengths of the boreholes were varied from 20 metres and up to 1000 metres.

We have also studied rock surfaces. The rock surfaces are analysed for fracture traces. A studied rock surface is called a window. All the analysed windows are horizontal; they correspond to horizontal rock outcrops. The geometrical shape of the windows studied is circular. The radii of the windows were varied from 4 metres and up to 150 metres. For the boreholes and the windows, the number of realisations of the rock mass were varied between 500 and 1000. Hence, for every borehole length and rock surface area studied, a large number of different realisations of the fracture network were analysed.

2.5 Correction for sampling bias – the Terzaghi correction

One-dimensional sampling is sampling along a straight line (a scanline). Such sampling of fracture orientation in a three-dimensional fracture system will introduce an orientation sampling bias. The bias follows from the fact that the probability for intersecting a fracture depends on the angle between the sampling line and the fracture, as well as on the area of the fracture. For compensation of this sampling bias /Terzaghi, 1965/ proposed the application of a geometrical correction factor based on the observed angle between the sampling line and the normal to a particular fracture. In this study, such a correction is called "Terzaghi correction". The highest probability for intersection (between fracture and sampling line) occurs when the fracture-plane is at right angle to the sampling line (borehole), if the fracture-plane is parallel to the sampling line (borehole), the probability for intersection is zero (except if the sampling line is at the fracture). Any direction of sampling line will therefore produce a sample that is biased to contain a lower amount of fractures than the actual amount. The reduced sample size can be compensated for by assigning a weighting factor to those fractures that are sampled. For a large sample size this weighting will serve to balance the orientation sampling bias introduced by linear sampling. The Terzaghi correction is not a perfect correction that will remove all the sampling bias, but it is an efficient method for minimising the systematic sampling bias that follows from sampling a three-dimensional fracture system by use of a one-dimensional sampling line. For a more thorough discussion of this we refer to /Terzaghi, 1965/ or /Priest, 1993/. A presentation of the methodology of the discussed correction and theoretical examples are given in Appendix B.

In this study all the fracture orientation data, derived from sampling the boreholes, are corrected for sampling bias by use of the Terzaghi correction. Hence, the analyses presented in Sections 3 and 4, are based on orientation data that has been corrected by use of Terzaghi correction, as presented in Appendix B. However, no Terzaghi correction was included in the analysis of the data gathered form horizontal rock surfaces.

2.6 Classification of observed fractures into fracture sets

A fracture set contains a number of fractures that are grouped together, as they demonstrate some tendency to have similar properties, e.g. orientation. In this study the fractures are grouped into three sets, dependent on fracture orientation. The orientations of the fractures of a fracture set may follow some stochastic distribution, in which some orientations are more likely than other orientations. An example of such a distribution is the Fisher spherical distribution. In this study the distribution of the fracture orientation within a fracture set follows a Fisher spherical distribution (see Section 2.8).

Since the fracture orientations are steered by a statistical distribution (Fisher), a simulated fracture may have any orientation though some orientations are more likely than other orientations. When several sets are superimposed, it is difficult to know the set to which a particular fracture belongs. There are several methods to group fractures in a heterogeneous sample or population into homogeneous sets. In this study such algorithms were not used. Instead each fracture was marked with its proper set identity since this is known at the generation of the fracture. In a real situation, different methods and algorithms for identifying and delimiting sets will be necessary to ensure objective set identifications. Different methods for identification of fracture sets will produce different results. The reason why we have used the known true fracture set identity and not applied a fracture set identification algorithm is because we do not want the efficiency of the fracture set identification algorithm to influence the result of the study. The efficiency of different methods for identification of fracture sets is an interesting topic, but it is not included in this study.

2.7 Aspects of the applied statistical tests – accepted deviations

The purpose of sampling the discontinuities (fractures) of the rock mass by use of boreholes and by mapping of rock surfaces (rock outcrops) is to estimate some unknown properties of the rock mass. From a statistical point of view, the unknown properties of the rock mass are the properties of a population studied; we will call these properties the parameters or the true properties. The samples will produce estimates of the population; these estimates are called the sample properties. Generally the sample properties deviate somewhat from the true properties, dependent (among other things) on size of samples etc. This study is about calculating these deviations for different sizes of sample and to analyse them statistically. By sample size we mean the number of observations that a sample is based upon. For example, a short borehole may produce a sample based on the observations of 50 fractures, while a long borehole may produce a sample based on the observations of 500 fractures; the sample size is larger for the long borehole.

In reality when observing fractures in boreholes and on outcrops, and when predicting properties of the rock mass based on these observations, it is impossible to exactly calculate how much the sample properties deviate from those of the population, as the properties of the population are unknown. Nevertheless, considering the purpose of a real investigation there are probably some demands on accuracy, which correspond to an acceptable deviation in estimated properties. Decisions and conclusions are founded on the sample properties, hence large deviations between the sample properties and the true properties are often not acceptable, but small deviations are acceptable as such deviations are of no practical importance. (For the average mean direction of a fracture set, such an acceptable deviation between the true properties and sample properties is perhaps 5 to 15 degrees or even larger, dependent on the purpose of the investigation.)

This study is not about calculating the necessary sample sizes or acceptable deviations to be used in site investigations for a nuclear repository. Actually in practise, the concept of acceptable deviations is not very useful, because it assumes the knowledge of the parameters of the population (the unknown true properties), which is normally not known. The necessary amount of information that needs to be gathered at a site investigation, is best calculated based on statistical analysis at different stages of sampling (preliminary and confirmatory sampling) in combination with safety analysis calculations (sensitivity analyses of such calculations). Theoretically, the necessary sample sizes and acceptable uncertainties (and acceptable deviations) in estimation of the true properties (parameters) of the rock mass depend on the properties of the investigated site and the results of safety analyses calculations. Large uncertainties could be accepted for parameters with little importance in the safety analysis, or for remote rock volumes that carries small importance in the safety analysis. While parameters and rock volumes that the safety analysis calculations has identified as being important for the performance of the investigated site, such parameters and volumes needs to be investigated in more detail to produce reliable estimates with a small amount of uncertainty.

In this study, the properties of the rock mass are known, hence it is possible to calculate the deviation between sample properties and the true properties. The analysis of the sample properties is carried out as a statistical hypothesis testing. The hypothesis testing is based on a calculated test variable of the samples and the corresponding properties of the population. The statistical tests are based on a large number of different realisations of the fracture networks (500–1000). The large numbers of realisations (samples) are necessary to obtain reliable results (statistics).

Primarily this study concerns tests in which the calculated deviation between sample properties and true properties is compared to different selected acceptable deviations of the test variable studied (first category of tests). The acceptable deviations are called the test criterions. However, this study also includes tests that do not directly correspond to a selected acceptable deviation, but to a given level of confidence in estimating the true properties (second category of tests). The difference between these two types of test should be noted. The purpose of the first category of tests is to determine when the size of the sample is large enough to produce an acceptable estimate of the true properties (e.g deviation <= 15 degrees), with a certain probability (e.g. >= 90%). For the first category of tests, we select the acceptable deviation between sample and true properties and the acceptable deviation is the same regardless of size of sample (length of borehole).

The purpose of the second category of tests is to demonstrate the probability for a given hypothesis of the properties of the population, to be rejected or accepted, at a certain selected level of confidence (e.g. 99%). In this study the true properties are known, and therefore the hypothesis concerning the properties of the population is equal to the known true properties of the population. We know that this is a correct hypothesis, but due to sampling bias etc, it will not necessarily be confirmed by the samples. The test will tell us the probability for rejection or acceptance of this hypothesis of the rock mass, at a certain selected level of confidence, and size of sample.

For the first category of tests, the selected acceptable deviation is constant for all sizes of sample; in the second category of tests the selected level of confidence is constant for all sizes of sample. The first category of tests are carried out as non-parametric tests, hence we make no assumptions regarding the statistical distributions of the properties of the studied fracture network or regarding systematic bias in the sampling procedure. The first category of tests could be considered as calculation of the sample size that is necessary to reach a confidence level, considering a given confidence interval. The confidence interval corresponds to the above-discussed acceptable deviation. The sample size corresponds to a length of borehole or size of area. The second category of tests are carried out as parametric tests, for which we assume that the orientation of the fractures of the studied network are according to Fisher distributions and that no sampling bias takes place; parametric tests are only applied in Sections 3.5 and 4.4.

When performing statistical tests, it is common that different sample sizes are selected beforehand, and for such an analysis a point estimate of an unknown parameter refers to different fixed sizes of sample. That is however not the case in this study. In this study the number of observed fractures (i) along a studied borehole or (ii) on a studied area, gives the sample size. Hence, for unknown boreholes or areas, the actual sample sizes are unknown, even if the lengths of the boreholes or sizes of areas are known, and the sample sizes are revealed when the samples are taken. The point estimates of this study refer not directly to different fixed sizes of sample, but to different fixed lengths of boreholes or sizes of area. On the average, the sample size increases with length of borehole and size of area. However, as the sample size will vary somewhat for a given borehole length or size of area, this variation will be a source of uncertainty added to the analyses of this study (an uncertainty that is not included when performing a "classical" point estimate with fixed sample sizes).

When studying the results of the tests it is important to remember that we are analysing a large number of samples that produce estimates of the true properties of the population. For each given borehole length or size of rock surface, the different estimates (500–1000 values for each borehole length or size of area) form different distributions of estimates. Hence, the statistical tests are applied to distributions of estimates corresponding to different lengths of borehole or areas of rock surfaces. It follows that the results of the tests depend on given test criterions, e.g. acceptable deviations, and of the properties of the analysed distributions. This is illustrated by Figure 2-7, the figure gives examples of theoretical distributions representing samples taken from different lengths of borehole (or rock surfaces), and how these distributions corresponds to given acceptable deviations.

Figure 2-7, give theoretical examples only; we will use the figure to illustrate some aspects of the interaction between test criterions and different distributions of estimates based on samples of different sizes. In the following discussion we assume that the studied estimates (the figure below) represents the acute angle between the mean orientation of a fracture set as given by a sample, and the known true orientation of the set. For such an example the true property of the population is a value equal to 0. Samples from a short borehole produces distribution D1, both the mean deviation from the true property and the variance is 'large' (Moments of D1: mean=12 deg., standard dev.= 7 deg.). Distribution D2 is produced by samples from a long borehole, both the mean deviation from the true property and the variance is 'small' (Moments of D2: mean=1 deg., standard dev.= 2 deg.). Consider an acceptable deviation from the true properties equal to plus/minus 10 degrees; for such a condition and for distribution

D1, 38% of the samples are within the boundaries, and for distribution D2, 99.9% of the samples are within the boundaries. Consider an acceptable deviation from the true properties equal to plus/minus 5 degrees; for distribution D1, 15% of the samples are within the boundaries, and for distribution D2, 97% of the samples are within the boundaries. Hence for the applied conditions (tests) and analysed distributions, the amount of accepted samples increases when the borehole length is increased.

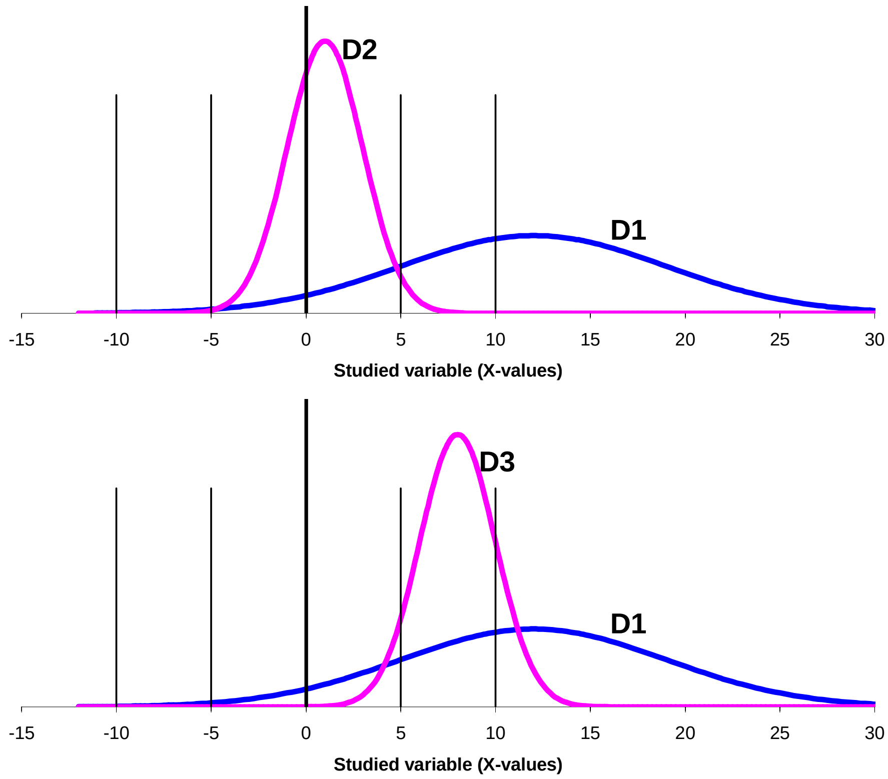


Figure 2-7. *Theoretical example of different distributions of estimates of a true property of a population. The true property of the population is a value equal to zero. Distribution D1 represents estimates based on samples from a short borehole, distributions D2 and D3 represents estimates based on samples form long boreholes.*

However it is important to note that in the figure above the mean values of the distributions deviate from the true property of the population (such behaviour is normally a result of a sampling bias). It follows that if the applied condition is strict and only very small deviations are accepted, the amount of accepted samples may decrease when the borehole length is increased, distribution D3 demonstrates this. Distribution D3 is produced by samples from a long borehole, the mean deviation from the true property is smaller than for D1, but it is not insignificant; the variance is much smaller than for D1 (Moments of D3: mean=8 deg., standard dev.= 2 deg.). Consider an acceptable deviation from the true properties equal to plus/minus 10 degrees; for such a condition and for distribution D1, 38% of the samples are within the boundaries, and for distribution D3, 83% of the samples are within the boundaries. Consider an acceptable deviation from the true properties equal to plus/minus 5 degrees; for distribution D1, 15% of the samples are within the boundaries, and for distribution D3, 6% of the samples are within the boundaries. Hence for the first condition (samples are accepted within plus/minus 10 degrees), the amount of accepted samples increases when the borehole length is increased; but for the second condition (samples are accepted within plus/minus 5 degrees), the amount of accepted samples decreases when the borehole length is increased. Considering the second condition, the decreasing number of accepted samples follows form a sampling bias, the bias makes the point estimate converge towards a value that is not the true value of the parameter studied.

2.8 Aspects of the Fisher spherical probability distribution

There are several probability distributions available for modelling spherical data, for example the Fisher distribution, which is the basic model for directions distributed unimodally with rotational symmetry, or the Watson distribution which is the basic model for undirected lines, axes, distributed with rotational symmetry in either bipolar or girdle form, see /Fisher et al, 1987/. Theoretically the Watson distribution is best suited for modelling the studied spherical data, as the studied data are undirected lines (axes) i.e. normals to fracture planes. However, many statistical tests and procedures are available for the Fisher distribution and therefore it is commonly used also for undirected lines; the applicability of the Fisher distribution for undirected lines is discussed below.

In this study we will use the Fisher probability distribution, both for generation of the fracture population studied and for parametric analysis of the fracture population. The Fisher distribution is an important distribution in the analysis of spherical data, as it is the basic model for directions distributed unimodally with rotational symmetry. The Fisher distribution is defined for the whole sphere, and it serves generally as an all-purpose probability model for directions in space and for directional measurement errors, much as for the normal distribution for observations on the line. The Fisher distribution is given by (i) parameters defining the direction of a modal vector, the distribution is symmetric about this vector, and (ii) a shape parameter called kappa. The larger the value of kappa the more the distribution is concentrated towards the direction of the modal vector. Kappa is often called the dispersion parameter, but actually it is a concentration parameter, since the larger the value of kappa the more the concentrated the distribution. For the Fisher distribution, the density of probability is greatest in the direction of the modal vector and is least in the opposite direction.

Cones are often used when describing the properties of a Fisher distribution. Such cones have their narrow ends at the origin of the unit sphere, and they are oriented about the modal vector. On the surface of the unit sphere the cones form circular intersections. The opening (θ) of such a cone is normally given from centre to side of cone (from modal vector to side). For the Fisher distribution, considering angles relative to the modal vector, the probability density distribution and the cumulative probability are as follows /Priest, 1993/. For a detailed mathematical presentation of the distribution we refer to /Fisher et al, 1987; Fisher, 1953; Mardia, 1972/.

Probability density distribution Cumulative probability

$$\mathbf{f}(\theta) = \frac{\mathbf{K}\sin(\theta)\ \mathbf{e}^{\mathbf{K}\cos(\theta)}}{\mathbf{e}^{\mathbf{K}} - \mathbf{e}^{-\mathbf{K}}} \quad , \quad \mathbf{P}(<\theta) = \frac{\mathbf{e}^{\mathbf{K}} - \mathbf{e}^{\mathbf{K}\cos(\theta)}}{\mathbf{e}^{\mathbf{K}} - \mathbf{e}^{-\mathbf{K}}} \qquad 2\text{-}1$$

θ =Angle from centre of distribution. Opening of cone studied (centre to side).

$f(\theta)$=The probability density distribution

$P(<\theta)$=The cumulative probability within a cone of opening equal to θ

K=kappa (dispersion parameter)

The equation for cumulative probability above can also be looked upon as the probability that a random spherical data, following a Fisher distribution, makes an angle of less than θ with the modal vector. Examples of the cumulative probability for three different Fisher distributions are given in Figure 2-8.

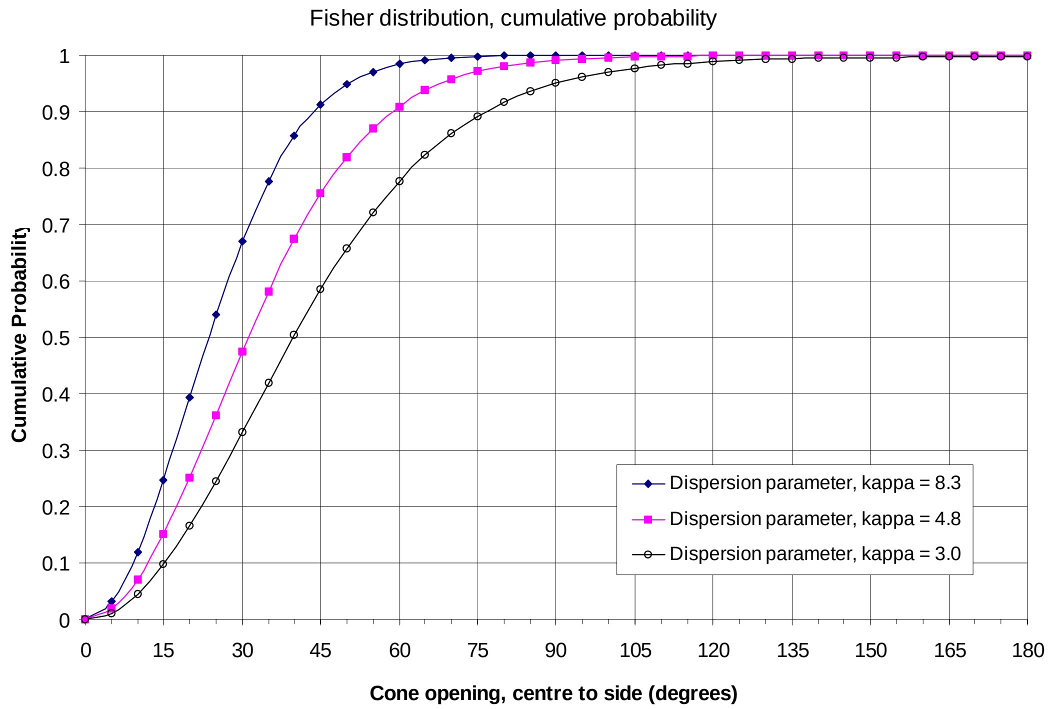


Figure 2-8. *Three examples of Fisher distributions. The figure gives the cumulative probability within cones of different openings, minimum cone opening is 0 degrees (no opening) and maximum opening is 180 degrees (whole sphere).*

When studying the equations and figure above one should note an important aspect of spherical distributions. The probability density function of a spherical distribution (given as a function of the cone opening) is not directly proportional to the concentration of data on the surface of a unit sphere. This is because the probability density function is given as a function of an angle θ (cone opening); but the concentration of modelled data depends on both (i) the probability density function and (ii) the size of the surface area on the unit sphere within the cone opening studied. For example, a Fisher distribution has its greatest concentration at θ equal to zero, which corresponds to the centre of the distribution, and at θ equal to zero the probability density function is equal to zero (as the cone opening is zero).

The applicability of the Fisher distribution for modelling undirected lines needs to be considered. Undirected lines occur as spherical data simultaneously in the lower and upper hemispheres, as the data is undirected. When using the Fisher distribution for modelling of spherical data of this type, it is important to note that the Fisher distribution is defined for directed lines and for the whole sphere. For values of kappa that are not to small (larger than ca. 5), nearly all of the spherical data of a Fisher distribution falls in one half-sphere about the mean direction of the distribution studied; and for such a situation the Fisher distribution is suitable for modelling undirected lines. The Fisher distribution can be used for representing undirected lines as long as only an insignificant amount of the distribution falls outside of a half-sphere centred about the mean direction (modal vector) of the distribution.

For a Fisher distribution with a kappa value equal to 5, the amount between cone openings equal to 0 and 90 degrees is equal to 99.3 percent of the distribution, and 0,7 percent of the distribution takes place between cone openings equal to 90 and 180. For kappa values smaller than three, the amount between cone openings equal to 90 and 180 degrees is larger than 5 percent of the distribution. This means that for kappa values smaller than 5 (and especially for kappa values smaller than 3), a perhaps not negligible amount of the studied Fisher distribution will fall outside of a half-sphere located about the mean direction of the distribution studied. This must be considered when modelling, analysing and visualising Fisher distributions with kappa values smaller than 5. Especially when using a Fisher distribution for representation of spherical data of axes-type, as such data are undirected and consequently occur simultaneously in both the upper and lower hemispheres.

3 Estimation of fracture-set mean direction from borehole data

3.1 Fracture set orientation and the acute angle

In this study, a fracture set contains a number of fractures that are grouped together as they demonstrate a tendency to have a similar orientation. Three vectors define the mean directions of the fracture sets of the population. These three vectors are called the modal vectors, and in this study these vectors are known.

The average orientation of a sample of fractures can also be represented by a vector. The orientation of such a vector is not necessarily the same as the orientation of the modal vector of the corresponding fracture set. The orientation of a sample is calculated based on two different methods, which both produces vectors with orientations that are very close, but of different sizes: (i) the eigenvalues method /Mardia, 1972/ and (ii) the resultant vector method. Both methods are presented in Appendix A. To ensure a correct result when applying the resultant vector method it is important to transform the undirected spherical data (axes) into a consistent set of directed data (vectors). If this is not done, the resultant vector method may under certain conditions produce a resultant vector that does not reflect the overall orientation of the studied fracture group (see Figure A-2 in Appendix A). In this study, the vector that is derived from the eigenvalues method is called ”the representative vector” and the vector that is derived from the resultant vector method is called “the resultant vector”. For the tests presented below, the eigenvalues method was used for calculation of sample orientation.

When comparing the two methods, the eigenvalues method is the best method for calculating the mean direction of a sample, as it is are more robust method than the resultant vector method. However, the resultant vector method should not be forgotten, because the length of the resultant vector (given by the resultant vector method) is often used when calculating the dispersion of the fracture sample. The best approach is to first apply the eigenvalues method to derive a good estimate of the mean direction, and based upon the direction of the representative vector, apply the resultant vector method for calculation of the resultant vector. If the resultant vector method is constrained by the results of the eigenvalues method, it will produce correct results.

The smallest angle between (i) the modal vector of the population and (ii) a representative vector (or resultant vector) of a sample is called the acute angle. It is possible to calculate acute angles as the modal vectors of the population are known. The acute angle is directly proportional to the deviation in estimation of the mean direction of a fracture set of the population. For a sample with a representative vector that has exactly the same direction as the modal vector, the acute angle is zero.

3.2 Point estimates and the acute angle

The fractures that intersect the borehole studied are samples of the fracture population. The properties of the sample are estimates of the properties of the population. The observed fractures are classified into three groups, one group for each theoretical fracture set. After the classification each fracture set is studied one by one, separate from the other sets. The test presented below is conducted for each fracture set separately.

From a statistical point of view, the calculation of the average orientation of the fractures of the samples is a point estimate of the orientation of the population. In this study, the acute angle is the sample variable studied. The acute angle is a function of the properties of the samples (i.e. the distribution of the observed fractures) and the known modal vector of the population. The efficiency of the point estimate of the variable studied increases with size of sample (number of observed fractures), and size of sample increases with length of borehole. This is demonstrated in 4.1 and in Figure 3-2.

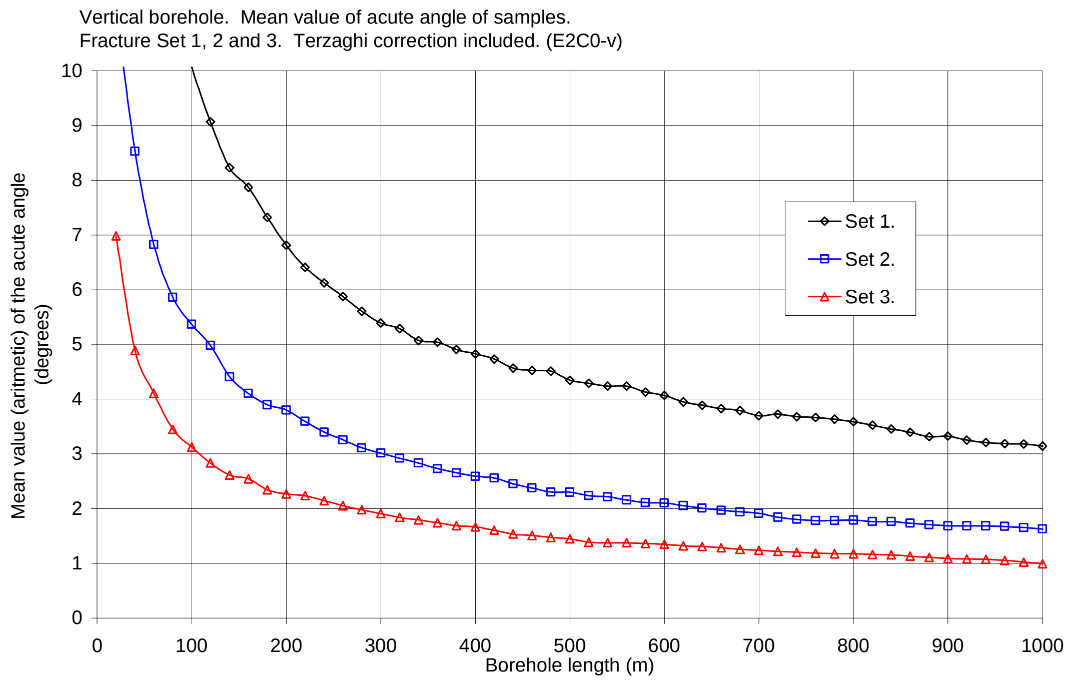
Sampling along a straight borehole (a scanline) is a one-dimensional sampling; such sampling of fracture orientation in a three-dimensional fracture system will introduce an orientation sampling bias. The bias follows from the fact that the probability for intersecting a fracture depends on the angle between the sampling line and the fracture, as well as on the area of the fracture. For compensation of this sampling bias we have applied a geometrical correction factor based on the observed angle between the sampling line and the normal to a particular fracture, such a correction is called Terzaghi correction (see Appendix B). In this study all the fracture orientation data, derived from sampling the boreholes, are corrected for sampling bias by use of the Terzaghi correction, the correction is not perfect and some bias will remain in the samples.

Considering a vertical borehole of length 1000 m, the acute angels are:

SET 1: Mean acute angel of samples = 3.14 degrees
SET 2: Mean acute angel of samples = 1.63 degrees
SET 3: Mean acute angel of samples = 0.99 degrees

Considering an inclined borehole of length 1000 m, the acute angels are:

SET 1: Mean acute angel of samples = 3.11 degrees
SET 2: Mean acute angel of samples = 1.56 degrees
SET 3: Mean acute angel of samples = 1.68 degrees



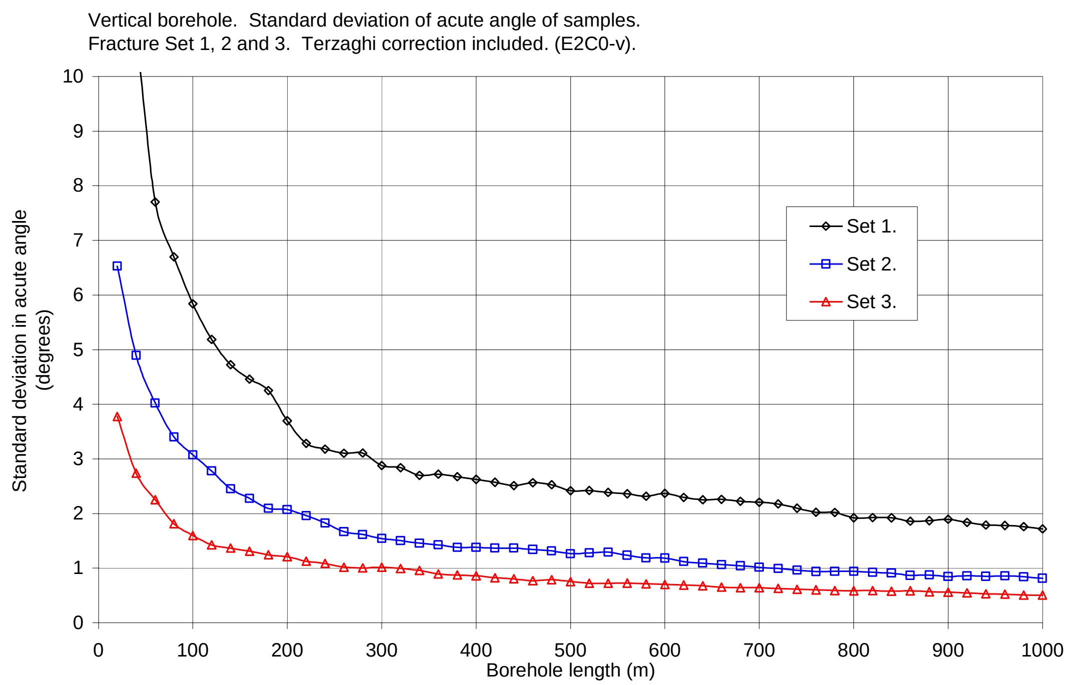


Figure 3-1. *Vertical borehole. The efficiency of the point estimate of the acute angle, given as the mean (upper figure) and standard deviation (lower figure) of the acute angle of the samples at different lengths of borehole. The calculations are based on 900 realisations of different boreholes, for each length studied*

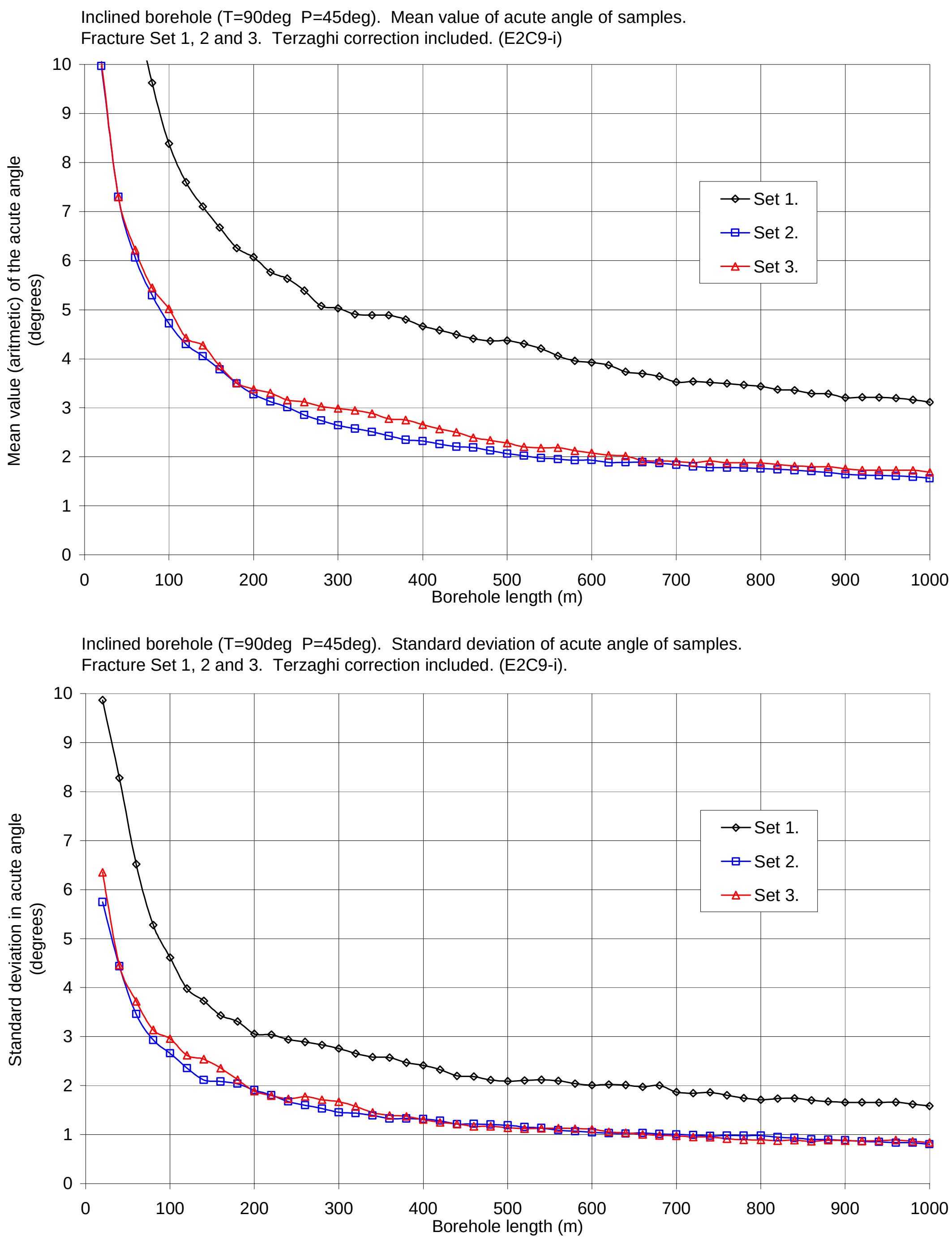


Figure 3-2. *Inclined borehole. The efficiency of the point estimate of the acute angle, given as the mean (upper figure) and standard deviation (lower figure) of the acute angle of the samples at different lengths of borehole. The calculations are based on 900 realisations of different boreholes, for each length studied.*

3.3 Types of tests

The analysis of the point estimate of the orientation is carried out as a statistical hypothesis testing. The hypothesis testing is based on the acute angle of the samples and given criterion of significance and confidence levels. Two different types of test have been performed: tests as regard acceptable deviations and tests as regard confidence levels (confidence intervals and confidence cones). Tests that correspond to selected constant values of acceptable deviations are presented in Section 3.4. Tests that correspond to selected confidence levels are presented in Section 3.5.

3.4 Hypothesis testing considering acceptable deviations

3.4.1 Purpose of test

The purpose of the test is to determine when the size of the sample is large enough to produce an acceptable estimate of the studied parameter, with a certain probability. This can also be stated in the following way: the calculation of the sample size that is necessary to reach a confidence level, considering a given confidence interval. The confidence interval is the same thing as a test criterion (an acceptable deviation). The sample size corresponds to length of borehole.

3.4.2 Null hypothesis, acceptable deviations and criterion of significance

The analysis of the point estimate of the fracture set orientations is based on hypothesis testing. The hypothesis testing is based on the sample variable studied (the acute angle) and given criterions of significance. The null hypothesis (H_0) is that the orientation derived from a sample is a good estimate of the true orientation of the population. For the tests presented in this section, the criterions of significance correspond to selected values of acceptable deviations. If the acute angle of a sample is larger than these acceptable deviations, the sample is rejected.

We have studied three different criterions that correspond to three different levels of significance.

First criterion: $H_{0\ (A<=15\ deg)}$ is rejected if Acute angle >= 15 degrees.

Second criterion: $H_{0\ (A<=10\ deg)}$ is rejected if Acute angle >= 10 degrees.

Third criterion: $H_{0\ (A<=\ 5\ deg)}$ is rejected if Acute angle >= 5 degrees.

The result of the analysis is presented as the percentage of accepted the samples, which is approximately the same thing as the probability for correct estimation, considering the different selected criterions.

3.4.3 Results for a vertical borehole

Results for Set 1

Examples of results for Set 1 are as follows (see Figure 3-3). At a borehole length larger than 140 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (A <=15\ deg)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (A <=15\ deg)}$ is less than 10 percent, if the length of the borehole is larger than 140 meters. And finally, if the borehole has a length larger than 140 meters, the probability is larger than 90 percent that the deviation in estimated orientation is less than 15 degrees.

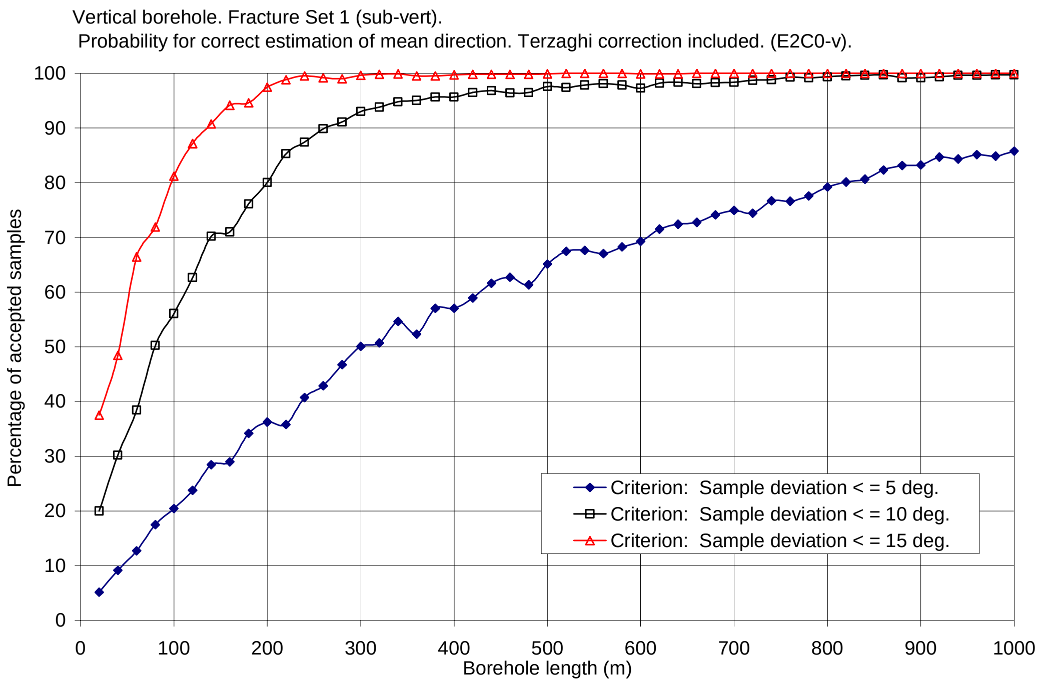


Figure 3-3. *Vertical borehole. Fracture set 1. Hypothesis testing for selected acceptable deviations in predicted orientation. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 2

Examples of results for Set 2 are as follows (see Figure 3-4). At a borehole length larger than 50 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (A <=15\ deg)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (A <=15\ deg)}$ is less than 10 percent, if the length of the borehole is larger than 50 meters. And finally, if the borehole has a length larger than 50 meters, the probability is larger than 90 percent that the deviation in estimated orientation is less than 15 degrees.

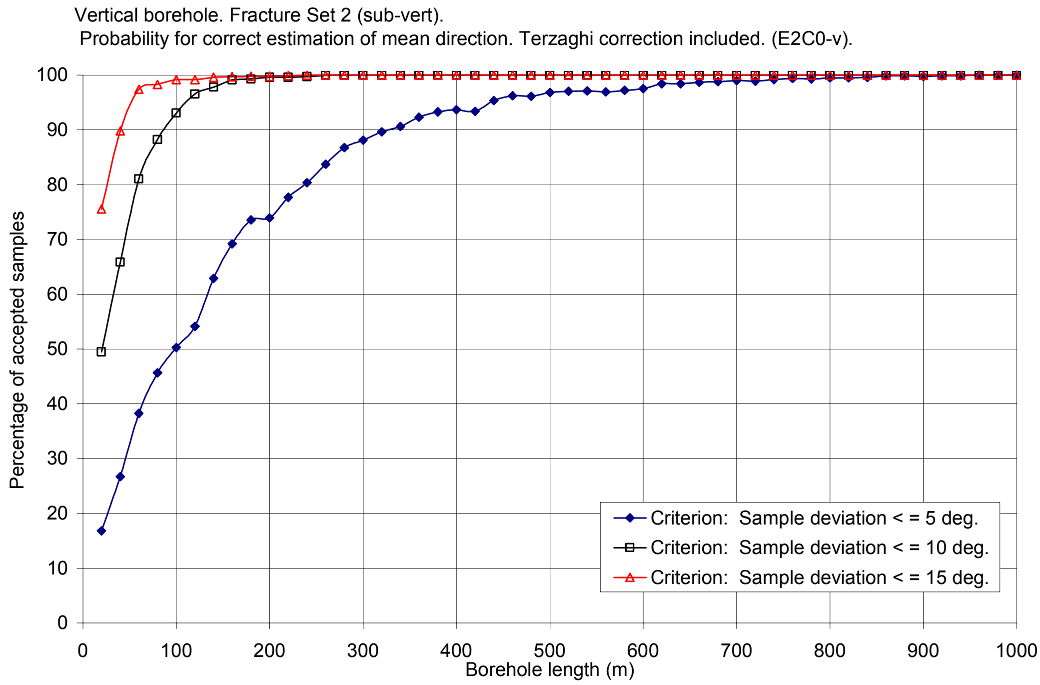


Figure 3-4. *Vertical borehole. Fracture set 2. Hypothesis testing for selected acceptable deviations in predicted orientation. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 3

Examples of results for Set 3 are as follows (see Figure 3-5). At a borehole length larger than 20 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (A\ <=15\ deg)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (A\ <=15\ deg)}$ is less than 10 percent, if the length of the borehole is larger than 20 meters. And finally, if the borehole has a length larger than 20 meters, the probability is larger than 90 percent that the deviation in estimated orientation is less than 15 degrees.

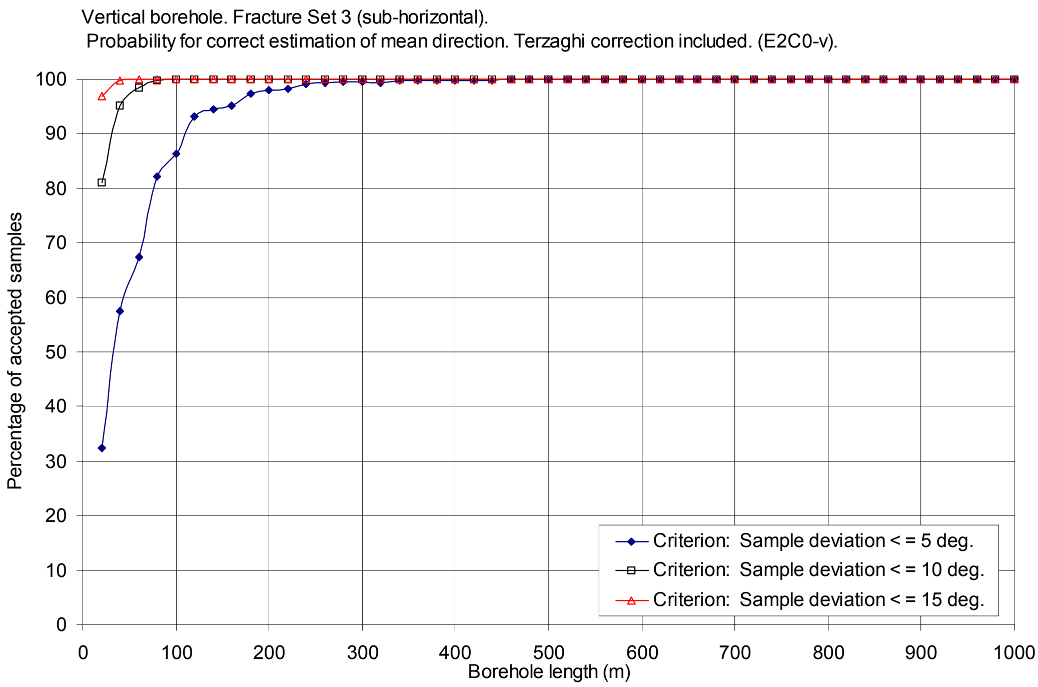


Figure 3-5. *Vertical borehole. Fracture set 3. Hypothesis testing for selected acceptable deviations in predicted orientation. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

3.4.4 Results for an inclined borehole

Results for Set 1

Examples of results for Set 1 are as follows (see Figure 3-6). At a borehole length larger than 90 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (A <=15\ deg)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (A <=15\ deg)}$ is less than 10 percent, if the length of the borehole is larger than 90 meters. And finally, if the borehole has a length larger than 90 meters, the probability is larger than 90 percent that the deviation in estimated orientation is less than 15 degrees.

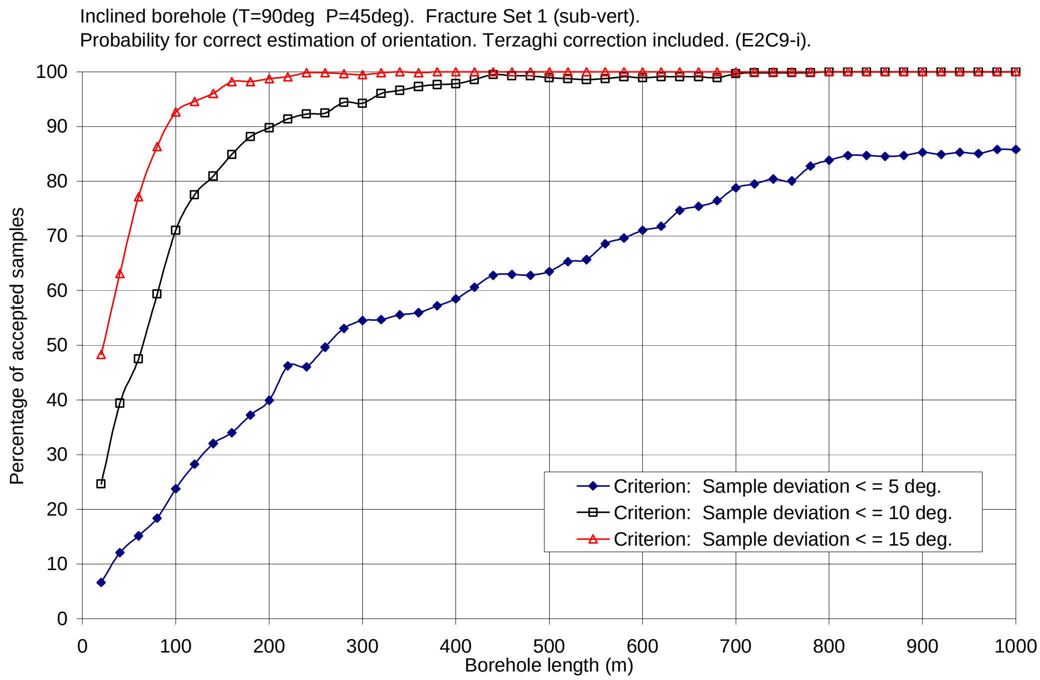


Figure 3-6. *Inclined borehole. Fracture set 1. Hypothesis testing for selected acceptable deviations in predicted orientation. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 2

Examples of results for Set 2 are as follows (see Figure 3-7). At a borehole length larger than 35 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (A <=15\ deg)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (A <=15\ deg)}$ is less than 15 percent, if the length of the borehole is larger than 35 meters. And finally, if the borehole has a length larger than 35 meters, the probability is larger than 90 percent that the deviation in estimated orientation is less than 15 degrees.

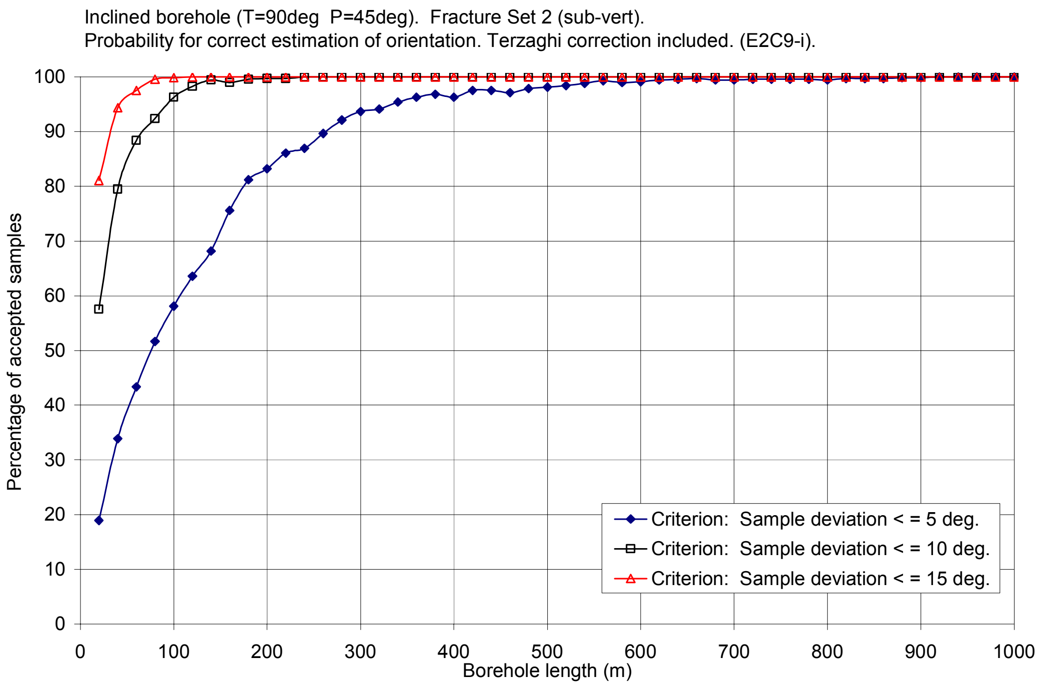


Figure 3-7. *Inclined borehole. Fracture set 2. Hypothesis testing for selected acceptable deviations in predicted orientation. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 3

Examples of results for Set 3 are as follows (see Figure 3-8). At a borehole length larger than 35 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (A <=15\ deg)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (A <=15\ deg)}$ is less than 10 percent, if the length of the borehole is larger than 35 meters. And finally, if the borehole has a length larger than 35 meters, the probability is larger than 90 percent that the deviation in estimated orientation is less than 15 degrees.

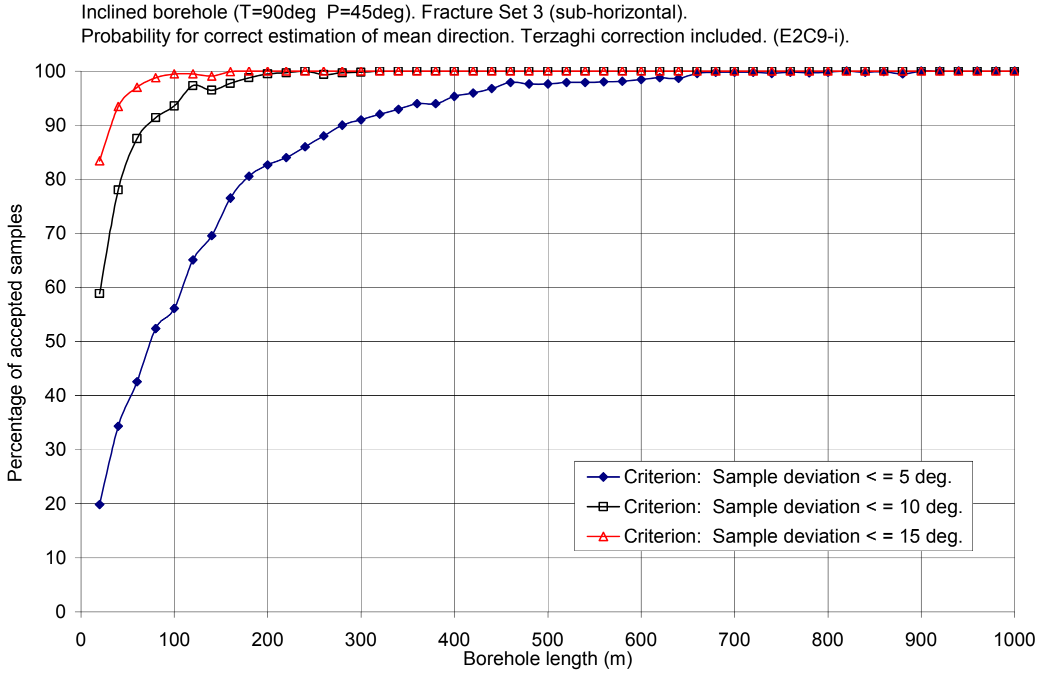


Figure 3-8. *Inclined borehole. Fracture set 3. Hypothesis testing for selected acceptable deviations in predicted orientation. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

3.5 Parametrical hypothesis testing considering Fisher distributions and confidence cones

3.5.1 Purpose

A test that assumes that the analysed population is distributed according to a known probability distribution is called a parametrical test. We have conducted such tests, and for these tests we have assumed that the orientations of the fractures of the population is distributed according to Fisher distributions. This is a correct assumption as the population was generated according to Fisher distributions. The purpose of these tests is to demonstrate the remaining bias of the sampling procedure (sampling in boreholes), the bias that remains after application of Terzaghi correction. This will be demonstrated by analysing the probability for a selected hypothesis of the properties of the population, to be rejected or accepted, at a certain selected level of confidence. In this study the population is known, and the hypothesis of the properties of the population is set equal to the known true properties of the population. The test will tell us the probability for rejection or acceptance of this correct hypothesis of the rock mass, at a certain selected level of confidence. In this section confidence cones will be used as a part of the hypothesis testing.

3.5.2 Confidence cones

A confidence interval for a parameter is an interval of values computed from a sample, which includes the unknown value of the parameter with some specified probability. The probability that a confidence interval will cover the unknown parameter value is the confidence level. A confidence cone is the same thing as a confidence interval, except that the region computed to cover the unknown parameter is not an interval, but of a conical or other specified shape. Hence, the concept of confidence cones corresponds to the concept of confidence intervals. Consider a Fisher distribution, a confidence cone for an unknown modal vector (of the Fisher dist.) is centred on the representative vector of the sample studied. The confidence cone has an opening governed by some confidence level and by both the size (number of fractures) and the dispersion (kappa) of the sample. In this study, the confidence cones are calculated based on the assumption that the dispersion of the population is unknown.

Based on these assumptions, the openings of the confidence cones are calculated by use of methods given by /Fisher et al, 1987/, these methods are based on the work of the following authors: Initially /Fisher, 1953/ considered point estimate of mean direction and dispersion. /Watson, 1956/ and /Watson and Williams, 1956/ derived an exact procedure for calculation of a confidence cone for the mean direction and a procedure for calculation of an interval for kappa. /Stephens, 1962, 1967/ provided tables enabling the Watson and Williams procedure to be implemented, /Stephens, 1967/ also gives the theory and tables for exact interval estimation of kappa. General summaries of these procedures are given by /Mardia, 1972/.

The sizes of cone openings are demonstrated in Figure 3-9 and Figure 3-10. The figures demonstrates that the opening (acceptable deviation) depend on both the calculated dispersion of the samples (kappa) as well as of the number of fractures in the sample, and, of course, by the given confidence level. It is also demonstrated by these two figures that in relation to the selected acceptable deviations, as discussed in Section 3.4 (5–15 degrees), the cone openings are small, especially for samples containing more than approximately 100 fractures. The cone opening is reduced further as the number of fractures in the sample increases. For samples containing more than approximately 100 fractures, the cone opening is small even if the confidence level is set as very large. This is correct, because a large (unbiased) sample is expected to produce a good estimate that is close to the true value of the population.

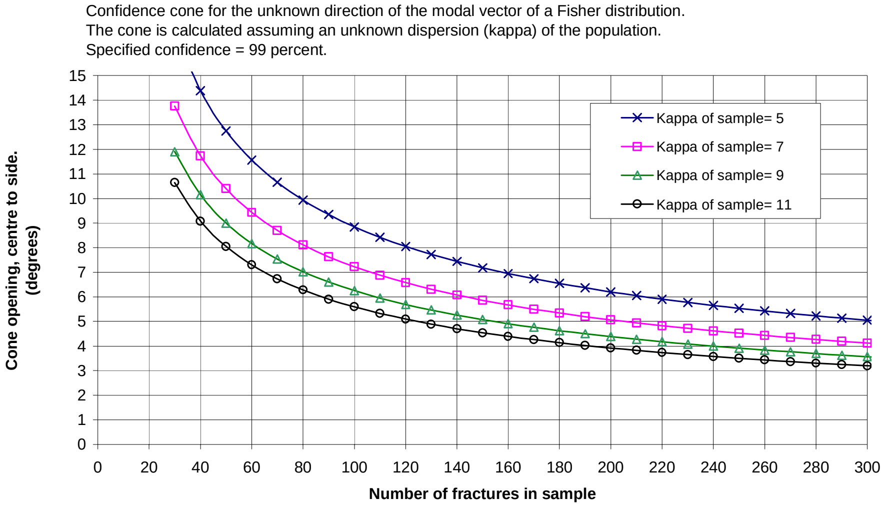


Figure 3-9. *Example of confidence cones for an unknown modal vector. Specified confidence = 99%.*

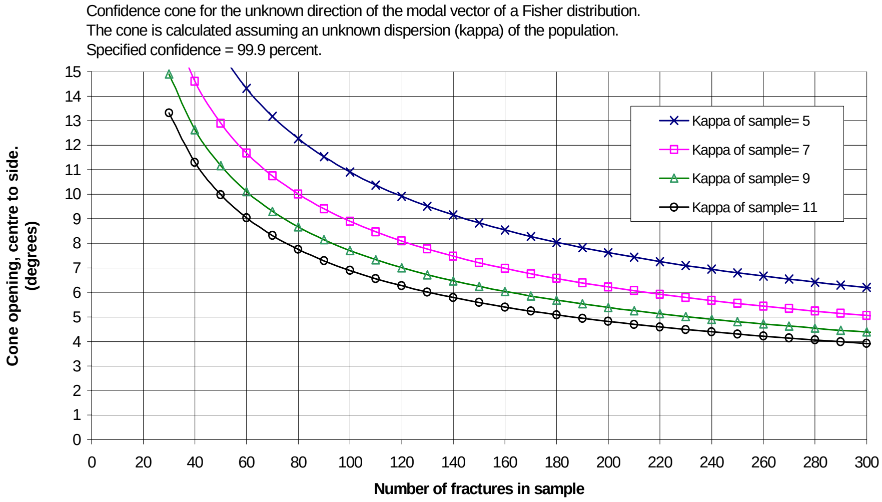


Figure 3-10. *Example of confidence cones for an unknown modal vector. Specified confidence = 99.9%.*

3.5.3 Null hypothesis and level of confidence

The analysis of the point estimate of the orientations is carried out as a statistical hypothesis testing. The hypothesis testing is based on the sample variable studied (the acute angle) and given levels of confidence. The null hypothesis (H_0) is that the

the mean direction of the population, as estimated by the samples, are equal to the known true mean direction of the population. We know that this is a correct hypothesis, but due to sampling bias etc it will not necessarily be confirmed by the samples. For a studied sample, rejection of the hypothesis will take place if the modal vector of the population (as defined by the acute angle) is outside of a confidence cone centred on the representative vector of the sample (i.e. the opening of the confidence cone, centre to side, is smaller than the acute angle of the sample).

The confidence level should be selected in a way that the probability for rejection of the hypothesis is small if the hypothesis is true. We have studied three different levels of confidence, 95, 99 and 99.9 percent. The hypothesis tests are as follows:

First confidence level 95% $H_{0\ (C=95\%\ )}$:

The hypothesis $H_{0\ (C=95\%\ )}$ is rejected if the modal vector of the population does not fall inside a confidence cone calculated for a confidence level of 95%

Second confidence level 99% $H_{0\ (C=99\%\ )}$:

The hypothesis $H_{0\ (C=99\%\ )}$ is rejected if the modal vector of the population does not fall inside a confidence cone calculated for a confidence level of 99%

Third confidence level 99.9% $H_{0\ (C=99.9\%\ )}$:

The hypothesis $H_{0\ (C=99.9\%\ )}$ is rejected if the modal vector of the population does not fall inside a confidence cone calculated for a confidence level of 99.9%

3.5.4 Results

For these tests (Section 3.5), the acceptable deviation in estimation of the true value, as given by the confidence cones, decreases as the number of fractures in the samples increases. Furthermore, the acceptable deviation will also vary dependent on the calculated dispersion of the fractures of the sample. Hence, the acceptable deviation is not a constant value. For small samples, the acceptable deviation is large, and for large samples, the acceptable deviation is small.

The test will tell us the probability for rejection or acceptance of the hypothesis of the rock mass properties, for different borehole lengths, and at selected levels of confidence. (As previously stated, the hypothesis is that the orientation of the population is equal to the known true orientation; we know that this is a correct hypothesis, but due to sampling bias etc it will not necessarily be confirmed by the samples.) The efficiency of the point estimate of the acute angle increases with size of sample, but the cone opening (acceptable deviation) decreases as the number of fractures in the sample increases. Therefore the percentage of accepted samples does not increase with borehole length, as for the previous tests (Section 3.4). Theoretically, if the samples were taken without sampling bias from perfect Fisher distributions, the probability for acceptance of the hypothesis should be equal to the confidence level, regardless of borehole length. The results for confidence levels 99% and 99.9% are given in Figure 3-11 and in Figure 3-12. For all fracture sets and especially for the inclined borehole, the results show a large amount of rejected samples. This is a consequence of a systematic bias in the point estimate of the acute angle. This bias comes from the fact that a borehole is a one-dimensional line that samples a three-dimensional fracture network. The applied Terzaghi correction, which is supposed to remove this bias, is not perfect and some aspects of the bias remain in the samples.

Results for vertical borehole

The results demonstrates that for Set 1 and 2, the probability for rejection of the correct hypothesis is larger than the prescribed level (100%-confidence level), at the confidence levels of 99 percent and 99.9 percent (this follows from the aforementioned systematic sampling-bias). However, for Set 3 which is not much influenced by sampling bias (as it is a sub-horizontal fracture set sampled by a vertical borehole), the probability for rejection of the correct hypothesis is close to the theoretically expected value.

At a confidence level of 99 percent (Figure 3-11).

For Set 1 and Set 2, the probability for acceptance of the hypothesis is between 50 and 60 percent regardless of borehole length. For Set 3 the probability for acceptance of the hypothesis is close to 96–97 percent regardless of borehole length. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99 percent.

At a confidence level of 99.9 percent (Figure 3-11):

For Set 1 and Set 2, the probability for acceptance of the hypothesis is between 70 and 75 percent regardless of borehole length. For Set 3 the probability for acceptance of the hypothesis is close to 99 percent regardless of borehole length. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99.9 percent.

Results for inclined borehole

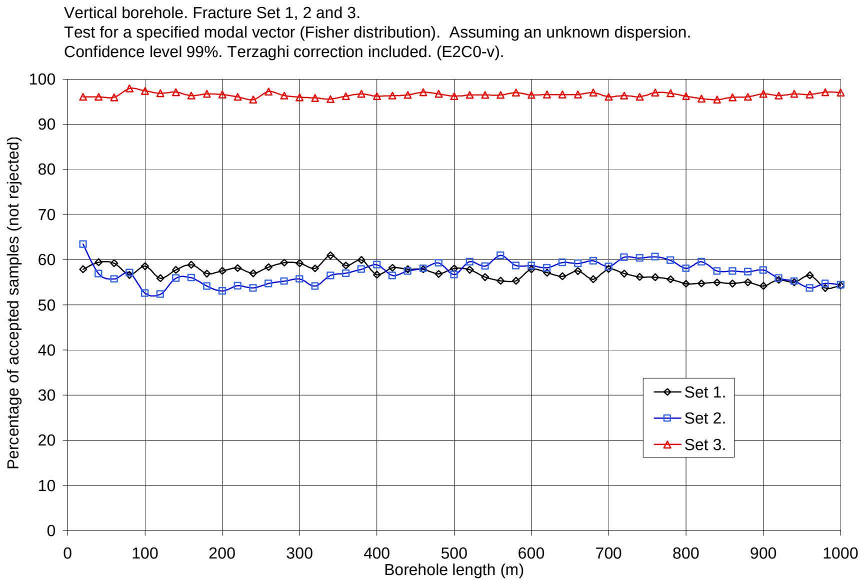
These results demonstrates that for all three sets, the probability for rejection of the correct hypothesis is larger than the prescribed level (100%-confidence level), at the confidence levels of 99 percent and 99.9 percent. This follows from the aforementioned systematic sampling-bias.

At a confidence level of 99 percent (Figure 3-12).

For Set 1 and Set 2, the probability for acceptance of the hypothesis is between 50 and 60 percent, for boreholes of lengths larger than 300 m. For Set 3 the probability for acceptance of the hypothesis is close to 70 percent, for boreholes of lengths larger than 300 m. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99 percent.

At a confidence level of 99.9 percent (Figure 3-12):

For Set 1 and Set 2, the probability for acceptance of the hypothesis is between 65 and 75 percent, for boreholes of lengths larger than 300 m. For Set 3 the probability for acceptance of the hypothesis is close to 85 percent, for boreholes of lengths larger than 300 m. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99.9 percent.



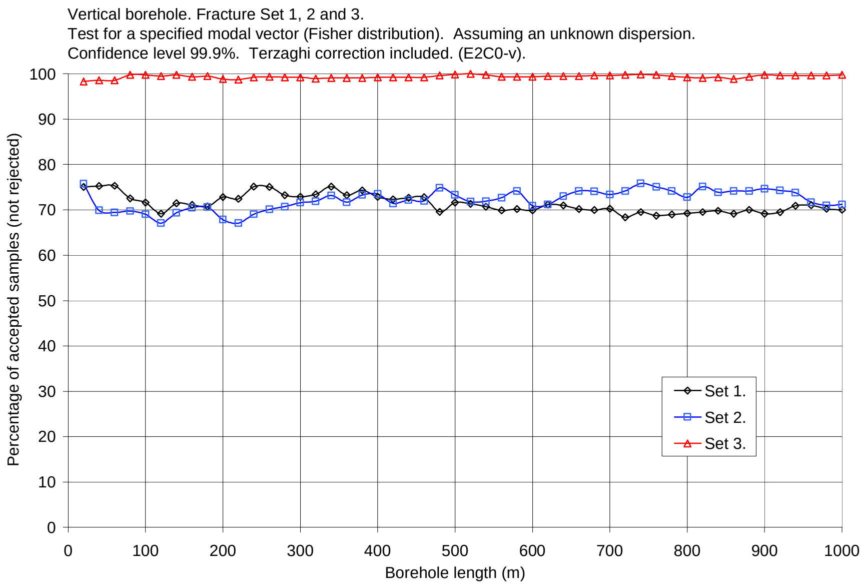
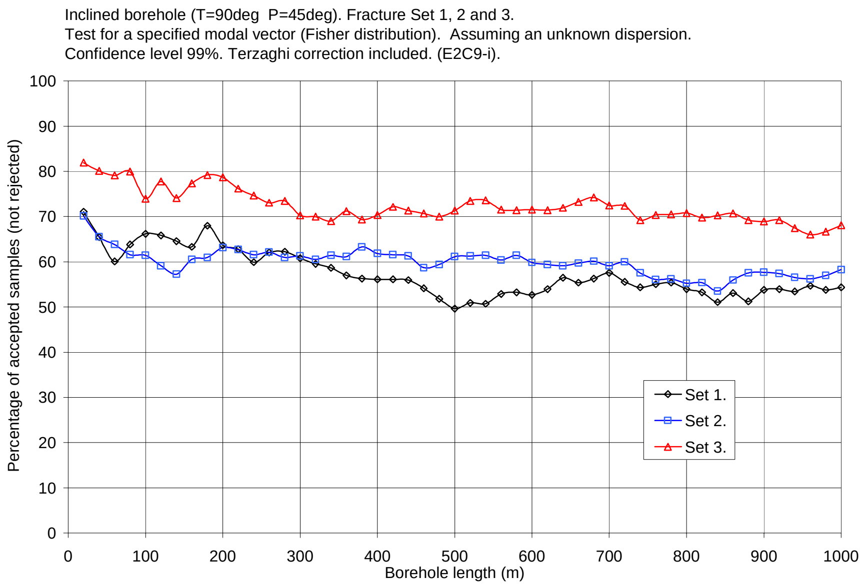


Figure 3-11. *Vertical borehole. Hypothesis testing considering orientation of fracture sets by use of confidence cones. The figure gives the percentage of accepted samples (probability for an accepted sample). Tested hypothesis is: the orientation of the fracture set studied is equal to the true orientation of the population. Confidence levels are 99 percent (upper figure) and 99.9 percent (lower figure).*



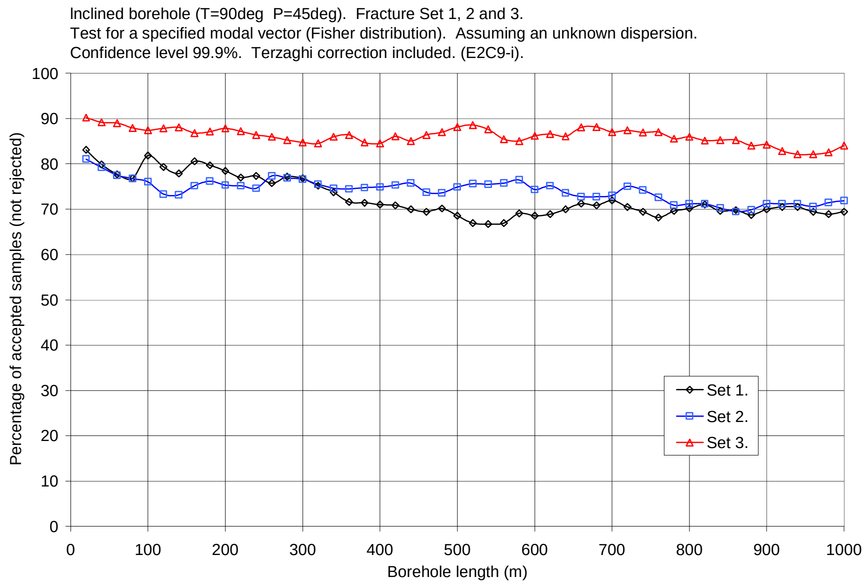


Figure 3-12. *Inclined borehole. Hypothesis testing considering orientation of fracture sets by use of confidence cones. The figure gives the percentage of accepted samples (probability for an accepted sample). Tested hypothesis is: the orientation of the fracture set studied is equal to the true orientation of the population. Confidence levels are 99 percent (upper figure) and 99.9 percent (lower figure).*

4 Estimation of fracture set dispersion from borehole data

4.1 Fracture set and dispersion

A fracture set is a number of fractures that are grouped together as they demonstrate some tendency to have a similar orientation. The orientations of the fractures of a fracture set may follow some stochastic distribution, in which some orientations are more likely than other orientations. An example of such a distribution is the Fisher distribution. The poles of the fractures of a fracture set have a tendency (weak or strong) to form some sort of cluster on a spherical projection. (It is also possible to group fractures into a fracture set based on the conclusion that the fractures have no tendency for a similar orientation).

The dispersion of a fracture set is a measure of the concentration (or spread) of the fracture orientations about some mean direction. For the Fisher distribution the dispersion parameter is called “kappa”, and it is actually a concentration parameter, since the larger the value of kappa the more the distribution is concentrated towards a mean direction. The kappa parameter corresponds to a Fisher distribution, and for other distributions there are other measures of dispersion. Examples of general dispersion parameters are the SR1 and SR2 parameters (discussed below) In this study we have analysed three different dispersion parameters, the kappa parameter (Fisher distributions) and the SR1 and SR2 parameters.

4.2 Estimated dispersion based on the SR1 parameter

4.2.1 Methodology – eigenvalues parameters and dispersion

The mean direction of a group of fractures can be calculated based on the eigenvalues method, as proposed by /Mardia, 1972/; this method is discussed in Appendix A. The method will provide us with a representative vector, and the direction of this vector is the mean direction of the group of fractures studied. In addition the method will provide us with three eigenvalues (L1, L2 and L3), these three values provide direct information about the distribution of the group of fractures studied.

Based on the eigenvalues, two different dispersion parameters are calculated, as proposed by /Woodcock, 1977/, these two parameters are called, SR1 and SR2, they are calculated as follows.

$$SR1 = LN(L1/L2) \quad \text{and} \quad SR2 = LN(L2/L3) \qquad \text{4-1}$$

If SR2 is small (approximately < 1) this is an indication that the poles of the fracture set studied forms a circular cluster on a spherical projection. Considering the population of fractures studied, the distribution of orientations of the fracture sets studied are Fischer distributions, which forms circular clusters on a spherical projection, it follows that the three different SR2-values corresponding to the three different fracture sets, are all equal to zero (for the population). This is confirmed by the results of our analysis of large samples, which demonstrate small values of SR2. For samples (Terzaghi corrected) taken at a borehole length equal to 1000 m, the mean SR2 values are: 0.15 for Set 1, 0.12 for Set 2 and 0.09 for Set 3. Hence, the small values of SR2 indicate that the distributions of the fracture sets studied are circular on a unit sphere.

For circular fracture sets, the SR1 parameter is a measure of the dispersion of the set. In analogy with the concentration parameter of a Fisher distribution (kappa), the larger the value of SR1 the more the distribution (the fractures) is concentrated towards a mean direction. Small values of SR1 indicates a large dispersion.

We have derived a relationship between the kappa parameter of a Fisher distribution and the corresponding SR1 parameter (see Appendix _A). This relationship is applicable for Fisher distributions, it is not to be used on samples containing a limited number of fractures.

$$SR1 = \ln\left(\frac{(K-1)^2 + 1}{K-1}\right) \qquad 4\text{-}2$$

K = Kappa of Fisher distribution

By use of Eq. 5-2, above, we have calculated the following values of SR1, to demonstrate the relationship between kappa and SR1:

Kappa = 5 corresponds to SR1 = 1.45

Kappa = 7 corresponds to SR1 = 1.82

Kappa = 9 corresponds to SR1 = 2.09

The SR1 value of the population is also calculated based on Eq. 5-2, the equation yields the following SR1 values:

SET 1: Fisher kappa of population = 4.84 → SR1 of population = 1.411

SET 2: Fisher kappa of population = 8.35 → SR1 of population = 2.013

SET 3: Fisher kappa of population = 8.33 → SR1 of population = 2.010

4.2.2 Point estimate and the SR1 parameter

The fractures that intersect the borehole studied are samples of the fracture population. The properties of the sample are estimates of the properties of the population. The observed fractures are classified into three groups, one group for each theoretical fracture set. After the classification each fracture set is studied one by one, separate from the other sets. The test presented below is conducted for each fracture set separately.

From a statistical point of view, the calculation of the SR1 variable, based on samples representing the population, is a point estimate of the dispersion parameter SR1 of the population. The SR1 variable is the sample variable studied. The efficiency of the point estimate of the variable studied increases with the size of the sample (number of observed fractures) and the size of the sample increases with the length of the borehole. This is demonstrated in Figure 4-1 and Figure 4-2 (below).

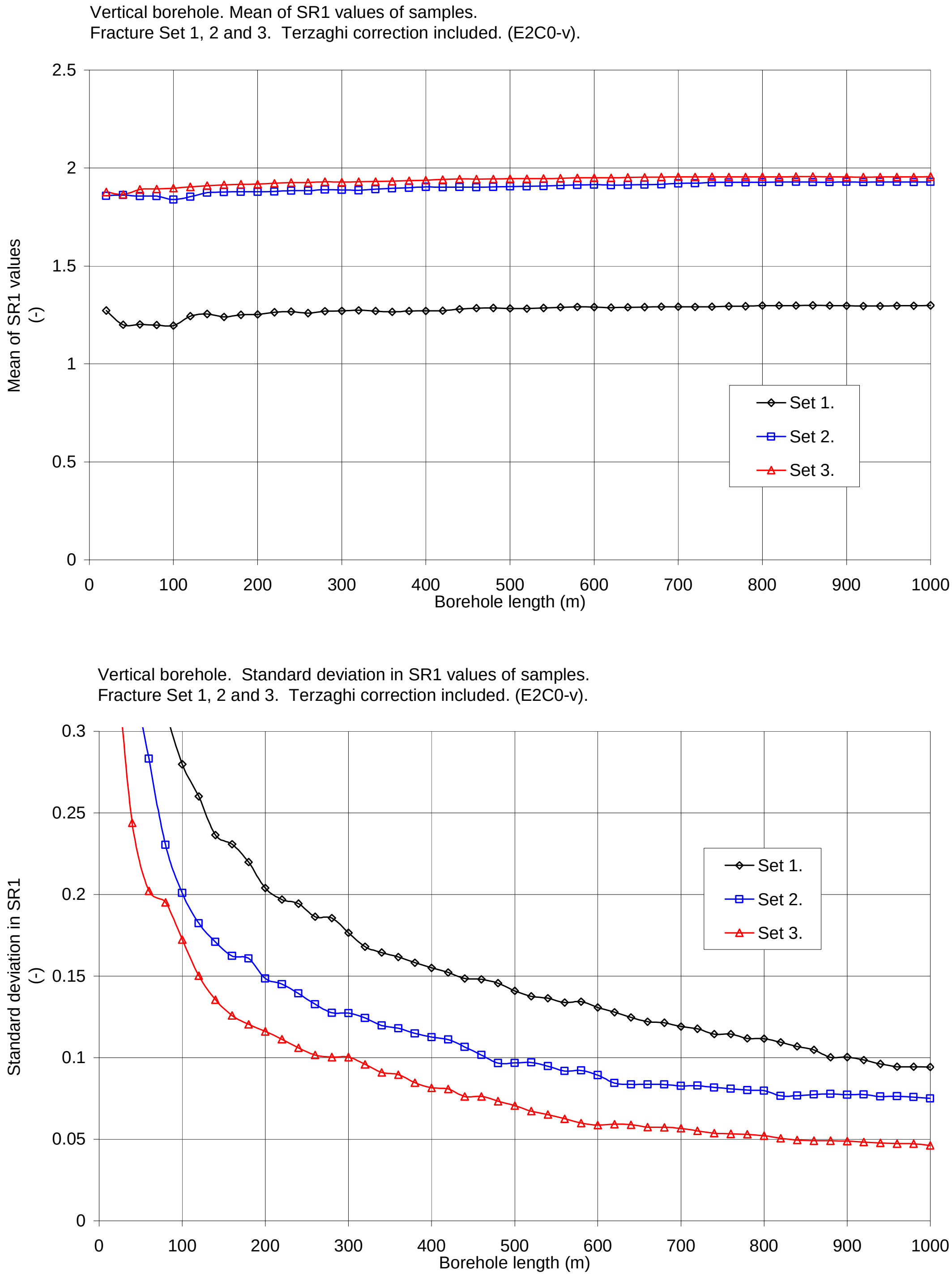
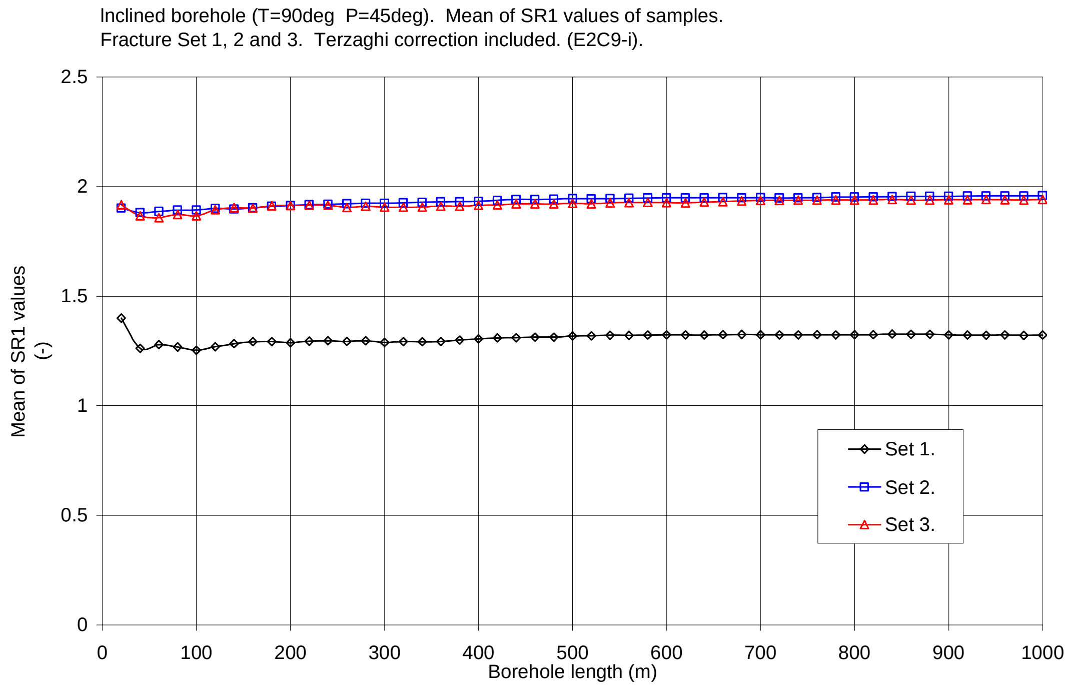


Figure 4-1. *Vertical borehole. The efficiency of the point estimate of the SR1 variable, given as the mean (upper figure) and standard deviation (lower figure) of the SR1 variable of the samples at different lengths of borehole. The calculations are based on 900 realisations of different boreholes, for each length studied.*



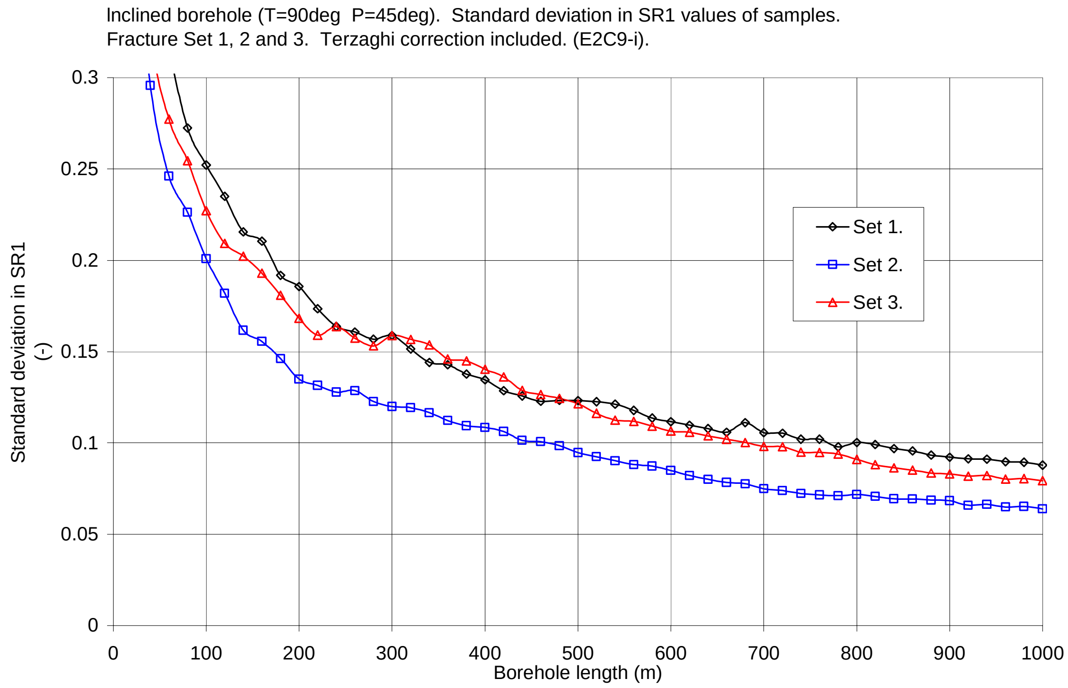


Figure 4-2. *Inclined borehole. The efficiency of the point estimate of the SR1 variable, given as the mean (upper figure) and standard deviation (lower figure) of the SR1 variable of the samples at different lengths of borehole. The calculations are based on 900 realisations of different boreholes, for each length studied.*

Sampling along a straight borehole (a scanline) is a one-dimensional sampling; such sampling of fracture orientation in a three-dimensional fracture system will introduce an orientation sampling bias. The bias follows from the fact that the probability for intersecting a fracture depends on the angle between the sampling line and the fracture, as well as on the size of the fracture. For compensation of this sampling bias we have applied a geometrical correction factor based on the observed angle between the sampling line and the normal to a particular fracture, such a correction is called Terzaghi correction (see Appendix B). In this study all the fracture orientation data, derived from sampling the boreholes, are corrected for sampling bias by use of the Terzaghi correction. Terzaghi correction is not perfect and some bias will remain in the samples.

Considering a vertical borehole of length 1000 m, the deviation between the mean SR1 of the samples and the SR1 of the population is as follows:

SET 1:	SR1 sample = 1.30	SR1 population = 1.41	Difference = 7.9%
SET 2:	SR1 sample = 1.93	SR1 population = 2.01	Difference = 4.0%
SET 3:	SR1 sample = 1.96	SR1 population = 2.01	Difference = 2.7%

Considering an inclined borehole of length 1000 m, the deviation between the mean SR1 of the samples and the SR1 of the population is as follows:

SET 1:	SR1 sample = 1.32	SR1 population = 1.41	Difference = 6.2%
SET 2:	SR1 sample = 1.96	SR1 population = 2.01	Difference = 2.6%
SET 3:	SR1 sample = 1.94	SR1 population = 2.01	Difference = 3.4%

4.2.3 Hypothesis testing of SR1 parameter considering acceptable deviations

Purpose of test

The purpose of the test is to determine when the size of the sample is large enough to produce an acceptable estimate of the studied parameter, with a certain probability. This can also be stated in the following way: the calculation of the sample size that is necessary to reach a confidence level, considering a given confidence interval. The confidence interval is the same thing as a test criterion (an acceptable deviation). The sample size corresponds to length of borehole.

Null hypothesis, acceptable deviations and criterion of significance

The analysis of the point estimate of the fracture set dispersion is based on hypothesis testing. The hypothesis testing is based on the sample variable studied (the SR1 variable) and given criterions of significance. The null hypothesis (H_0) is that the dispersion parameter derived from a sample is a good estimate of the true parameter of the population. For the tests presented in this section, the criterions of significance correspond to selected values of acceptable deviations. If the deviation between a SR1 value derived from a sample and the true SR1 value is larger than these acceptable deviations, the sample is rejected. We have studied three different criterions that correspond to three different levels of significance.

First criterion: $H_{0\ (SR1\_deviation <=15\%)}$ is rejected if:

ABS [SR1 $_{(sample)}$ – SR1 $_{(population)}$] <= 0.15 * SR1 $_{(population)}$

Second criterion: $H_{0\ (SR1\_\ deviation <=10\%)}$ is rejected if:

ABS [SR1 $_{(sample)}$ – SR1 $_{(population)}$] <= 0.10 * SR1 $_{(population)}$

Third criterion: $H_{0\ (SR1\_\ deviation <=5\%)}$ is rejected if:

ABS [SR1 $_{(sample)}$ – SR1 $_{(population)}$] <= 0.5 * SR1 $_{(population)}$

The result of the analysis is presented as the percentage of accepted the samples, which is approximately the same thing as the probability for correct estimation, considering the different selected criterions

4.2.4 Results for a vertical borehole

Results for Set 1

Examples of results for Set 1 are as follows (see Figure 4-3): At a borehole length larger than 1100 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (SR1\_\ deviation <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (SR1\_\ deviation <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 1100 meters. And finally, if the borehole has a length larger than 1100 meters, the probability is larger than 90 percent that the deviation in estimated SR1 value is within plus/minus 15 percent of the true SR1 value of the population.

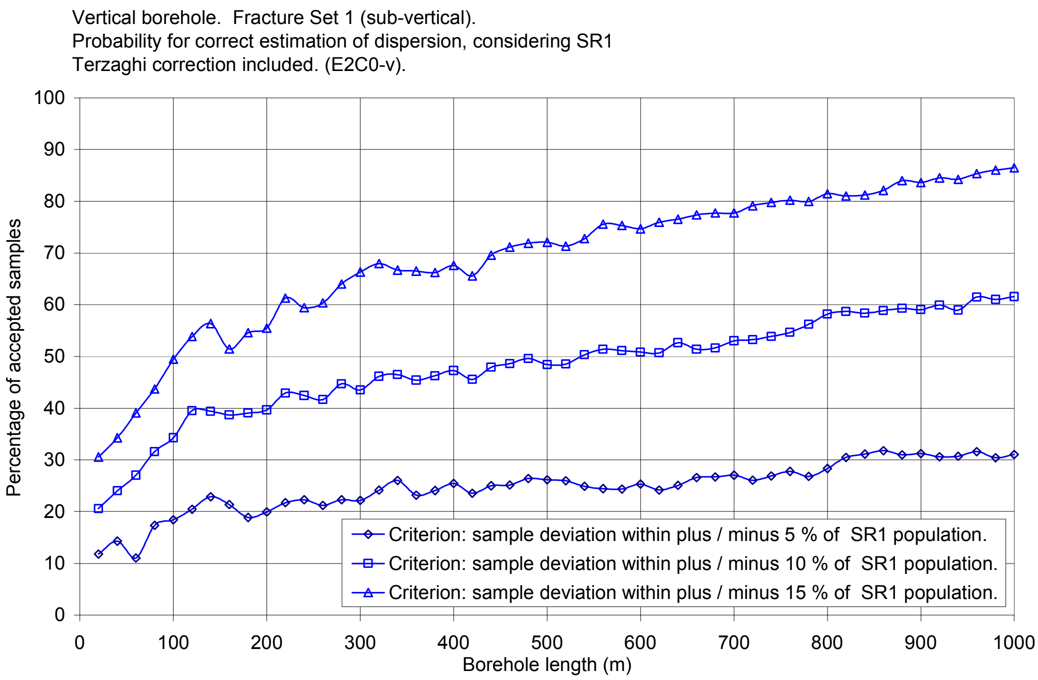


Figure 4-3. *Vertical borehole. Fracture set 1. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the SR1 parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 2

Examples of results for Set 2 are as follows (see Figure 4-4). At a borehole length larger than 250 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (SR1\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (SR1\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 250 meters. And finally, if the borehole has a length larger than 250 meters, the probability is larger than 90 percent that the deviation in estimated SR1 value is within plus/minus 15 percent of the true SR1 value of the population.

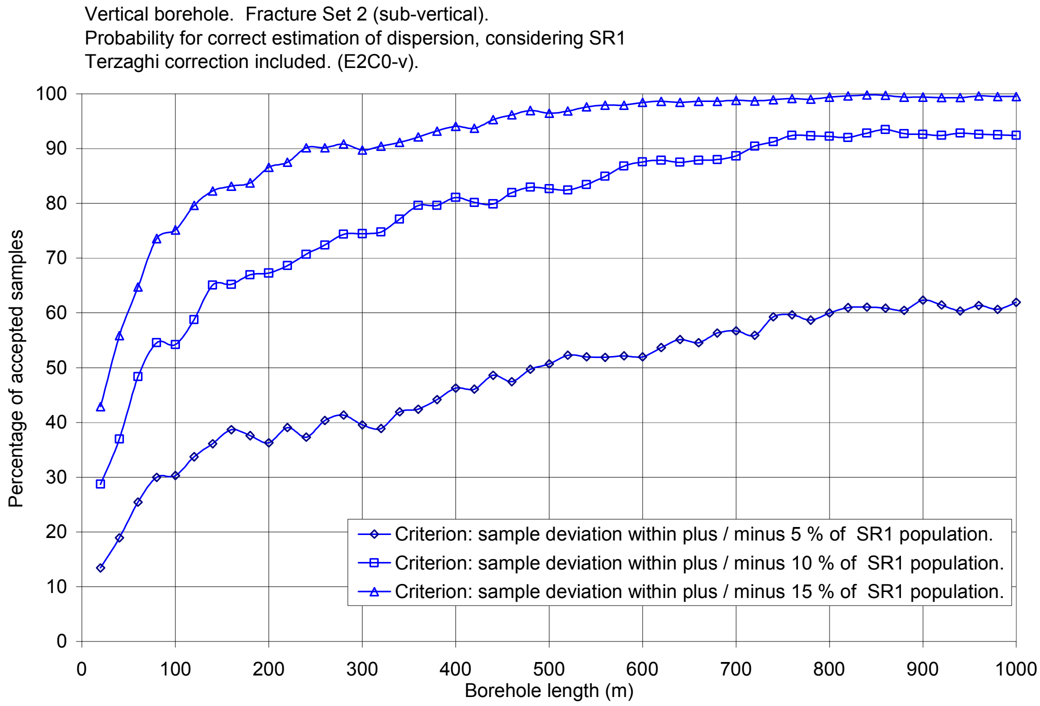


Figure 4-4. *Vertical borehole. Fracture set 2. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the SR1 parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 3

Examples of results for Set 3 are as follows (see Figure 4-5). At a borehole length larger than 100 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (SR1\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (SR1\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 100 meters. And finally, if the borehole has a length larger than 100 meters, the probability is larger than 90 percent that the deviation in estimated SR1 value is within plus/minus 15 percent of the true SR1 value of the population.

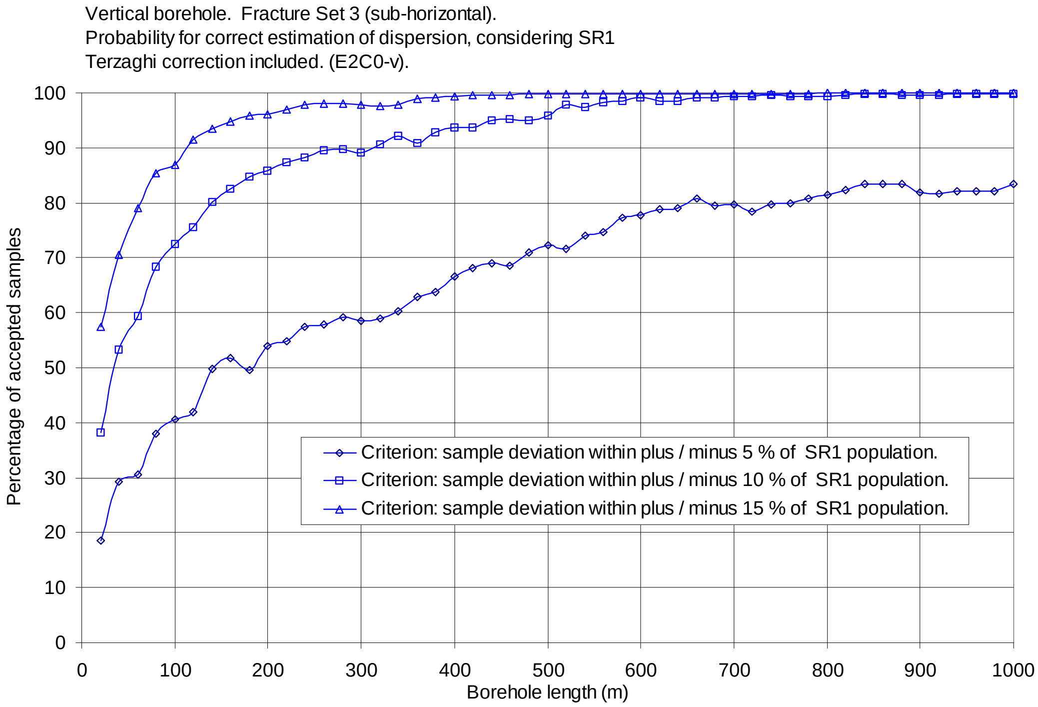


Figure 4-5. *Vertical borehole. Fracture set 3. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the SR1 parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

4.2.5 Results for an inclined borehole

Results for Set 1

Examples of results for Set 1 are as follows (see Figure 4-6): At a borehole length larger than 750 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (SR1\_deviation <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (SR1\_deviation <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 750 meters. And finally, if the borehole has a length larger than 750 meters, the probability is larger than 90 percent that the deviation in estimated SR1 value is within plus/minus 15 percent of the true SR1 value of the population.

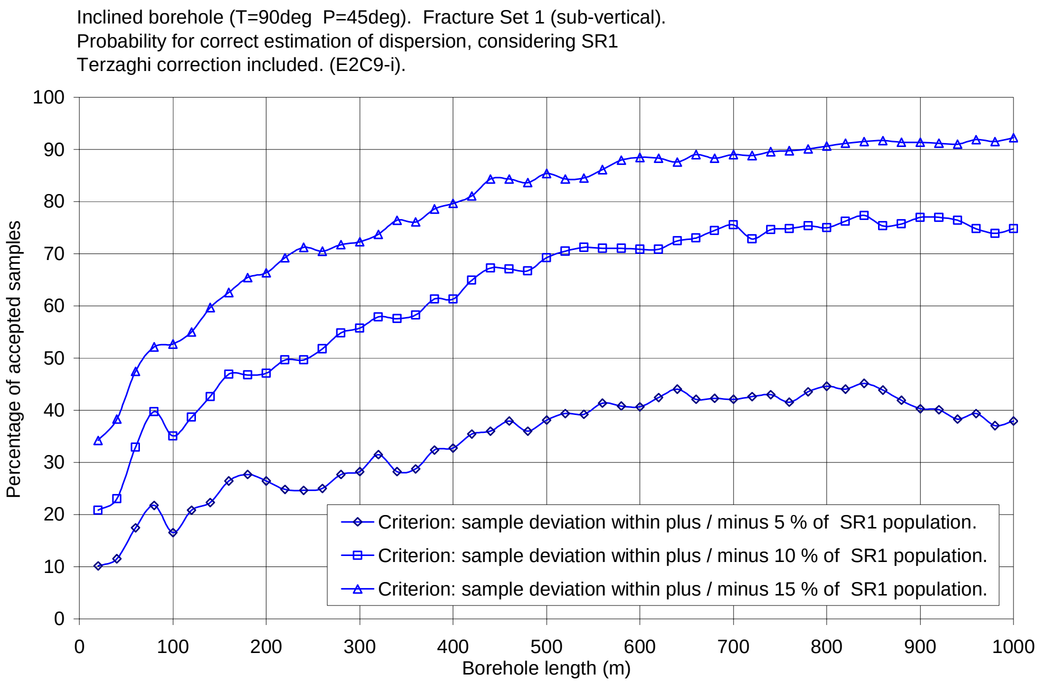


Figure 4-6. *Inclined borehole. Fracture set 1. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the SR1 parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 2

Examples of results for Set 2 are as follows (see Figure 4-7): At a borehole length larger than 170 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (SR1\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (SR1\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 170 meters. And finally, if the borehole has a length larger than 170 meters, the probability is larger than 90 percent that the deviation in estimated SR1 value is within plus/minus 15 percent of the true SR1 value of the population.

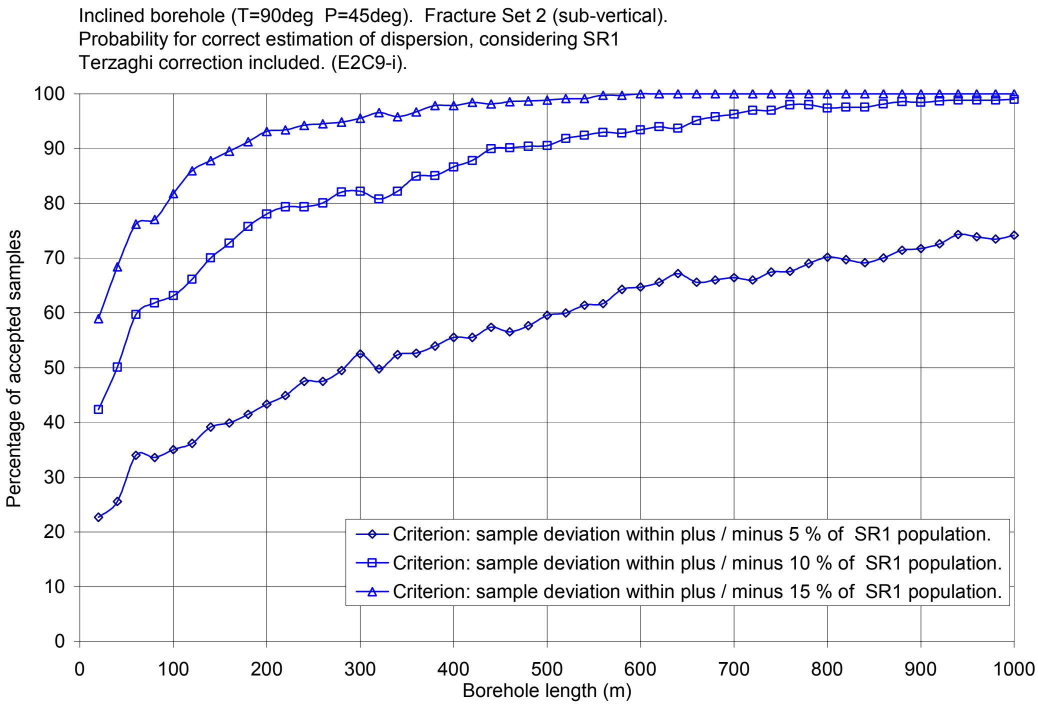


Figure 4-7. *Inclined borehole. Fracture set 2. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the SR1 parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 3

Examples of results for Set 3 are as follows. At a borehole length larger than 250 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (SR1\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (SR1\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 250 meters. And finally, if the borehole has a length larger than 250 meters, the probability is larger than 90 percent that the deviation in estimated SR1 value is within plus/minus 15 percent of the true SR1 value of the population.

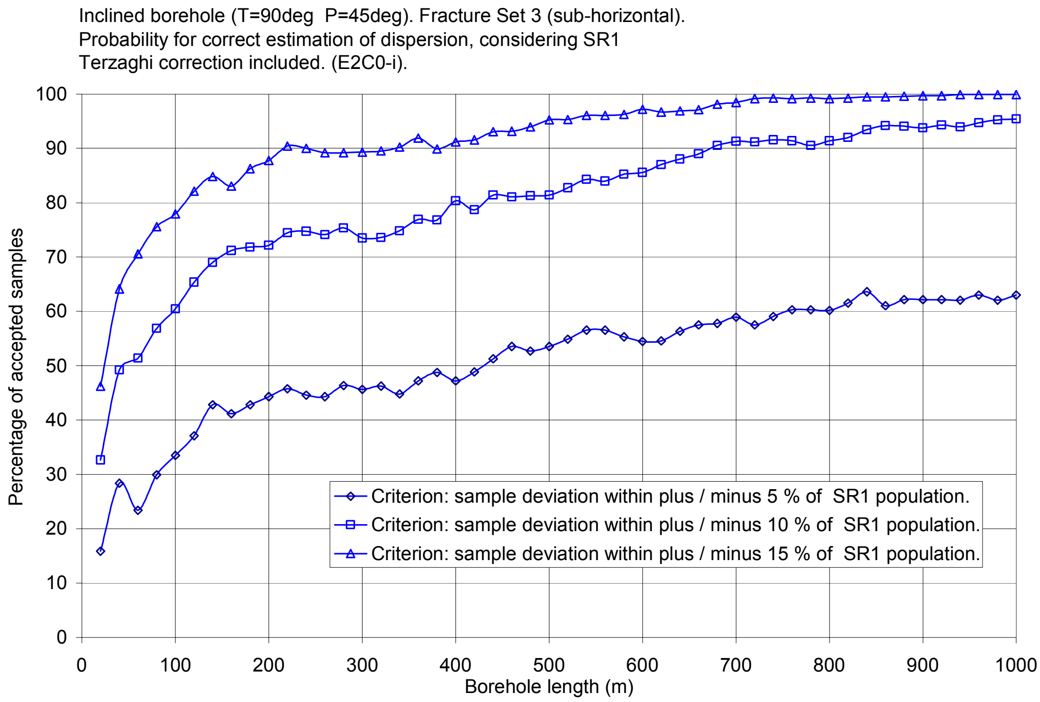


Figure 4-8. *Inclined borehole. Fracture set 3. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the SR1 parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

4.3 Estimated dispersion based on the kappa parameter

4.3.1 Methodology – resultant vectors and Fisher kappa parameter

The samples analysed in this study demonstrate that the orientations of the fractures have a circular distribution on a spherical projection; this is demonstrated by the calculated SR2 values (see Section 4.2.1). These results are as expected, as the fracture sets were generated by use of Fisher distributions. However, even if we had not known that the fracture sets were generated by use of Fisher distributions, the circular shape of the fracture clusters as revealed by the SR2 parameter, indicates that parametric tests against Fisher distributions are appropriate. The Fisher distribution is characterised by a modal vector (mean direction) and a concentration parameter called kappa, the distribution has a rotational symmetry about the modal vector. The larger the value of kappa the more the distribution (the fractures) is concentrated towards the modal vector.

The mean direction for a group of fractures can be calculated based on the resultant vector method, this method is discussed in Appendix A. The method will provide us with a resultant vector, and the direction of this vector is the mean direction of the group of fractures studied. In addition, the method will provide us with the length of the resultant vector (R); the length of the vector provides direct information about the distribution of the group of fractures studied. In analogy with the concentration parameter of a Fisher distribution (kappa), the larger the value of *R* the more the distribution (the fractures) is concentrated towards a mean direction.

The kappa values of the parent distribution -the population- can be estimated based on information provided by samples. /Fisher, 1953/ proposed a method for such an estimate based on a ML-estimation (maximum likelihood method). The method involves the length of the resultant vector and the number of fractures in the sample, as follows.

$$\frac{e^k + e^{-k}}{e^k - e^{-k}} - \frac{1}{k} = \frac{R}{M} \qquad \text{4-3}$$

k = Kappa of sample

R = Length of resultant vector (of sample)

M = Number of fractures in samplet

For values of kappa larger than ca. 5 the variable e^{-k} is negligible, and the equation above is reduced to

$$k \approx \frac{M}{M - R} \qquad \text{4-4}$$

The first equation is difficult to analytically solve for kappa; however, it is not difficult to solve the equation by use of numerical methods. In this study a sample is first tested by use of the reduced equation (Eq. 5-4). If the obtained kappa value is less than 8 the complete equation (Eq. 5-3) is solved numerically by use of the secant method, and if the obtained kappa is larger than 8 the reduced equation (Eq. 5-4) is used for calculation of kappa.

To ensure consistent results of the resultant vector method, it was constrained by the results of the eigenvalues method, as dicussed in Appendix A.

4.3.2 Point estimate and the kappa parameter

The fractures that intersect the borehole studied are samples of the fracture population. The properties of the sample are estimates of the properties of the population. The observed fractures are classified into three groups, one group for each theoretical fracture set. After the classification each fracture set is studied one by one, separate from the other sets. The test presented below is conducted for each fracture set separately.

From a statistical point of view, the calculation of the kappa variable, based on samples representing the population, is a point estimate of the parameter kappa of the population. Kappa is the sample variable studied. The efficiency of the point estimate of the variable studied increases with the size of the sample (number of observed fractures) and the size of the sample increases with the length of the borehole. This is demonstrated in Figure 4-9 and Figure 4-10.

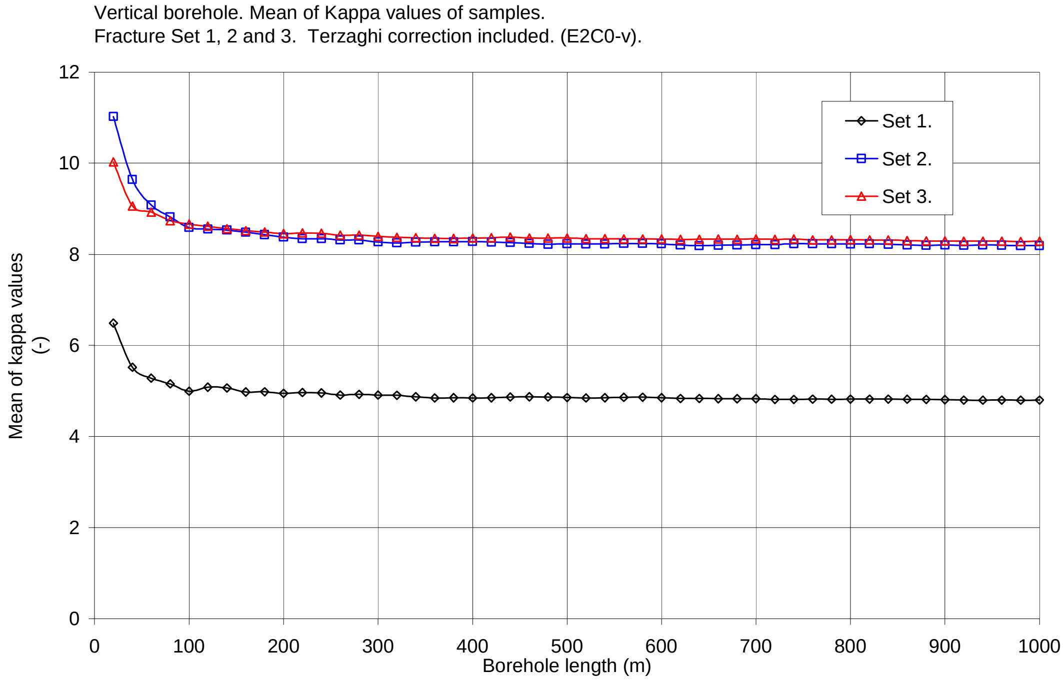
Sampling along a straight borehole (a scanline) is a one-dimensional sampling; such sampling of fracture orientation in a three-dimensional fracture system will introduce an orientation sampling bias. The bias follows from the fact that the probability for intersecting a fracture depends on the angle between the sampling line and the fracture, as well as on the size of the fracture. For compensation of this sampling bias we have applied a geometrical correction factor based on the observed angle between the sampling line and the normal to a particular fracture, such a correction is called Terzaghi correction (see Appendix B). In this study all the fracture orientation data, derived from sampling the boreholes, are corrected for sampling bias by use of the Terzaghi correction. The Terzaghi correction is not perfect and some bias will remain in the samples.

Considering a vertical borehole of length 1000 m, the final deviation between the mean kappa of the samples and the kappa of the population is as follows:

SET 1:	Kappa sample = 4.80	Kappa population = 4.84	Difference = 0.8%
SET 2:	Kappa sample = 8.19	Kappa population = 8.35	Difference = 1.9%
SET 3:	Kappa sample = 8.29	Kappa population = 8.33	Difference = 0.5%

Considering an inclined borehole of length 1000 m, the final deviation between the mean kappa of the samples and the kappa of the population is as follows:

SET 1:	Kappa sample = 4.89	Kappa population = 4.84	Difference = 1.0%
SET 2:	Kappa sample = 8.43	Kappa population = 8.35	Difference = 0.9%
SET 3:	Kappa sample = 8.43	Kappa population = 8.33	Difference = 1.2%



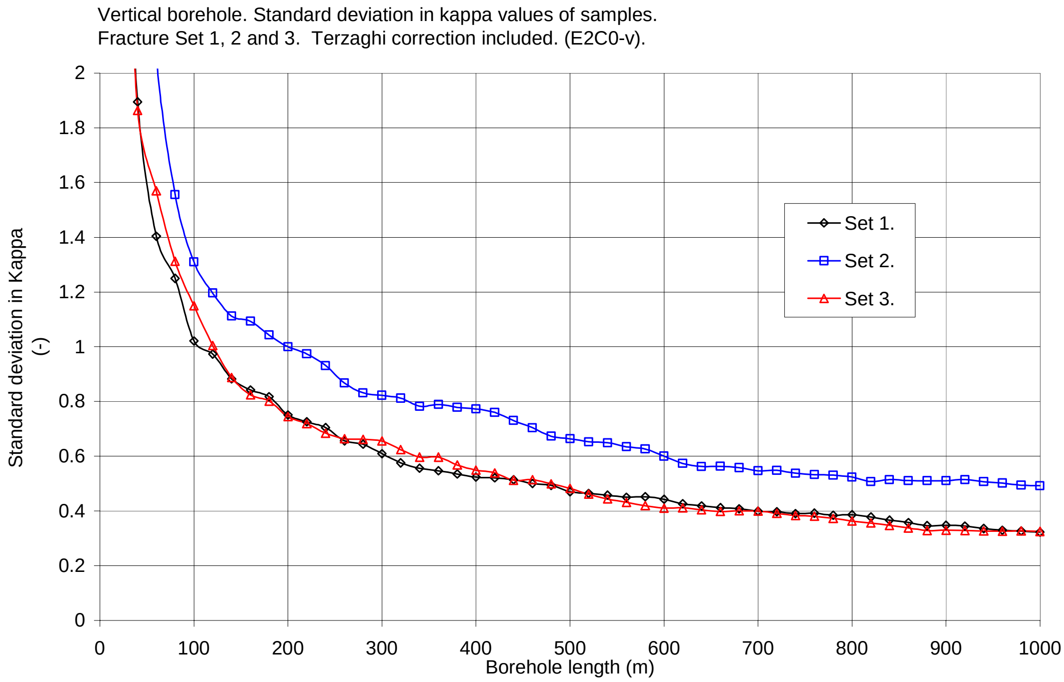
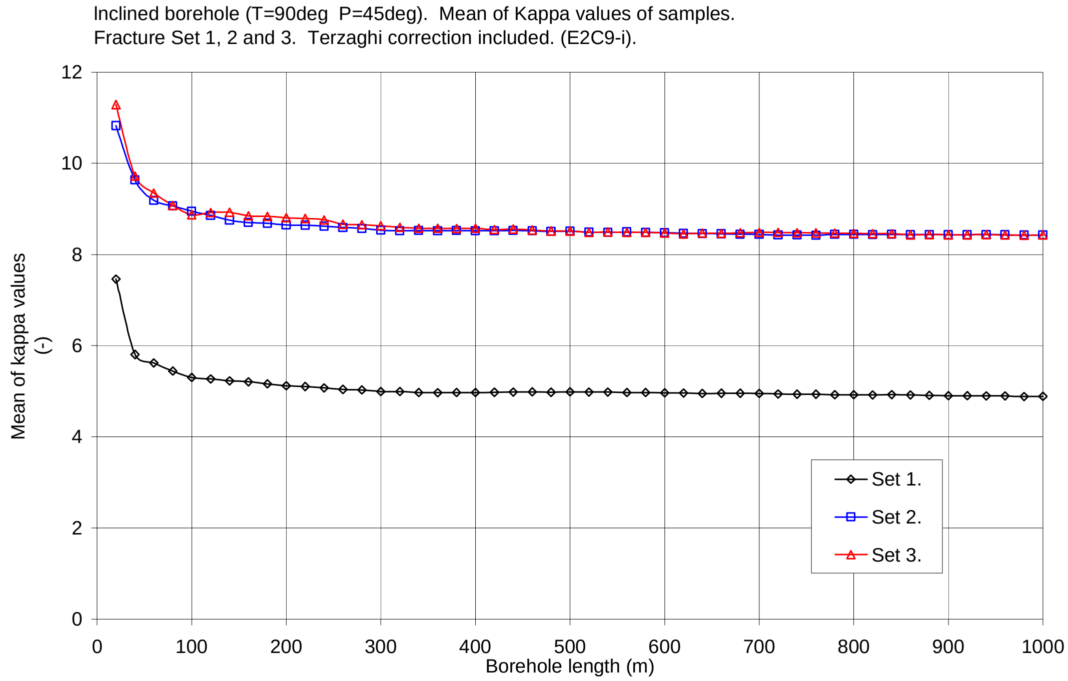


Figure 4-9. *Vertical borehole. The efficiency of the point estimate of the kappa variable, given as the mean (upper figure) and standard deviation (lower figure) of the kappa variable of the samples at different lengths of borehole. The calculations are based on 900 realisations of different boreholes, for each length studied.*



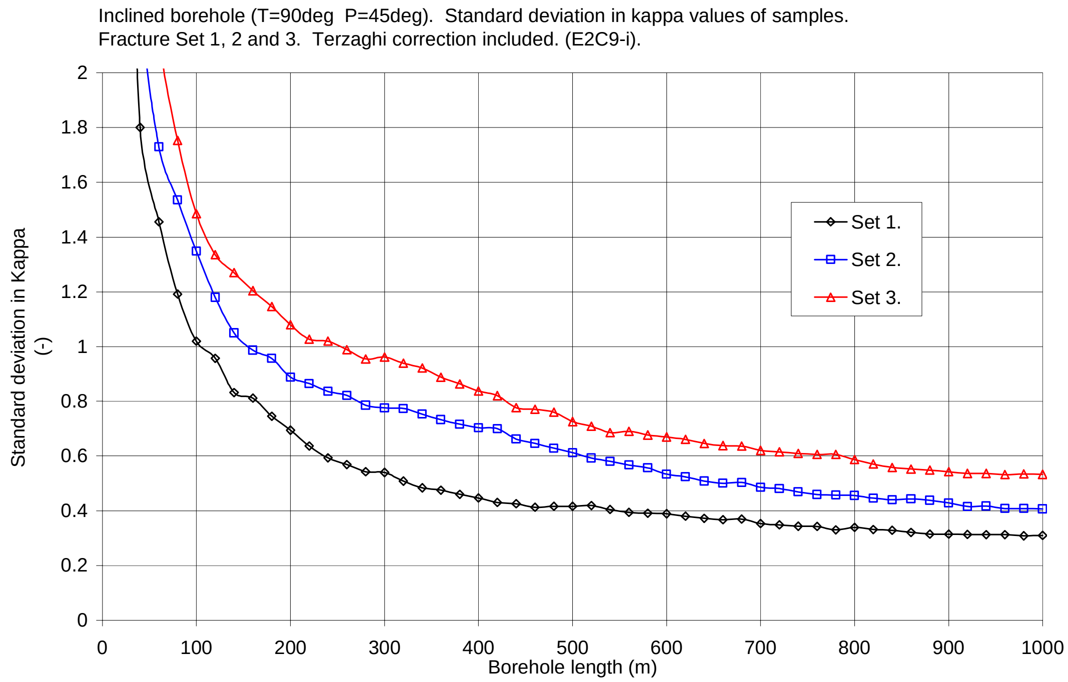


Figure 4-10. *Inclined borehole. The efficiency of the point estimate of the kappa variable, given as the mean (upper figure) and standard deviation (lower figure) of the kappa variable of the samples at different lengths of borehole. The calculations are based on 900 realisations of different boreholes, for each length studied.*

4.3.3 Hypothesis testing of the kappa parameter considering acceptable deviations

Purpose of test

The purpose of the test is to determine when the size of the sample is large enough to produce an acceptable estimate of the studied parameter, with a certain probability. This can also be stated in the following way: the calculation of the sample size that is necessary to reach a confidence level, considering a given confidence interval. The confidence interval is the same thing as a test criterion (an acceptable deviation). The sample size corresponds to length of borehole.

Null hypothesis, acceptable deviations and criterion of significance

The analysis of the point estimate of the fracture set dispersion is based on hypothesis testing. The hypothesis testing is based on the sample variable studied (the kappa variable) and given criterions of significance. The null hypothesis (H_0) is that the dispersion parameter derived from a sample is a good estimate of the true parameter of the population. For the tests presented in this section, the criterions of significance correspond to selected values of acceptable deviations. If the deviation between a kappa value derived from a sample and the true kappa value is larger than these acceptable deviations, the sample is rejected.

The kappa values of the population are as follows:

SET 1: Fisher kappa of population = 4.84

SET 2: Fisher kappa of population = 8.35

SET 3: Fisher kappa of population = 8.33

We have studied three different criterions of significance.

First criterion: $H_{0\ (Kappa\_deviation <=15\%)}$ is rejected if:

ABS [$Kappa_{(sample)} - Kappa_{(population)}$] <= 0.15 * $Kappa_{(population)}$

This first level corresponds to intervals between:

For SET 1: 4.11 to 5.57 which corresponds to an interval length of 1.45

For SET 2: 7.10 to 9.60 which corresponds to an interval length of 2.50

For SET 3: 7.08 to 9.58 which corresponds to an interval length of 2.50

Second criterion: $H_{0\ (Kappa\_ deviation <=10\%)}$ is rejected if:

ABS [$Kappa_{(sample)} - Kappa_{(population)}$] <= 0.10 * $Kappa_{(population)}$

Third criterion: $H_{0\ (Kappa\_ deviation <=5\%)}$ is rejected if:

ABS [$Kappa_{(sample)} - Kappa_{(population)}$] <= 0.5 * $Kappa_{(population)}$

The result of the analysis is presented as the percentage of accepted the samples, which is approximately the same thing as the probability for correct estimation, considering the different selected criterions

4.3.4 Results for a vertical borehole

Results for Set 1

Examples of results for Set 1 are as follows (see Figure 4-11): At a borehole length larger than 500 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (Kappa\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (Kappa\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 500 meters. And finally, if the borehole has a length larger than 500 meters, the probability is larger than 90 percent that the deviation in estimated Kappa value is within plus/minus 15 percent of the true Kappa value of the population.

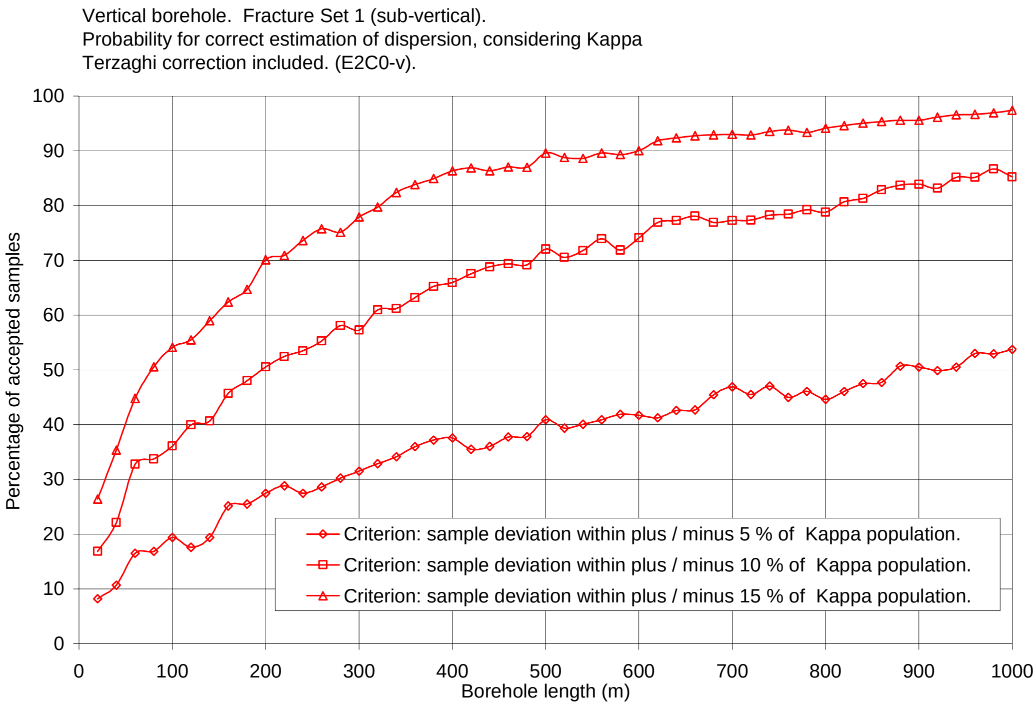


Figure 4-11. *Vertical borehole. Fracture set 1. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the kappa parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 2

Examples of results for Set 2 are as follows (see Figure 4-12): At a borehole length larger than 420 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (Kappa\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (Kappa\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 420 meters. And finally, if the borehole has a length larger than 420 meters, the probability is larger than 90 percent that the deviation in estimated Kappa value is within plus/minus 15 percent of the true Kappa value of the population.

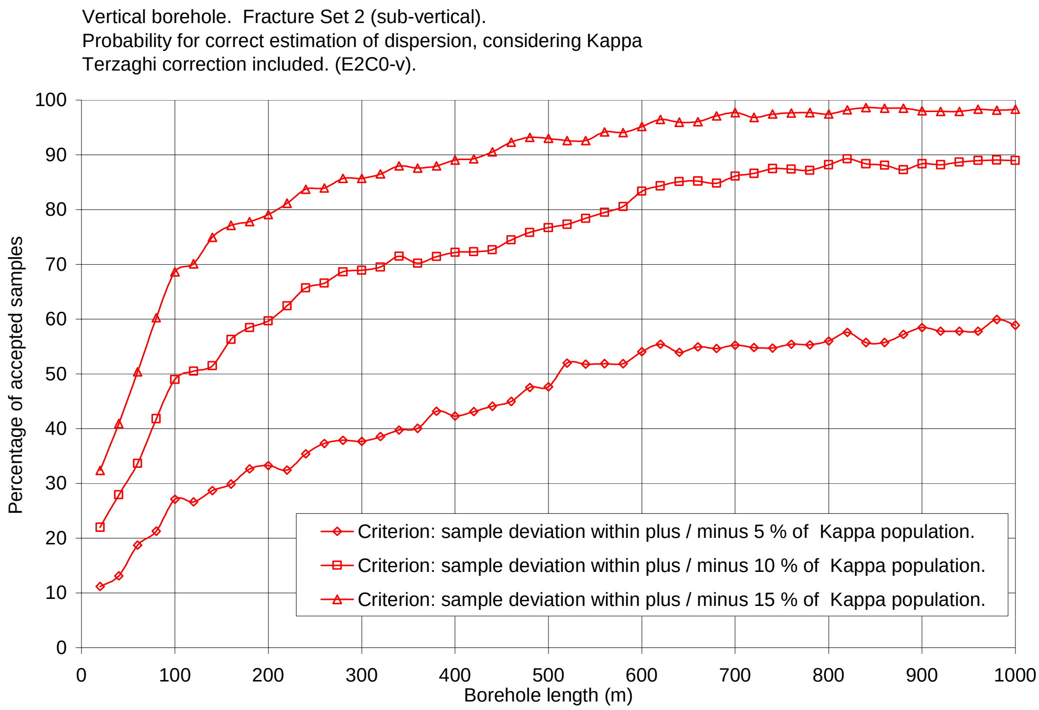


Figure 4-12. *Vertical borehole. Fracture set 2. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the kappa parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 3

Examples of results for Set 3 are as follows (see Figure 4-13): At a borehole length larger than 200 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (Kappa\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (Kappa\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 200 meters. And finally, if the borehole has a length larger than 200 meters, the probability is larger than 90 percent that the deviation in estimated Kappa value is within plus/minus 15 percent of the true Kappa value of the population.

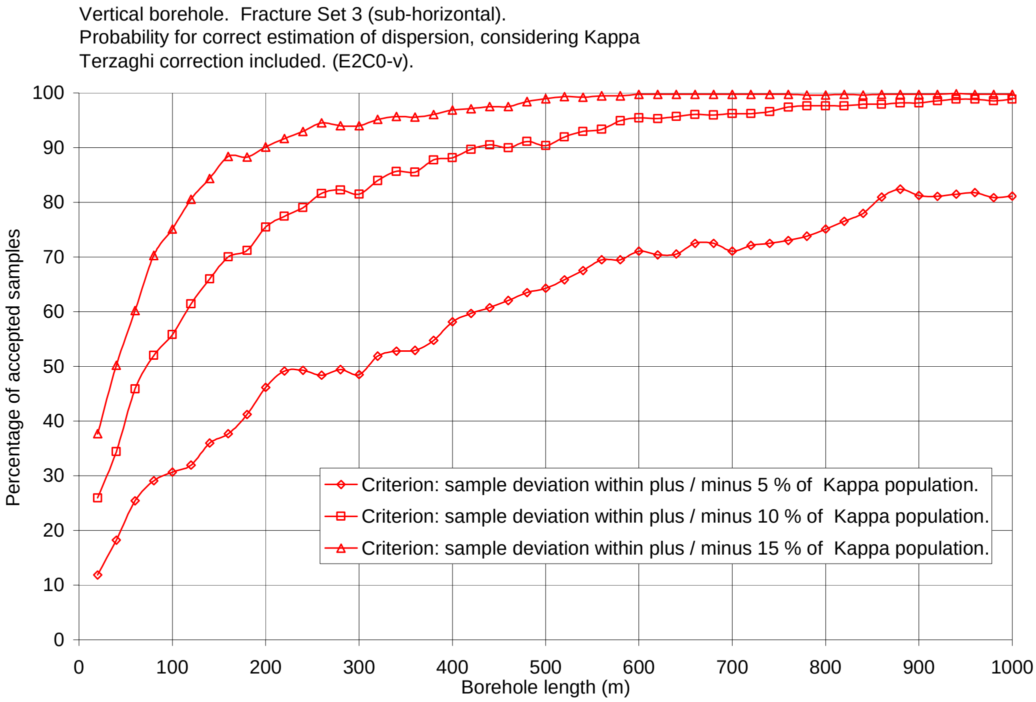


Figure 4-13. *Vertical borehole. Fracture set 3. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the kappa parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

4.3.5 Results for an inclined borehole

Results for Set 1

Examples of results for Set 1 are as follows: At a borehole length larger than 420 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (Kappa\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (Kappa\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 420 meters. And finally, if the borehole has a length larger than 420 meters, the probability is larger than 90 percent that the deviation in estimated Kappa value is within plus/minus 15 percent of the true Kappa value of the population.

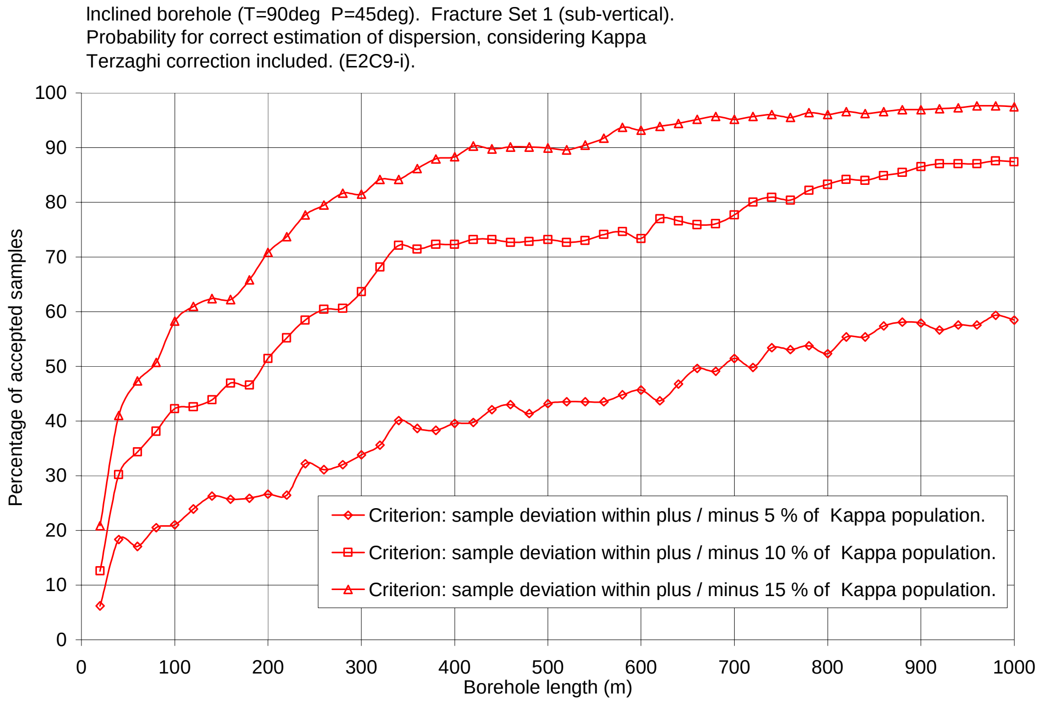


Figure 4.14. *Inclined borehole. Fracture set 1. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the kappa parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 2

Examples of results for Set 2 are as follows: At a borehole length larger than 360 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (Kappa\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (Kappa\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 360 meters. And finally, if the borehole has a length larger than 360 meters, the probability is larger than 90 percent that the deviation in estimated Kappa value is within plus/minus 15 percent of the true Kappa value of the population.

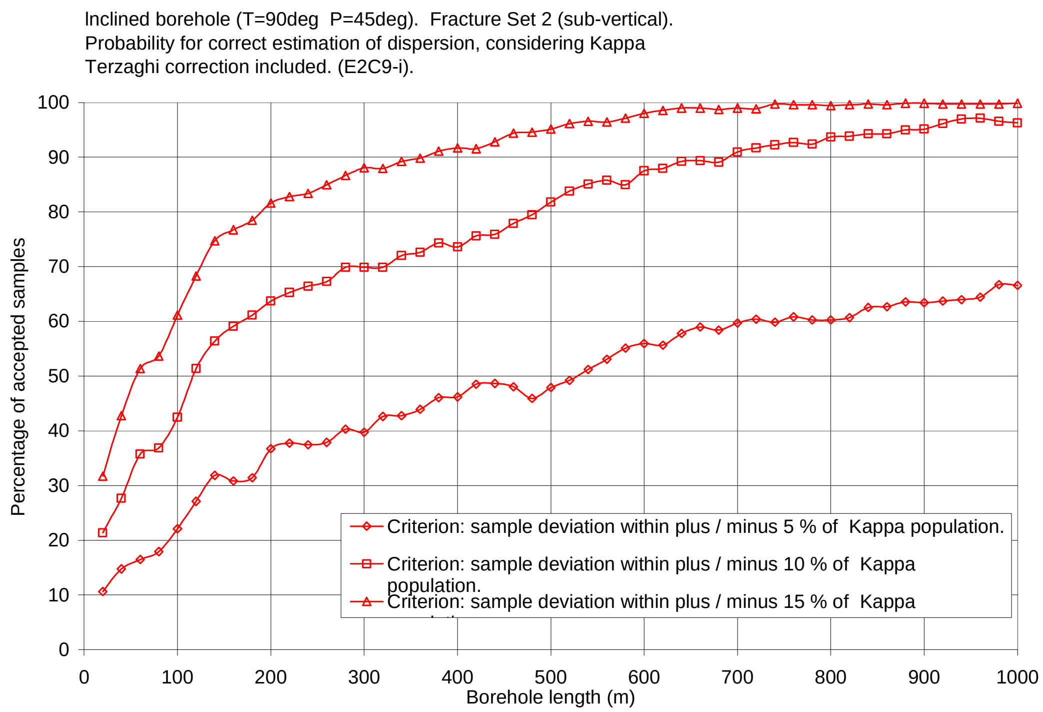


Figure 4-15. *Inclined borehole. Fracture set 2. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the kappa parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

Results for Set 3

Examples of results for Set 3 are as follows (see Figure 4-16): At a borehole length larger than 500 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (Kappa\_\ deviation\ <=15\%)}$). Or with other words, the probability that a sample deviates significantly considering $H_{0\ (Kappa\_\ deviation\ <=15\%)}$ is less than 10 percent, if the length of the borehole is larger than 500 meters. And finally, if the borehole has a length larger than 500 meters, the probability is larger than 90 percent that the deviation in estimated Kappa value is within plus/minus 15 percent of the true Kappa value of the population.

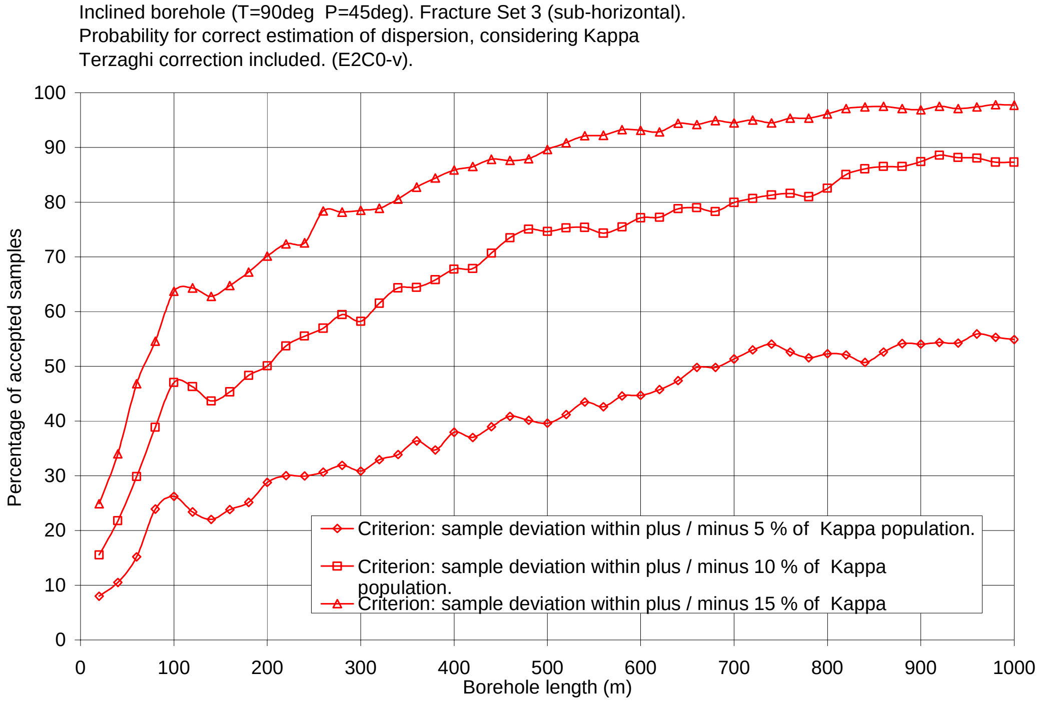


Figure 4-16. *Inclined borehole. Fracture set 3. Hypothesis testing for selected acceptable deviations in predicted dispersion, as given by the kappa parameter. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

4.4 Parametric hypothesis testing considering a confidence interval for the kappa parameter

4.4.1 Purpose

A test that assumes that the analysed population is distributed according to a known probability distribution is called a parametrical test. We have conducted such tests, and for these tests we have assumed that the orientations of the fractures of the population is distributed according to Fisher distributions. This is a correct assumption as the population was generated according to Fisher distributions. The purpose of these tests is to demonstrate the remaining bias of the sampling procedure (sampling in boreholes), the bias that remains after application of Terzaghi correction. This will be demonstrated by analysing the probability for a selected hypothesis of the properties of the population, to be rejected or accepted, at a certain selected level of confidence.
In this study the population is known, and the hypothesis of the properties of the population is set equal to the known true properties of the population. The test will tell us the probability for rejection or acceptance of this correct hypothesis of the rock mass, at a certain selected level of confidence. In this section confidence intervals will be used as a part of the hypothesis testing.

4.4.2 Confidence interval

A confidence interval for a parameter is an interval of values computed from a sample, which includes the unknown value of the parameter with some specified probability. The probability that a confidence interval will cover the unknown parameter value is the confidence level.

Hence, for a sample studied the confidence interval for the kappa parameter is centred on the kappa value derived from the sample. The size of an interval is governed by (i) some specified confidence level, (ii) by the calculated kappa value of the samples and by (iii) the number of fractures in the sample. In this study, the confidence intervals for the kappa parameter are calculated based on the assumption that the direction of the modal vector is unknown (mean direction of population is unknown).

Based on these assumptions, the sizes of the confidence intervals are calculated by use of methods given by /Fisher et al, 1987/, these methods are based on the work of the following authors: Initially /Fisher, 1953/ considered point estimate of mean direction and dispersion. /Watson, 1956/ and /Watson and Williams, 1956/ derived an exact procedure for calculation of a confidence cone for the mean direction and a procedure for calculation of an interval for kappa. /Stephens, 1962, 1967/ provided tables enabling the Watson and Williams procedure to be implemented, /Stephens, 1967/ also gives the theory and tables for exact interval estimation of kappa. General summaries of these procedures are given by /Mardia, 1972/.

Example of sizes of confidence intervals are demonstrated in Figure 4-17 and Figure 4-18. The figures demonstrates that the interval (acceptable deviation) depend on both the calculated kappa values of the samples as well as of the number of fractures in the sample, and of course by the given confidence level.

Selected acceptable deviations in kappa values were discussed in Section 4.3; for the first level, the acceptable deviation is plus/minus 15 percent of the kappa value of the population. Plus/minus 15 percent of kappa-population corresponds to the following interval lengths:

Set 1 = 1.5 Set 2=: 2.5 Set 3= 2.5.

A comparison between the intervals given above and the confidence intervals given in Figure 4-17 and Figure 4-18, reveals the following.

- At a confidence level of 95% the confidence intervals are smaller than the previously discussed acceptable deviation, presuming that the number of fractures in the sample is larger than: (i) approximately. 150 fractures if kappa of sample is 4.8 (Set 1) and (ii) approximately 160 fractures if kappa of sample is 8.3 (Set 2 and 3).

- At a confidence level of 99% the confidence intervals are smaller than the previously discussed acceptable deviation if the number of fractures in the sample is larger than: (i) approximately. 270 fractures if kappa of sample is.4.8 (Set 1) and (ii) approximately 290 fractures if kappa of sample is 8.3 (Set 2 and 3).

Hence, it is demonstrated by the two figures that in relation to the first level of selected acceptable deviations in kappa value, as discussed in Section 4.3, and for confidence levels less than 95% the confidence intervals are small for samples containing more than approximately 160 fractures.

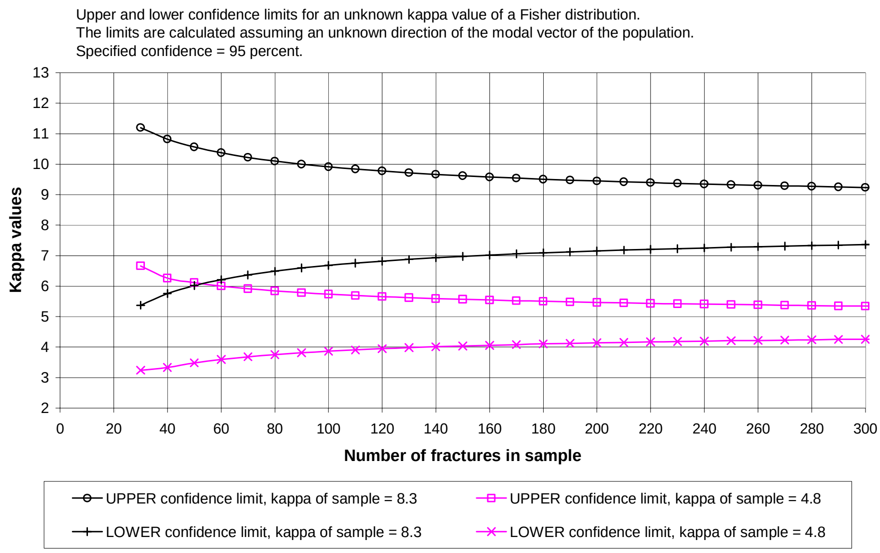


Figure 4-17. *Example of confidence interval for an unknown kappa value of a Fisher distribution. Specified confidence = 95%.*

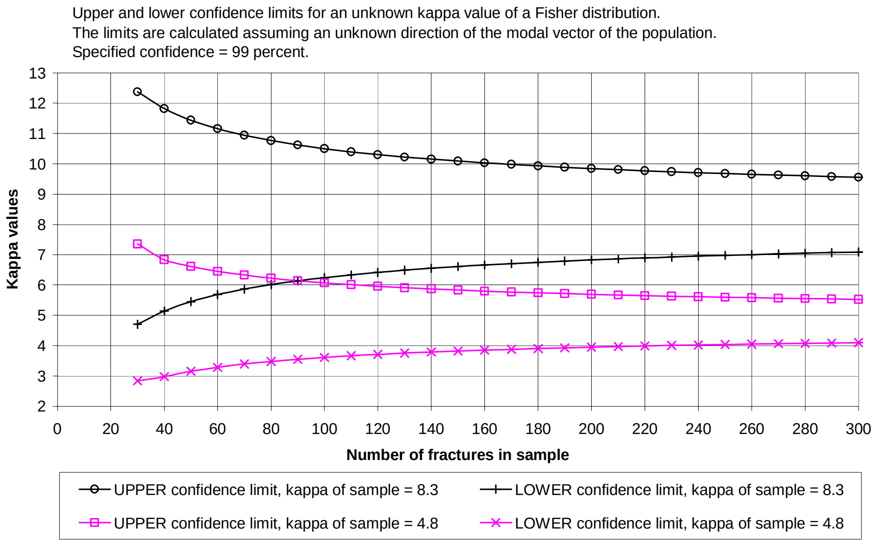


Figure 4-18. *Example of confidence interval for an unknown kappa value of a Fisher distribution. Specified confidence = 90%.*

4.4.3 Null hypothesis and level of confidence

The analysis of the point estimate of the dispersion (kappa) of the fracture orientation is carried out as a statistical hypothesis testing. The hypothesis testing is based on the sample variable studied (kappa) and given levels of confidence. The null hypothesis (H_0) is that the dispersion of the population, as estimated by the samples, are equal to the known true dispersion of the population. We know that this is a correct hypothesis, but due to sampling bias etc it will not necessarily be confirmed by the samples. For a studied sample, rejection of the hypothesis will take place if the known true kappa value of the population is outside of a confidence interval centred on the kappa value of the sample.

An illustration of the principles of the test is given in Figure 4-19. This figure presents confidence intervals at different borehole lengths for samples studied and the kappa value of the population (the hypothesis tested). For the presented case and confidence level, rejection of the hypothesis will take place for samples containing more than 320 fractures, because for samples of this size or larger, the upper limit of the confidence interval is less than the kappa value of the population (the hypothesis tested).

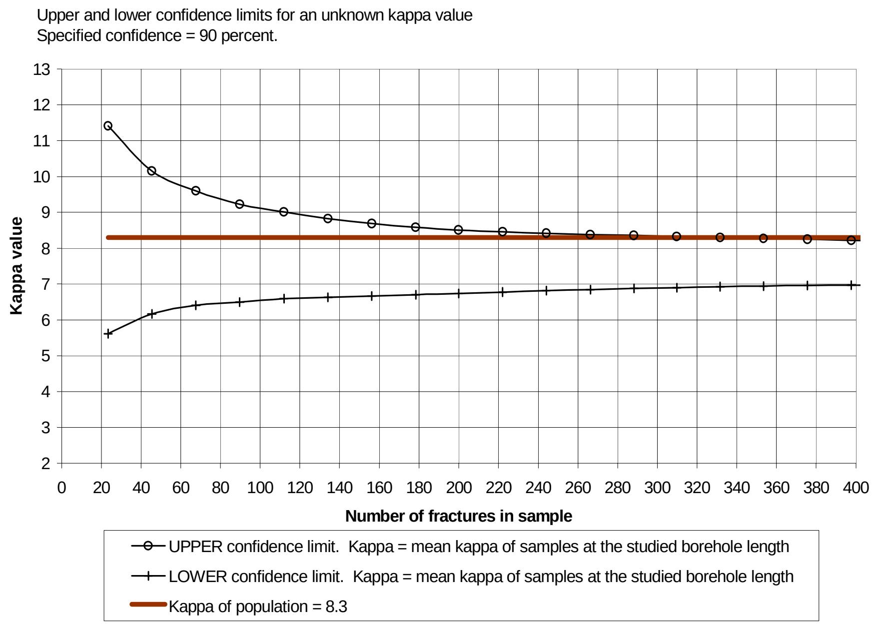


Figure 4-19. *Example demonstrating upper and lower confidence limits, as given by samples, and the test-value of an analysed hypothesis.*

The confidence level should be selected in a way that the probability for rejection of the hypothesis is small if the hypothesis is true. We have studied three different levels of confidence: 99, 99.9 and 99.99 percent. The hypothesis tests are as follows:

First confidence level 99% $H_{0\ (C=99\%)}$:

The hypothesis $H_{0\ (C=99\%)}$ is rejected if the kappa value of the population does not fall inside a confidence interval calculated for a confidence level of 99%.

Second confidence level 99.9% $H_{0\ (C=99.9\%)}$:

The hypothesis $H_{0\ (C=99.9\%)}$ is rejected if the kappa value of the population does not fall inside a confidence interval calculated for a confidence level of 99.9%.

Third confidence level 99.99% $H_{0\ (C=99.99\%)}$:

The hypothesis $H_{0\ (C=99.99\%)}$ is rejected if the kappa value of the population does not fall inside a confidence interval calculated for a confidence level of 99.99%.

For each confidence level, the result of the analysis is presented as the percentage of accepted samples at different borehole lengths.

4.4.4 Results

For these tests (Section 4.4), the acceptable deviation, as given by the confidence intervals, decreases as the number of fractures in a sample increases. Furthermore, the acceptable deviation (confidence interval) will also vary dependent on the calculated kappa value of the sample. Hence, the acceptable deviation is not a constant value. For small samples, the acceptable deviation is large, and for large samples, the acceptable deviation is small. The test will tell us the probability for rejection or acceptance of the studied hypothesis, for different borehole lengths, and at selected levels of confidence. (As previously stated, the hypothesis is that the dispersion of the population is equal to the known true dispersion; we know that this is a correct hypothesis, but due to sampling bias etc it will not necessarily be confirmed by the samples.) The efficiency of the point estimate of the kappa value increases with size of sample, but the confidence interval (acceptable deviation) decreases as the number of fractures in the sample increases. Therefore the percentage of accepted samples does not increase with borehole length, as for the previous tests of Chapter 4. Theoretically, if the samples were taken without sampling bias from perfect Fisher distributions, the probability for acceptance of the hypothesis should be equal to the confidence level, regardless of borehole length. The results for the confidence levels studied are given in Figure 4-20 through Figure 4-23.

The results show a large number of rejected samples. This is a consequence of a systematic bias in the point estimate of the kappa value. This bias follows from the fact that a borehole is a one-dimensional line that samples a three-dimensional fracture network. The applied Terzaghi correction, which removes most of this bias, is not perfect and some aspects of the bias remain in the samples.

Results for vertical borehole

The results demonstrate that for Set 1 and 2, the probability for rejection of the correct hypothesis is larger than the prescribed level (100%-confidence level), at the confidence levels of 99 percent and 99.9 percent. However for Set 3, which is not very much influenced by sampling bias (as it is a sub-horizontal fracture set sampled by a vertical borehole) the probability for rejection of the correct hypothesis is close to the theoretically expected value.

<u>At a confidence level of 99 percent (Figure 4-20):</u>

For Set 1 and Set 2, the probability for acceptance of the hypothesis is between 70 and 85 percent, regardless of borehole length. For Set 3 the probability for acceptance of the hypothesis is close to 96–97 percent regardless of borehole length. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99 percent.

<u>At a confidence level of 99.9 percent (Figure 4-21):</u>

For Set 1 and Set 2, the probability for acceptance of the hypothesis is between 80 and 90 percent regardless of borehole length. For Set 3 the probability for acceptance of the hypothesis is close to 99 percent regardless of borehole length. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99.9 percent.

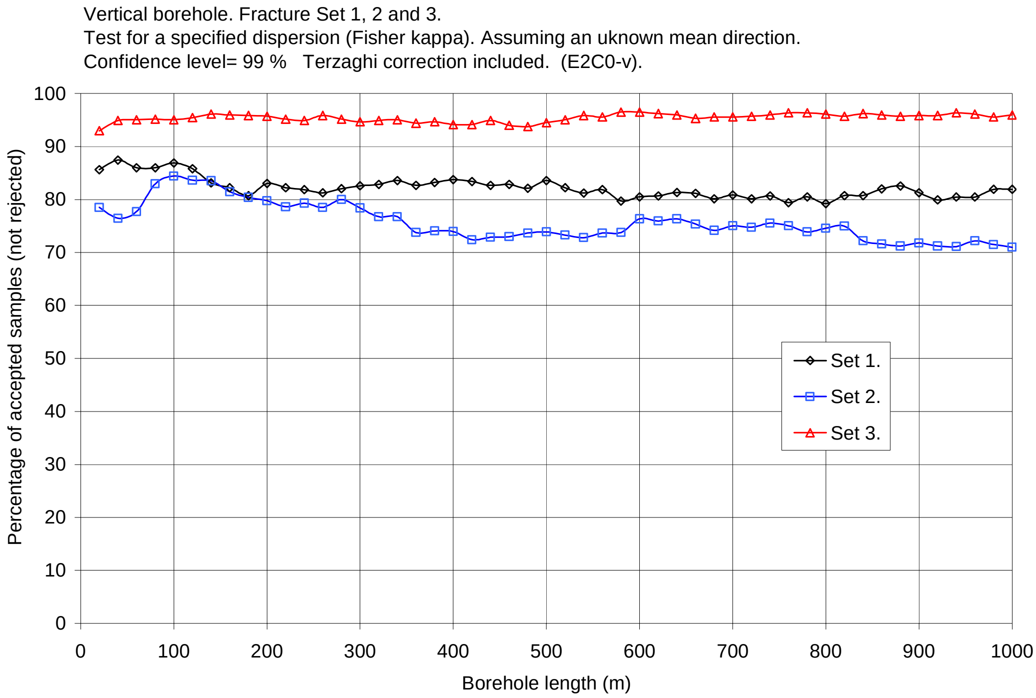


Figure 4-20. *Vertical borehole. Hypothesis testing considering dispersion (kappa-values) of fracture sets, by use of confidence intervals. The figure gives the percentage of accepted samples (probability for an accepted sample). Tested hypothesis: kappa value of the fracture set studied is equal to the true value of the population. Confidence level is 99 percent.*

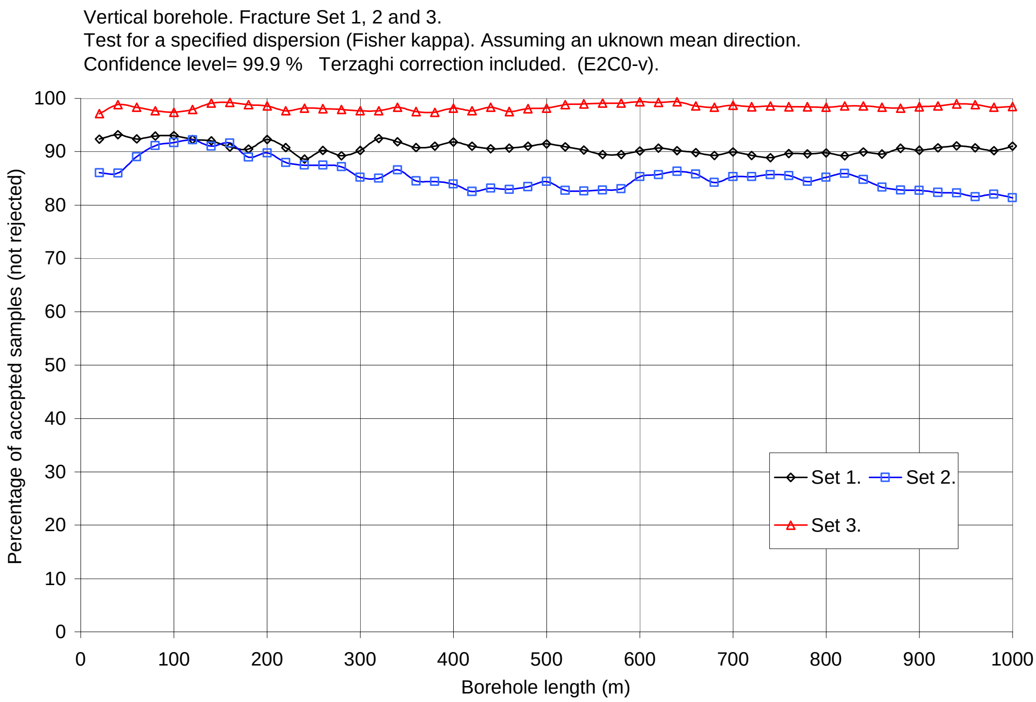


Figure 4-21. *Vertical borehole. Hypothesis testing considering dispersion (kappa-values) of fracture sets, by use of confidence intervals. The figure gives the percentage of accepted samples (probability for an accepted sample). Tested hypothesis: kappa value of the fracture set studied is equal to the true value of the population. Confidence level is 99.9 percent*

Results for inclined borehole

The results demonstrate that for all three sets, the probability for rejection of the correct hypothesis is larger than the prescribed level (100%-confidence level), at the confidence levels of 99 percent and 99.9 percent.

<u>At a confidence level of 99 percent (Figure 4-22):</u>

For all sets, the probability for acceptance of the hypothesis is between 70 and 90 percent, regardless of borehole length. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99 percent.

<u>At a confidence level of 99.9 percent (Figure 4-23):</u>

For all sets, the probability for acceptance of the hypothesis is between 90 and 95 percent, regardless of borehole length. Theoretically, if the samples were taken without sampling bias, the probability for acceptance of the hypothesis should be equal to the confidence level, which is 99.9 percent.

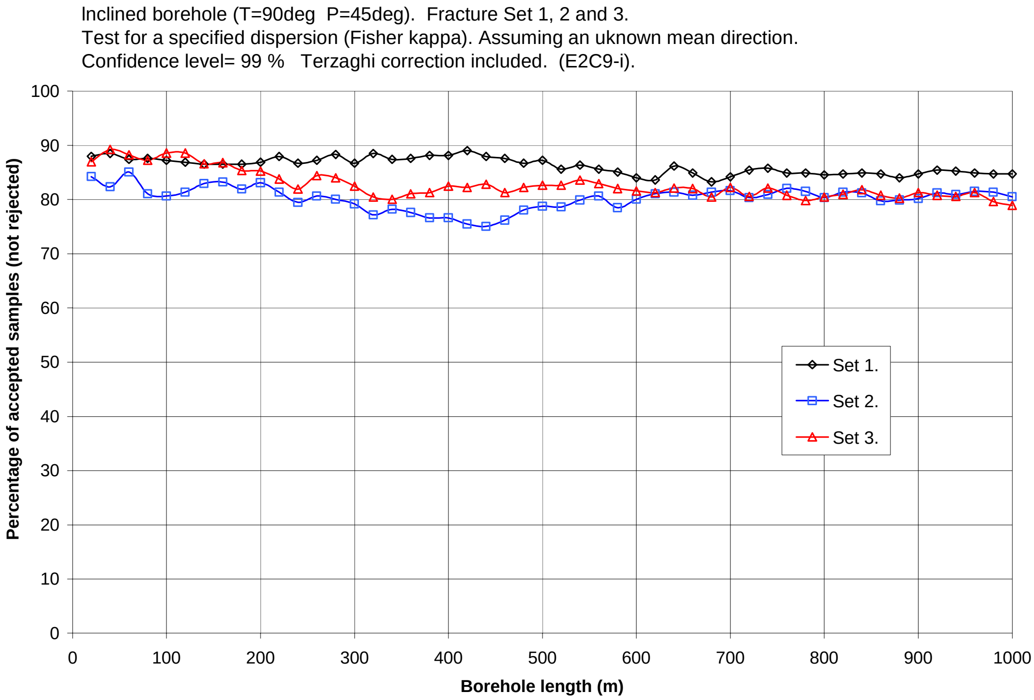


Figure 4-22. *Inclined borehole. Hypothesis testing considering dispersion (kappa-values) of fracture sets, by use of confidence intervals. The figure gives the percentage of accepted samples (probability for an accepted sample). Tested hypothesis: kappa value of the fracture set studied is equal to the true value of the population. Confidence level is 99 percent*

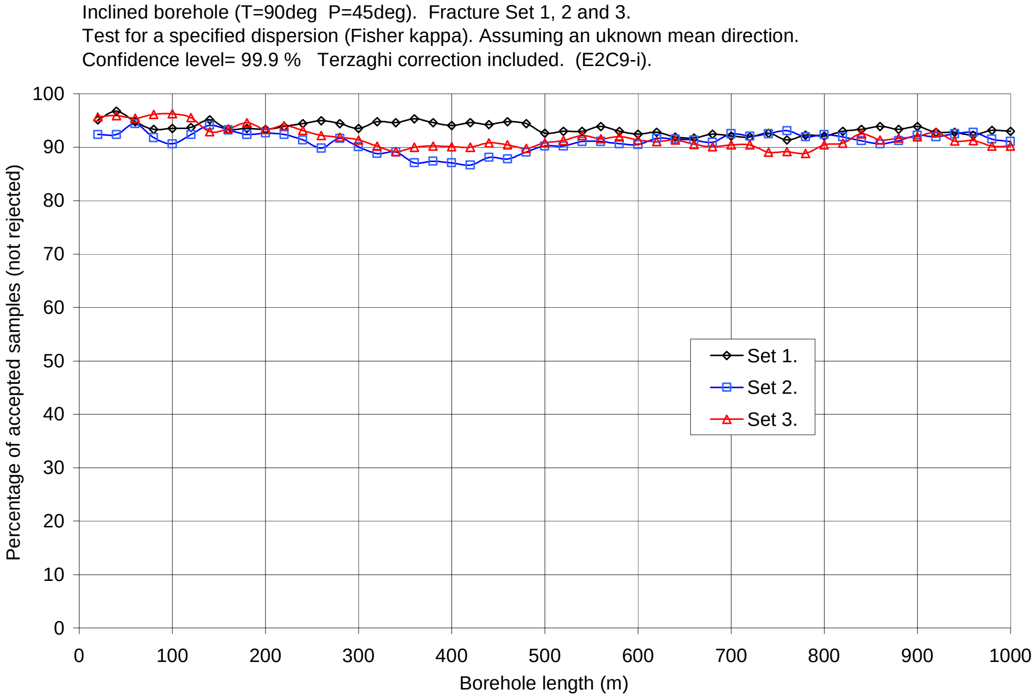


Figure 4-23. *Inclined borehole. Hypothesis testing considering dispersion (kappa-values) of fracture sets, by use of confidence intervals. The figure gives the percentage of accepted samples (probability for an accepted sample). Tested hypothesis: kappa value of the fracture set studied is equal to the true value of the population. Confidence level is 99.9%*

5 Estimation of fracture density from boreholes and rock surfaces

5.1 Measures of fracture density: *P10*, *P21* and *P32*

The number and sizes of the fractures in the rock mass are commonly described as the fracture density (or fracture intensity). Three different parameters are used to describe the fracture density: *P10*, *P21* and *P32*.

- The one-dimensional density is given by the *P10* parameter; it is equal to *number of fractures per unit length*, taken along a straight line.
- The two-dimensional density is given by the *P21* parameter; it is equal to *fracture trace-length per unit surface area*, taken over a surface.
- The three-dimensional density is given by the *P32* parameter; it is equal to *fracture surface area per unit rock volume*, taken over a volume.

The *P10* value is often called the fracture frequency and given as fractures per metre. As such a measure the *P10* value is included in estimations of rock mechanical properties. Both the *P10* and *P21* depend on the orientation of the line or plane considered for sampling of the fracture network (except in the case of an isotropic fracture network); therefore the most interesting parameter is the *P32*, which considers a volume and not a sampling line or a plane. Based on observations in boreholes and on rock surfaces (e.g. rock outcrops) it is possible to estimate the *P10* and *P21* parameters. It is however a more complicated to estimate the *P32* parameter as normally no direct observations can be made of the complete extension of fracture surfaces inside a studied volume of rock. Consequently, when estimating the *P32* parameter it has to be calculated based on other measurable properties.

For fracture networks in which the fracture orientations are not uniformly random, there is no simple direct relationship between these three density parameters. This is because the different density parameters describe the fracture density in different number of dimensions, and for each new dimension added, additional information of the fracture system is needed for calculation of the corresponding density parameter. However, when modelling fracture networks, the quota between modelled properties (*MODEL*) and true properties (*TRUE*) are the same for all tree parameters, as given below.

$$\frac{P10_{TRUE}}{P10_{MODEL}} = \frac{P21_{TRUE}}{P21_{MODEL}} = \frac{P32_{TRUE}}{P32_{MODEL}} \qquad \text{5-1}$$

This relationship can be used when analysing fracture networks. In numerical models properties of the rock mass can be estimated by a trial and error procedure, based on the equation above.

Another measure of fracture density that may look attractive is the number of fractures per unit surface area (*P20*), it is however not a very convenient measure as it is scale dependent. For a given fracture network, the number of fractures per unit surface area will decrease with size of the window studied.

5.2 Complete description of fracture network

The distribution of fracture orientation (direction and dispersion) together with the *P32* value is not enough to obtain a unique description of the fracture network of the rock mass. We also need information of the fracture size distribution, because a large number of small fractures may give raise to the same *P32* value as a small number of large fractures. (A complete model of the fracture network is also given by the distribution of fracture direction and dispersion together with the fracture size distribution and the number of fractures.) In addition to the above discussed, the fractures of the network may also be spatially correlated, if so the spatial correlation also needs to be analysed and quantified. In this study the fractures of the network has no spatial correlation and consequently no such analyses have been carried out.

5.3 Point estimate and test considering *P10* (fracture frequency) and boreholes

5.3.1 Introduction

The *P10* parameter is estimated from sampling of fractures along a straight line, i.e. a scan line or sampling line. The sampling line can be applied along a borehole, but it can also be applied along the surface of a rock outcrop or along a rock walls etc. However, in this study we have only considered *P10* values calculated from observations in boreholes. The *P10* value is often called the fracture frequency and given as fractures per metre. As such a measure the *P10* value is included in estimations of rock mechanical properties (e.g RQD, RMR, Q, etc). It should however be noted that the DFN-networks analysed in this study (the DFN 2 model of the Prototype Repository) were primarily aimed at representing the hydrogeological properties of the fractured rock mass and not necessarily the rock mechanical properties.

5.3.2 Point estimate of the *P10* value of the population

The fractures that intersect the studied borehole are samples of the fracture population. The properties of the samples are estimates of the properties of the population. The observed fractures are classified into three groups, one group for each fracture set. After the classification each fracture set is studied one by one, separate from the other sets. The test presented below is conducted for each fracture set separately.

The *P10*-parameter is conceptually different from the parameters defining the orientation and dispersion of the fracture sets studied. Orientation and dispersion are three-dimensional properties of the rock mass, whereas the *P10*-parameter is a one-dimensional property. As the *P10*-parameter is a one-dimensional property it depends on the direction of the one-dimensional sampling line used for determining the parameter. Hence the variation of the *P10*-parameter with orientation of sampling line is a true property of the rock mass and not a systematic sampling error.

The *P10* value of a sample is calculated as follows: the number of fractures in the sample (observed in the borehole) divided by the length of the studied section (length of the borehole). From a statistical point of view, the analysis is a point estimate of the variable *P10* and this variable is a function of the properties of the samples. The efficiency of the point estimate increases with the size of the sample (number of observed fractures) and the size of the sample increases with the length of the borehole (studied section). This is demonstrated in Figure 5-1.

Considering the vertical borehole, the point estimate produces the following results at a borehole length of 1000 metres:

Set 1: Mean *P10* = 0.29 Standard deviation *P10* = 5.5% of Mean *P10*

Set 2: Mean *P10* = 0.43 Standard deviation *P10* = 4.5% of Mean *P10*

Set 3: Mean *P10* = 0.85 Standard deviation *P10* = 3.4% of Mean *P10*

All Sets Mean *P10* = 1.57

Considering the inclined borehole, the point estimate produces the following results at a borehole length of 1000 metres:

Set 1: Mean *P10* = 0.46 Standard deviation *P10* = 4.9% of Mean *P10*

Set 2: Mean *P10* = 0.73 Standard deviation *P10* = 3.8% of Mean *P10*

Set 3: Mean *P10* = 0.55 Standard deviation *P10* = 4.3% of Mean *P10*

All Sets Mean *P10* = 1.74

The change in mean *P10* values with increasing borehole length is small for boreholes longer than 100 metres. It is possible to express this change as the derivative of *P10* with respect to borehole length (*L*). The derivative is approximated by a first order backward finite difference and calculated as follows.

$$\frac{\partial P10}{\partial L} \approx \frac{\Delta P10}{\Delta L} = \frac{P10_{(L)} - P10_{(L-\Delta L)}}{\Delta L}$$

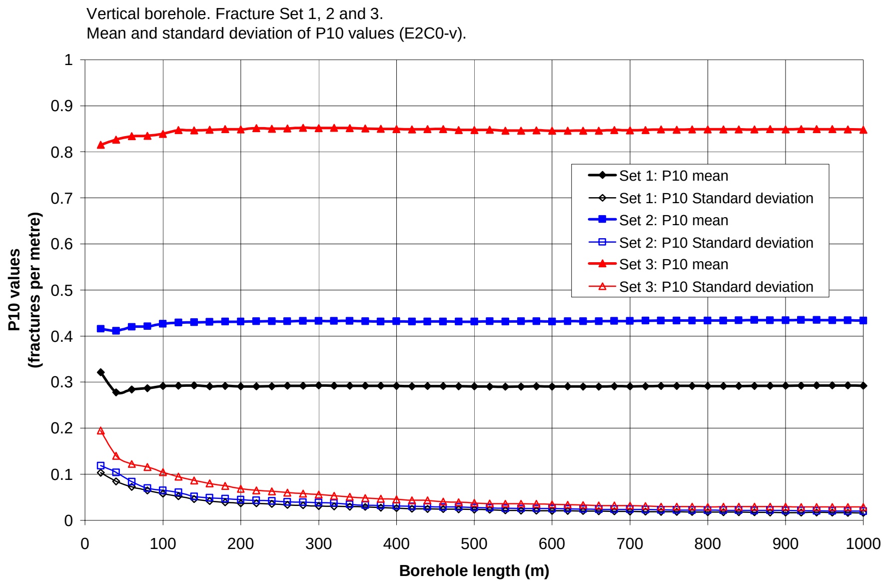
Considering the vertical borehole , the average derivative in mean *P10* with respect to borehole length, for the last five borehole lengths, is as follows.

(i) Set 1: $1.5\text{x}10^{-6}$, (ii) Set 2: $-7.3\text{x}10^{-6}$ and (iii) $-7.4\text{x}10^{-6}$

Considering the inclined borehole , the average derivative in mean *P10* with respect to borehole length, for the last five borehole lengths, is as follows

(i) Set 1: $-6.9\text{x}10^{-6}$, (ii) Set 2: $-1.1\text{x}10^{-5}$ and (iii) $-1.4\text{x}10^{-5}$

These results demonstrate that the mean *P10* values at 1000 metre of borehole are very stable, and the change in *P10* values that will come with longer boreholes are negligible. This is of interest, as we have no knowledge of the true *P10* values of the population. In the test below we will set the true *P10* values of the population as equal to the mean *P10* values at 1000 metre of borehole. By doing this we assume that the point estimate converges towards the true *P10* value of the population



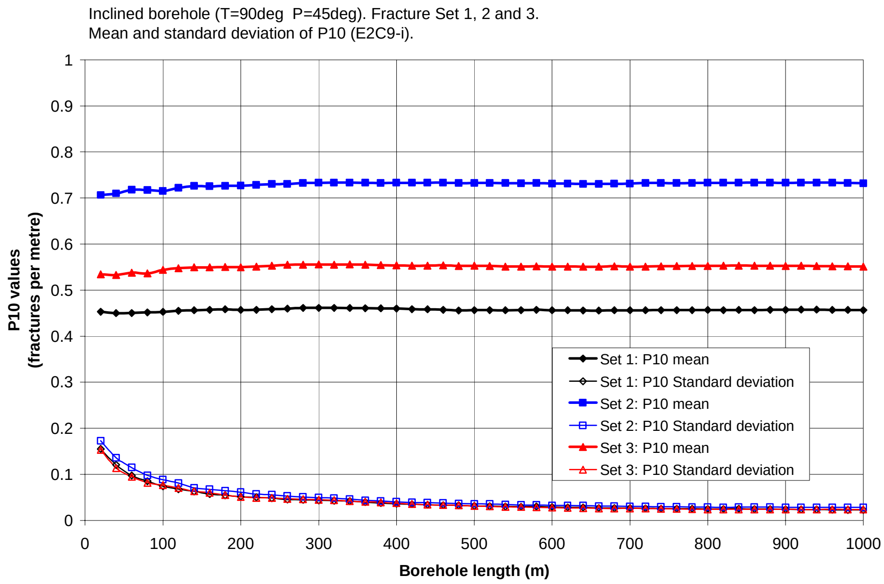


Figure 5-1. *Efficiency of the point estimate of mean and standard deviation of P10 values. The upper figure gives results for a vertical borehole; the lower figure gives results for an inclined borehole.*

5.3.3 Hypothesis testing considering *P10* and acceptable deviations

Purpose of test

The purpose of this test is to determine when the size of a sample is large enough to produce an acceptable estimate of the *P10* parameter of the population studied, with a certain probability. This can also be stated in the following way: the calculation of the sample size that is necessary to reach a confidence level, considering a given confidence interval. The confidence interval is the same thing as a test criterion (an acceptable deviation). The sample size corresponds to length of borehole.

Null hypothesis, acceptable deviations and criterion of significance

The samples were analysed by a statistical hypothesis testing. The hypothesis testing is based on the variable *P10* and given criterions of significance. A difficulty is that we do not know the correct *P10* value of the population studied. The established criterions of significance will therefore correspond to the mean *P10* value derived from the largest samples, called the simulated true *P10* value. This is an acceptable method as such mean values are very stable, as discussed above. It is however necessary to remember that when establishing criterions of significance in this way, we also assumes that the point estimate is not biased, and this is only the case if no sampling errors occur

The null hypothesis (H_0) is that a sample is a good representation of the true properties of the population. This hypothesis is accepted if not a significant deviation takes place in the *P10* value of the sample. The following criterions are used. The three criterions represent three different levels of significance.

First criterion: $H_{0\ (P10C\_deviation <=15\%)}$ is true if:

$$ABS[P10_{(sample)} - P10_{(mean\ sample\ at\ 1000m)}] <= 0.15 * P10_{(mean\ sample\ at\ 1000m)}$$

Second criterion: $H_{0\ (P10C\_deviation <=10\%)}$ is true if:

$$ABS[P10_{(sample)} - P10_{(mean\ sample\ at\ 1000m)}] <= 0.10 * P10_{(mean\ sample\ at\ 1000m)}$$

Third criterion: $H_{0\ (P10C\_deviation <=5\%)}$ is true if:

$$ABS[P10_{(sample)} - P10_{(mean\ sample\ at\ 1000m)}] <= 0.05 * P10_{(mean\ sample\ at\ 1000m)}$$

The results of the analysis are presented as the probability that a sample, at a certain borehole length, will fulfil the hypothesis considering three different criterions.

5.3.4 Results

Results considering a vertical borehole

The results are given in the figures below, for the three fracture sets and for the three different levels of significance.

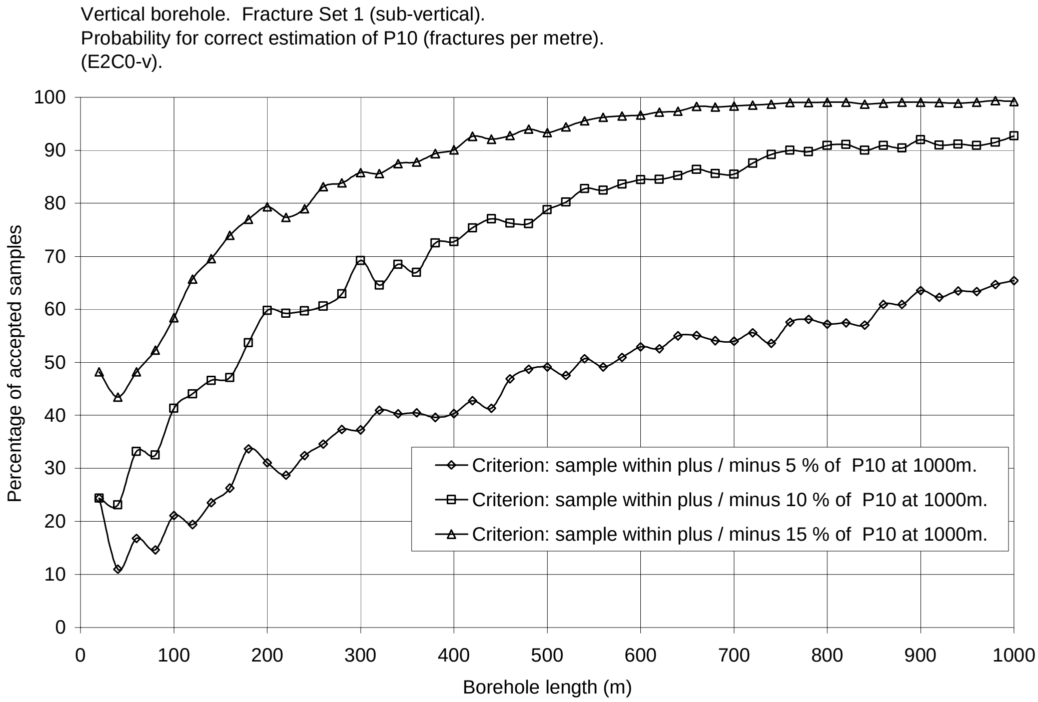


Figure 5-2. *Vertical borehole. Set 1. Hypothesis testing for selected acceptable deviations in predicted P10 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

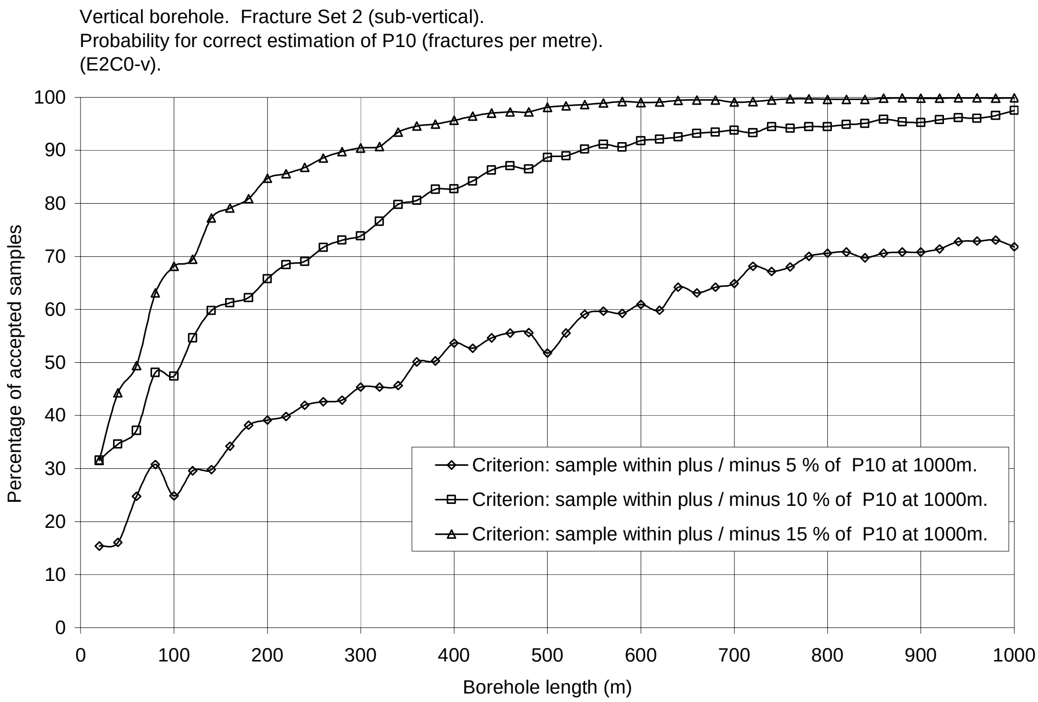


Figure 5-3. *Vertical borehole. Set 2. Hypothesis testing for selected acceptable deviations in predicted P10 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

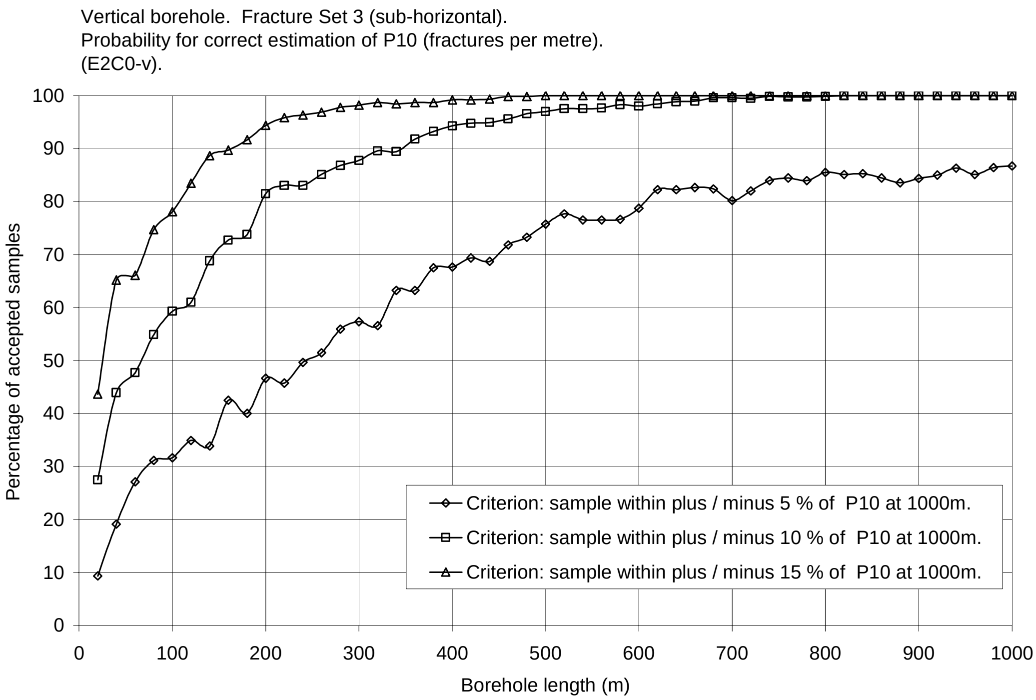


Figure 5-4. *Vertical borehole. Set 3. Hypothesis testing for selected acceptable deviations in predicted P10 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

- Examples of results for Set 1 are as follows (see Figure 5-2): For a vertical borehole with a length larger than 400 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P10\_\ \mathrm{deviation}\ <=15\%)}$). If the borehole has a length larger than 400 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population.

- Examples of results for Set 2 are as follows (see Figure 5-3): For a vertical borehole with a length larger than 300 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P10\_\ \mathrm{deviation}\ <=15\%)}$). If the borehole has a length larger than 300 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population.

- Examples of results for Set 3 are as follows (see Figure 5-4): For a vertical borehole with a length larger than 150 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P10\_\ \mathrm{deviation}\ <=15\%)}$). If the borehole has a length larger than 150 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population.

Results considering an inclined borehole

The results are given in the figures below, for the three fracture sets and for the three different levels of significance.

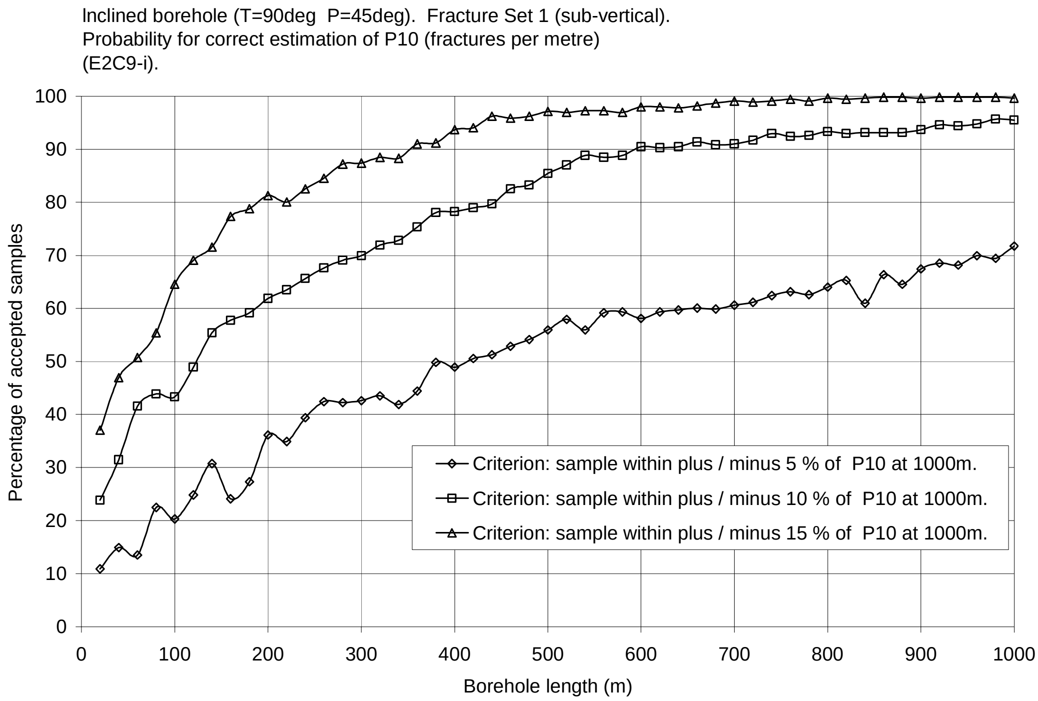


Figure 5-5. *Inclined borehole. Set 1. Hypothesis testing for selected acceptable deviations in predicted P10 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

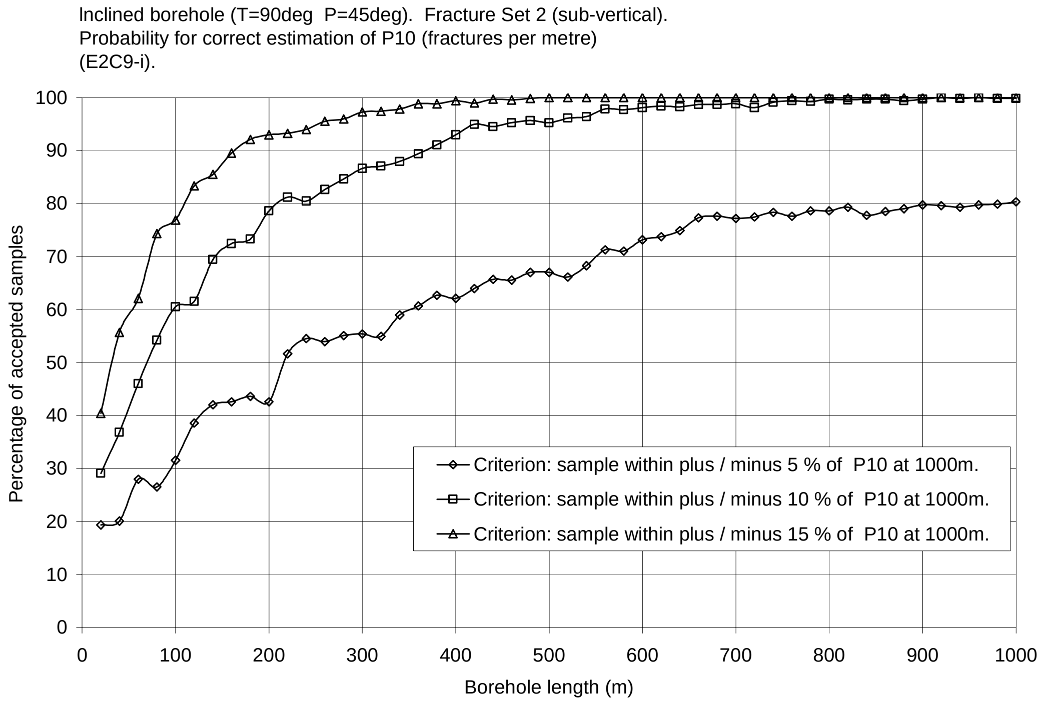


Figure 5-6. *Inclined borehole. Set 2. Hypothesis testing for selected acceptable deviations in predicted P10 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

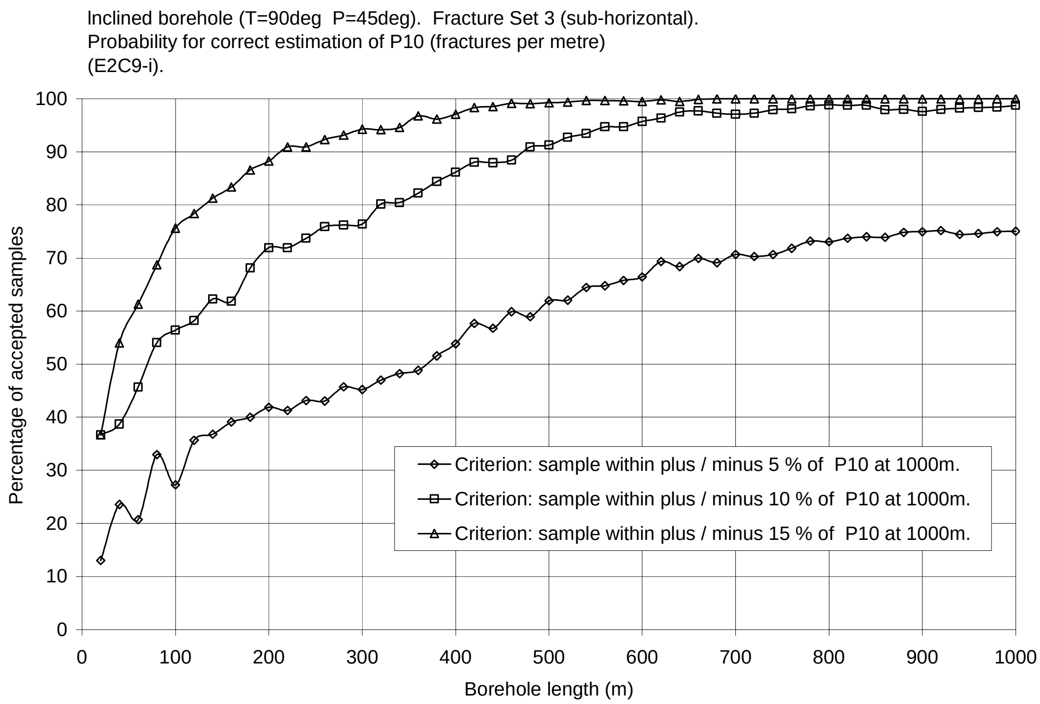


Figure 5-7. *Inclined borehole. Set 3. Hypothesis testing for selected acceptable deviations in predicted P10 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions*

- Examples of results for Set 1 are as follows (see Figure 5-5): For an inclined borehole with a length larger than 350 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P10\_deviation\ <=15\%)}$). If the borehole has a length larger than 350 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population.

- Examples of results for Set 2 are as follows (see Figure 5-6): For an inclined borehole with a length larger than 150 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P10\_deviation\ <=15\%)}$). If the borehole has a length larger than 150 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population.

- Examples of results for Set 3 are as follows (see Figure 5-7): For an inclined borehole with a length larger than 210 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P10\_deviation\ <=15\%)}$). If the borehole has a length larger than 210 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population.

5.4 Point estimate and test considering *P21* and horizontal rock surfaces

5.4.1 Introduction

The number and length of the fracture traces on a rock surface, is linked to the fracture density of the fracture population of the rock mass behind the surface. The two-dimensional fracture density is given by the *P21* parameter and it is defined as the *fracture trace-length per unit surface area*, taken over a surface. Based on the number and length of observed fracture traces on a surface it is possible to estimate the *P21* parameter of the rock mass, with consideration of the properties of the studied surface.

The DFN-model used in this study (as the base case) is the DFN 2 model presented in /Hermanson et al, 1999/. The main objective of the DFN 2 modelling was to establish a discrete fracture network model, representing the rock mass at the Prototype Repository, which could be used for simulation of groundwater flow. The DFN 2 model underestimates the total number of fractures in the rock mass at the Prototype Repository, as small fractures with minor or negligible hydraulic importance is not included in the model. We have therefore established an alternative DFN-model, which includes a larger number of small fractures, but has the same value of fracture density (*P32*-value). For this alternative DFN-model, the results considering the necessary sample sizes for reliable estimation of the *P21* parameter is slightly different. The results of the alternative model are presented in Chapter 8 (Limited Sensitivity Analysis).

5.4.2 Methodology

P21 is estimated from observations of fracture traces on rock surfaces (also called mapping of fracture traces on windows). In this study the analysed rock surface windows have been simulated based on the same DFN-network as was used for the analysis of the boreholes. The windows were analysed for total trace-length and distribution of trace-length. Examples of simulated fracture traces on circular horizontal windows are given in Figure 2-5.

The *P21* parameter is conceptually different from the parameters defining the orientation and dispersion of the fracture sets studied. Orientation and dispersion are three-dimensional properties of the rock mass, whereas the *P21*-parameter is a two-dimensional property. As the *P21*-parameter is a two-dimensional property it depends on the orientation and shape of the two-dimensional sampling surface (the window) used for determining the parameter. Hence, the variation of the *P21*-parameter with orientation and shape of sampling surface is a true property of the rock mass and not a systematic sampling error.

In this study, for eliminating the variation of the *P21*-parameter with shape of sampling surface, all the analysed trace-windows are of circular shape. The *P21*-parameter will however also vary with inclination (orientation) of the trace window; in this study all trace-windows are along the horizontal plane, e.g. corresponding to horizontal rock outcrops.

The *P21* estimate is calculated as follows:

$$P21 = \frac{L_{tot}}{A_w} \qquad \text{5-2}$$

L_{tot} = *Total length of all fracture traces on trace-window studied (Length).*

A_w = *Area of trace-window studied (Length^2)*

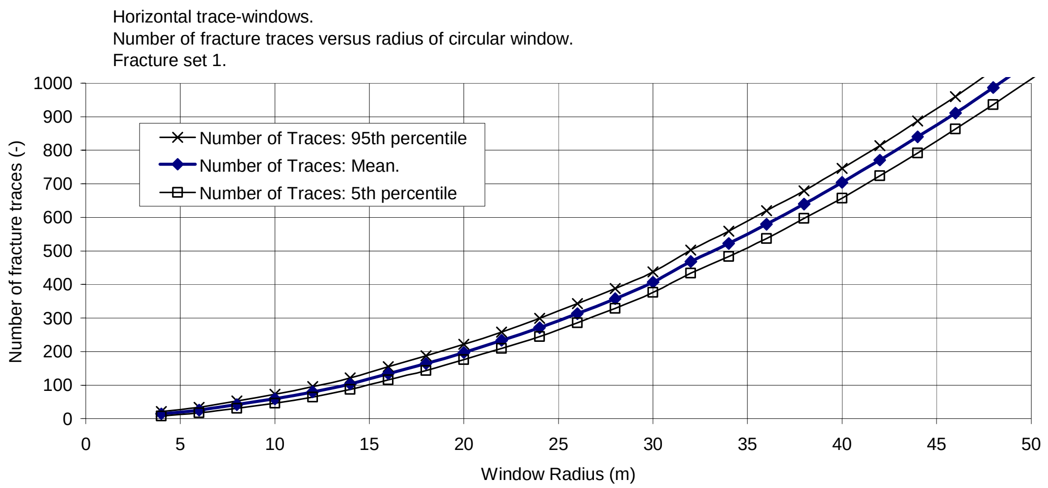
When analysing the length of fracture traces, a difficulty is how to handle fracture traces that continues outside of the studied window – traces with terminations that are not observable. In this study these fracture traces are included in the analysis and their observed length (censored length), within the window studied, gives their length. Fracture traces that terminate outside of the window studied are called *boundary-truncated traces* and their proportion to the total number of traces is analysed in Sec 5.4.7.

Another problem that may arise when mapping fracture traces is that fracture traces that are smaller than a certain size will not be observed correctly. Such traces are called *size-truncated traces*. In this study the analysed fracture traces are numerically generated and the truncation limit, regarding small traces, is set as small. Traces smaller than 0.025 metre are truncated and excluded from the analyses.

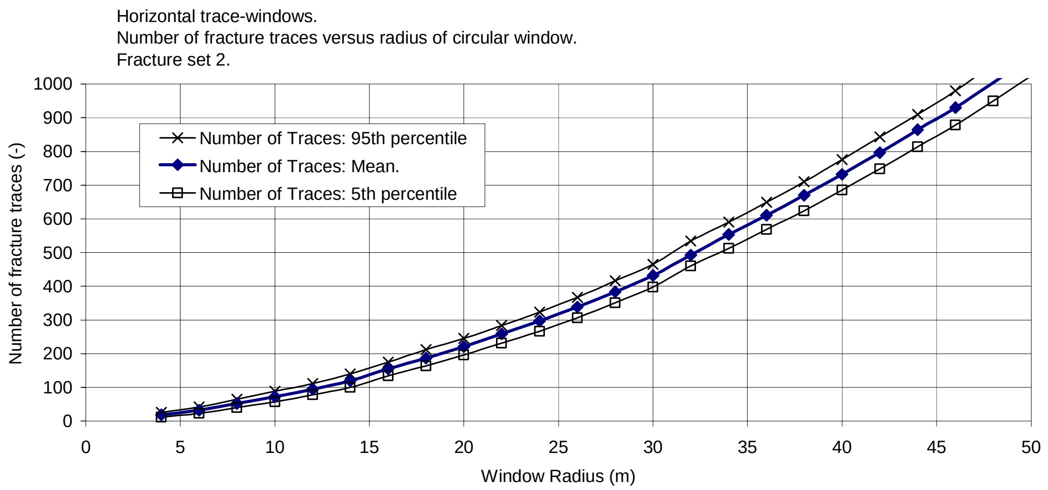
Another cause for bias when in practice estimating a *P21* values is that the localisation of the two-dimensional surface (the window studied) is not picked at random, but given by circumstances that will influence the observed fracture size distribution. For example it is likely that naturally occurring rock outcrops corresponds to rock masses with a higher resistance to weathering etc than the average rock mass, and it follows that it is likely that such rock masses carries fewer fractures than the average rock mass. This is however not a cause for bias in this study, as all the windows studied are randomly generated and the properties observed are unbiased as regards the quality of rock mass.

5.4.3 Number of fracture traces on a horizontal circular window

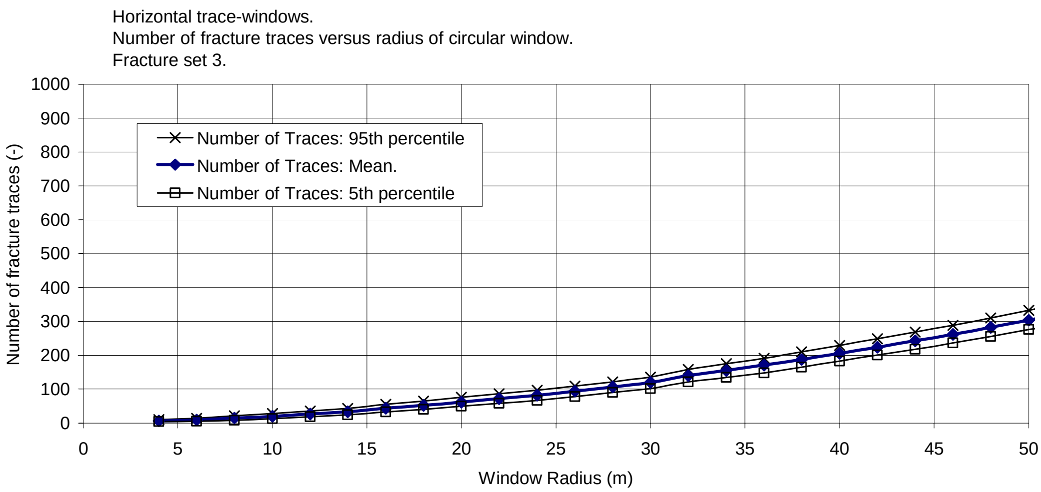
The average number of fracture traces on a surface varies with properties of the fracture set studied, but also on the orientation, shape and size of the window studied. In addition to this the average number of traces on a surface does not vary in a linear way with radius (or side) of a window. This follows from the relationship between area and radius (or side), which is not a linear relationship. The number of traces on a surface is of interests as the point estimates depend on the sample size, and the number of traces observed on the windows studied give the sample size. Considering the windows studied, which are horizontal and circular, the average number of traces, as well as the 5th and 95th percentiles, are given in Figure 5-8, below.



SET 1



SET 2



SET 3

Figure 5-8. *Number of fracture traces on circular horizontal windows, considering the three fracture sets studied.*

5.4.4 Point estimate of the *P21* value of the population

The fracture traces that occur on the trace-windows studied are samples of the properties of the fracture population. The properties of the sample are estimates of the properties of the population.

In this study each fracture was marked with its proper set identity since this is known at the generation of the fracture. In a real situation, different methods and algorithms for identifying and delimiting sets will be necessary to ensure objective set identifications. In the analysis presented below, the fracture traces are divided into three different sets, based on the known Set ID of each fracture that creates a trace. The results of the analysis are given for each fracture sets separately.

From a statistical point of view, the analysis is a point estimate of the variable *P21* and this variable is a function of the properties of the samples. The efficiency of the point estimate increases with the size of the sample and the size of the sample increases with the size of the trace-window. This is demonstrated in Figure 5-9, Figure 5-10 and Figure 5-11.

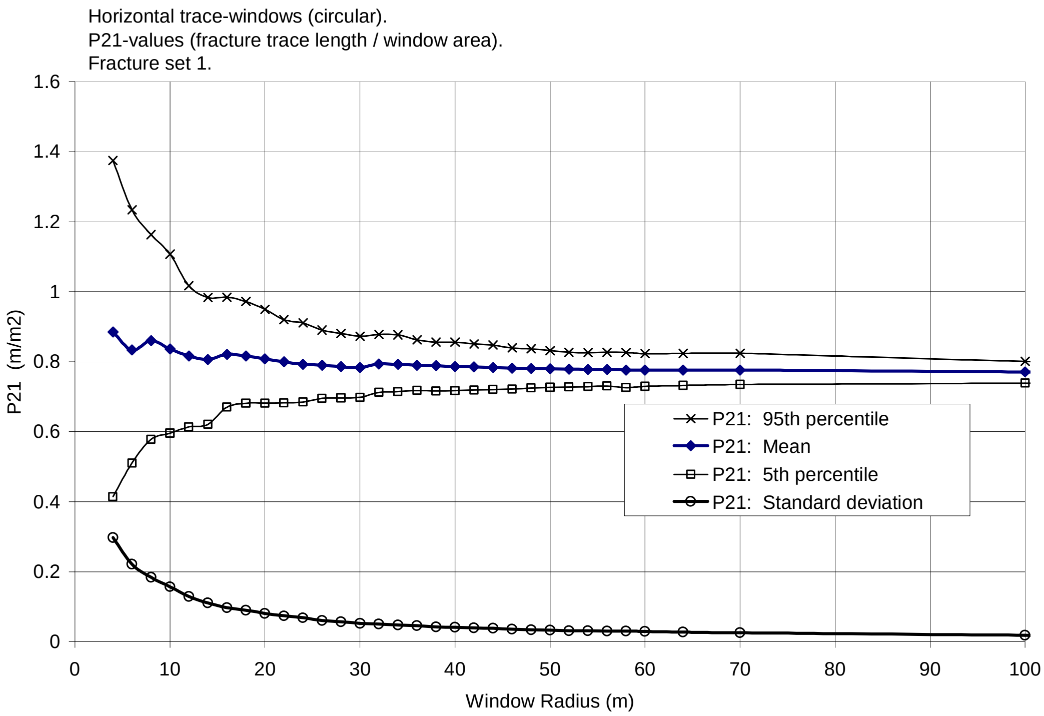


Figure 5-9. *Fracture set 1. The efficiency of the point estimate of the P21 parameter by use of horizontal circular windows (rock outcrops).*

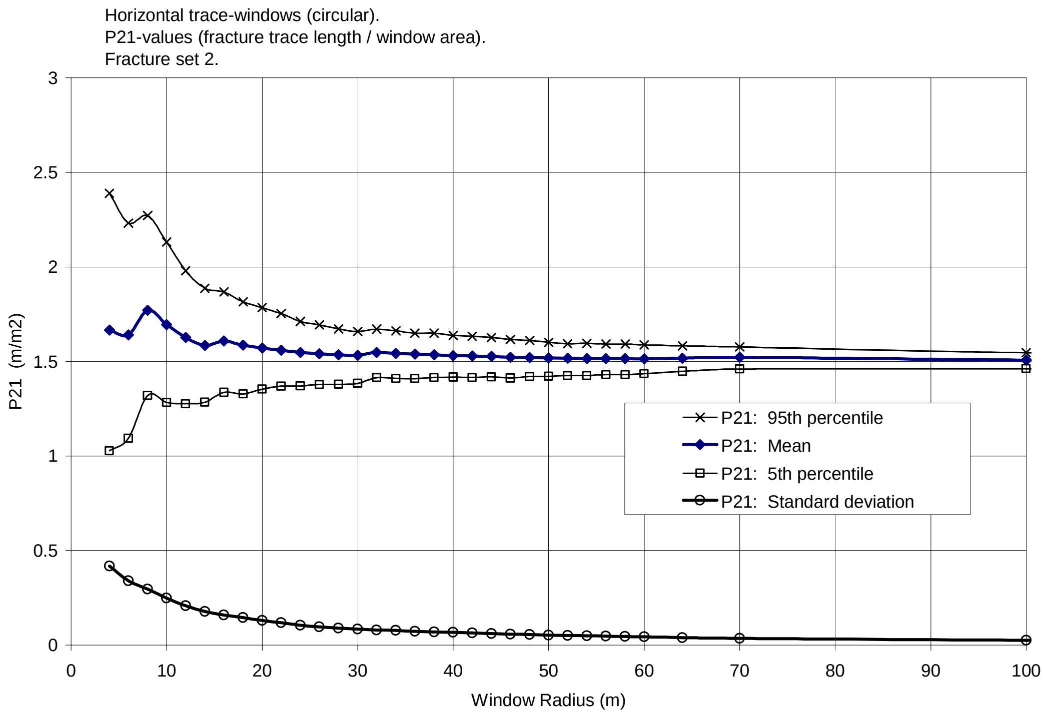


Figure 5-10. *Fracture set 2. The efficiency of the point estimate of the P21 parameter by use of horizontal circular windows (rock outcrops)*

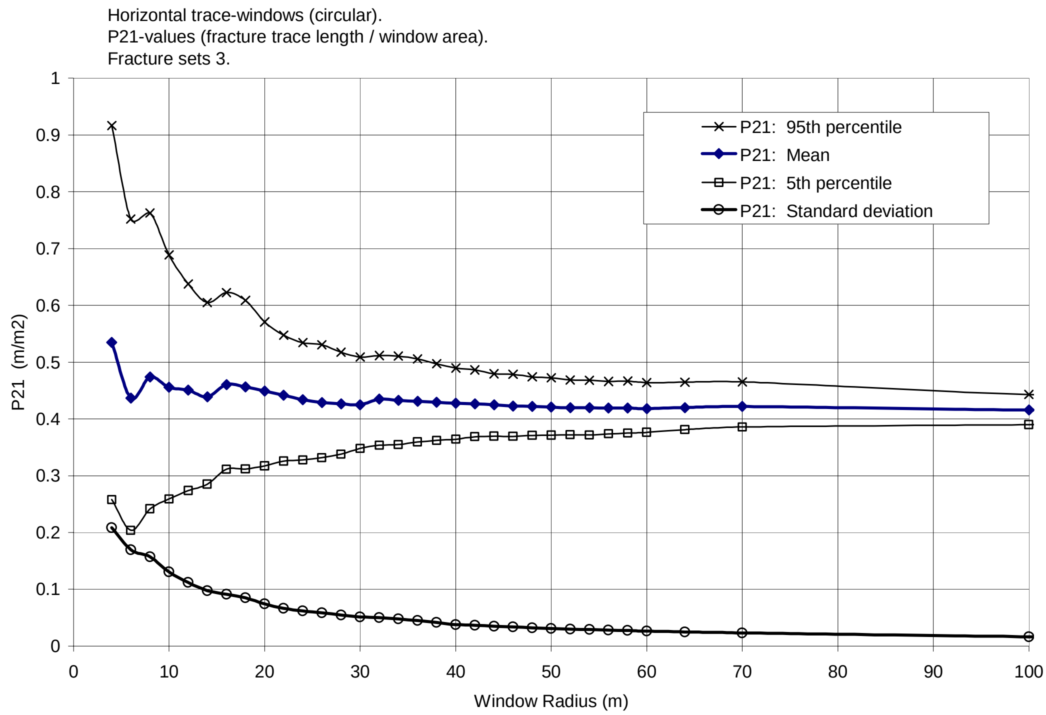


Figure 5-11. *Fracture set 3. The efficiency of the point estimate of the P21 parameter by use of horizontal circular windows (rock outcrops)*

Considering a horizontal circular trace-window of radius 150 m (area = 22500π m^2), the point estimate produces the following results:

P21 Set 1: Arithmetic mean *P21* = 0.77 Standard deviation *P21* = 0.68% of Mean *P21*

P21 Set 2: Arithmetic mean *P21* = 1.50 Standard deviation *P21* = 0.81% of Mean *P21*

P21 Set 3: Arithmetic mean *P21* = 0.41 Standard deviation *P21* = 0.38% of Mean *P21*

The change in mean *P21* values with increasing window size is small for windows of radius larger than 20 m. It is possible to express this change as the derivative of *P21* with respect to window area (*A*) or window radius (*R*). The derivative is approximated by a first order backward finite difference and calculated as follows.

$$\frac{\partial P21}{\partial A} \approx \frac{\Delta P21}{\Delta A} = \frac{P21_{(A)} - P21_{(A-\Delta A)}}{\Delta A}$$

$$\frac{\partial P21}{\partial L} \approx \frac{\Delta P21}{\Delta L} = \frac{P21_{(L)} - P21_{(L-\Delta L)}}{\Delta L}$$

Considering the radius of horizontal circular windows, the average derivative in mean *P21* with respect to window radius, for the last three analysed windows (R= 100m, 125m and 150m) are as follows:

(i) Set 1: -3.8×10^{-5}

(ii) Set 2: -2.0×10^{-4}

(iii) Set 3: -6.6×10^{-5}

These results demonstrate that the mean *P21* values for a circular window of radius 100 m or larger is very stable, and the change in *P21* values that will come with larger windows are negligible. This is of interest, as we have no knowledge of the true *P21* value of the population. In the test below we will set the true *P21* values of the population as equal to the mean *P21* values for a circular window of radius 150 m, we will call this value the "simulated true *P21* value. By doing this we assume that the point estimate converges towards the true *P21* value of the population

5.4.5 Hypothesis testing considering *P21* and acceptable deviations

Purpose of test

The purpose of this test is to determine when the size of a sample is large enough to produce an acceptable estimate of the *P21* parameter of the population studied, with a certain probability. This can also be stated in the following way: the calculation of the sample size that is necessary to reach a confidence level, considering a given confidence interval. The confidence interval is the same thing as a test criterion (an acceptable deviation). The sample size corresponds to area or radius of studied window.

Null hypothesis, acceptable deviations and criterion of significance

The samples were analysed by a statistical hypothesis testing. The hypothesis testing is based on the variable *P21* and given criterions of significance. A difficulty is that we do not know the correct *P21* value of the population studied. The established criterions of significance will therefore correspond to the mean *P21* value derived from a very large sample (the simulated true *P21* value) This is an acceptable method as the mean values are very stable at such a large sample (this is discussed above). The null hypothesis (H_0) is that a sample is a good representation of the true properties of the population. This hypothesis is rejected if a large (significant) deviation takes place in the *P21* value of the sample compared to the simulated true *P21* value of the population. The following criterions are used. The three criterions represent three different levels of significance.

First criterion: $H_{0\ (P21\_\text{deviation} <=15\%)}$ is rejected if:

$\text{ABS}[P21_{(\text{sample})} - P21_{(\text{simulated true})}] >= 0.15 * P21_{(\text{simulated true})}$

Second criterion: $H_{0\ (P21\_\text{deviation} <=10\%)}$ is rejected if:

$\text{ABS}[P21_{(\text{sample})} - P21_{(\text{simulated true})}] >= 0.10 * P21_{(\text{simulated true})}$

Third criterion: $H_{0\ (P21\_\text{deviation} <=5\%)}$ is rejected if:

$\text{ABS}[P21_{(\text{sample})} - P21_{(\text{simulated true})}] >= 0.05 * P21_{(\text{simulated true})}$

The results of the analysis are presented as the probability that a sample, at a certain borehole length, will fulfil the hypothesis considering three different criterions.

5.4.6 Results of hypothesis testing

The fracture traces are divided into three different sets, based on the known Set ID of each fracture that creates a trace. The results are given for each fracture sets separately. The results are given in the figures below.

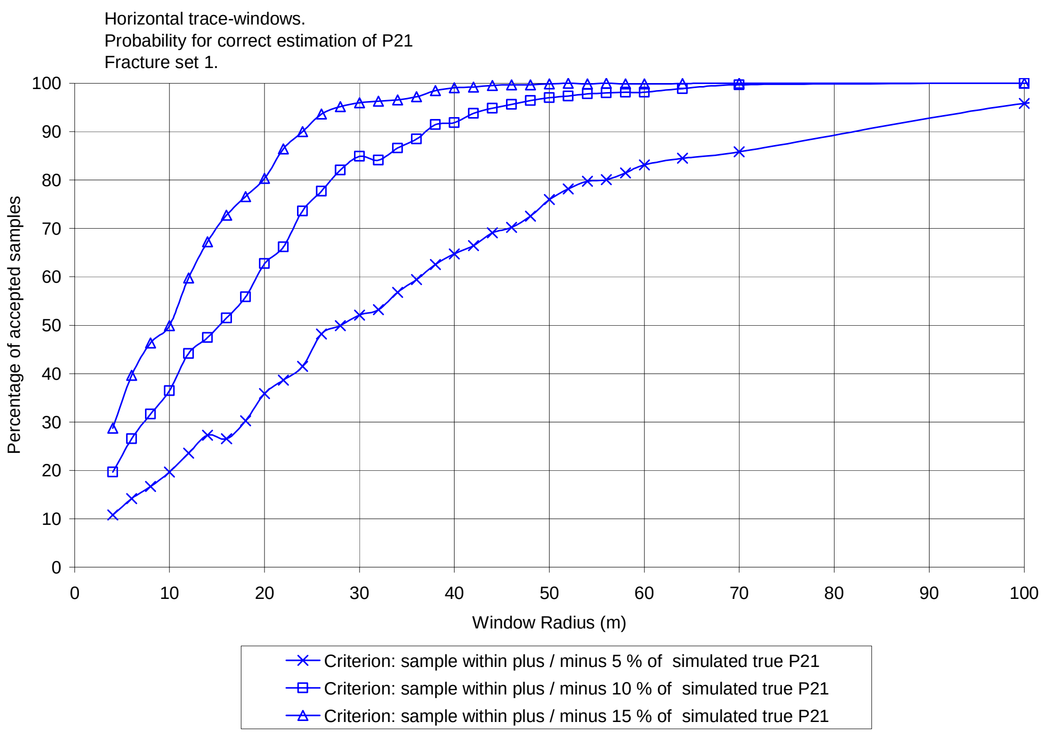


Figure 5-12. *Set 1. Horizontal circular window. Hypothesis testing for selected acceptable deviations in predicted P21 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

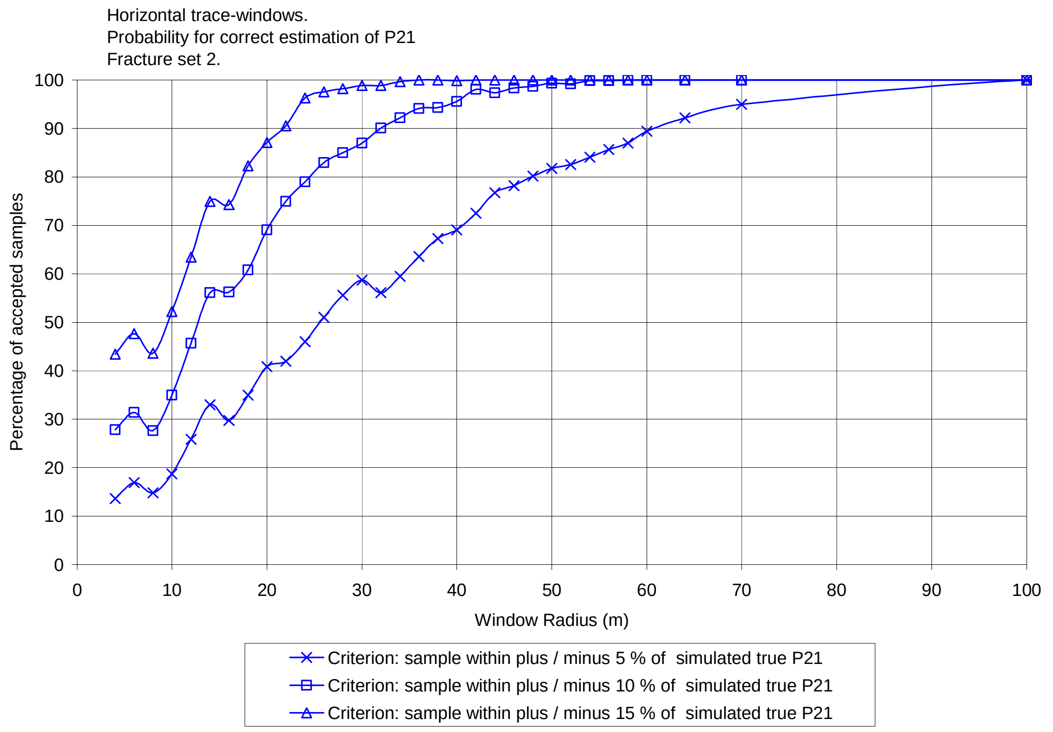


Figure 5-13. *Set 2. Horizontal circular window. Hypothesis testing for selected acceptable deviations in predicted P21 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions*

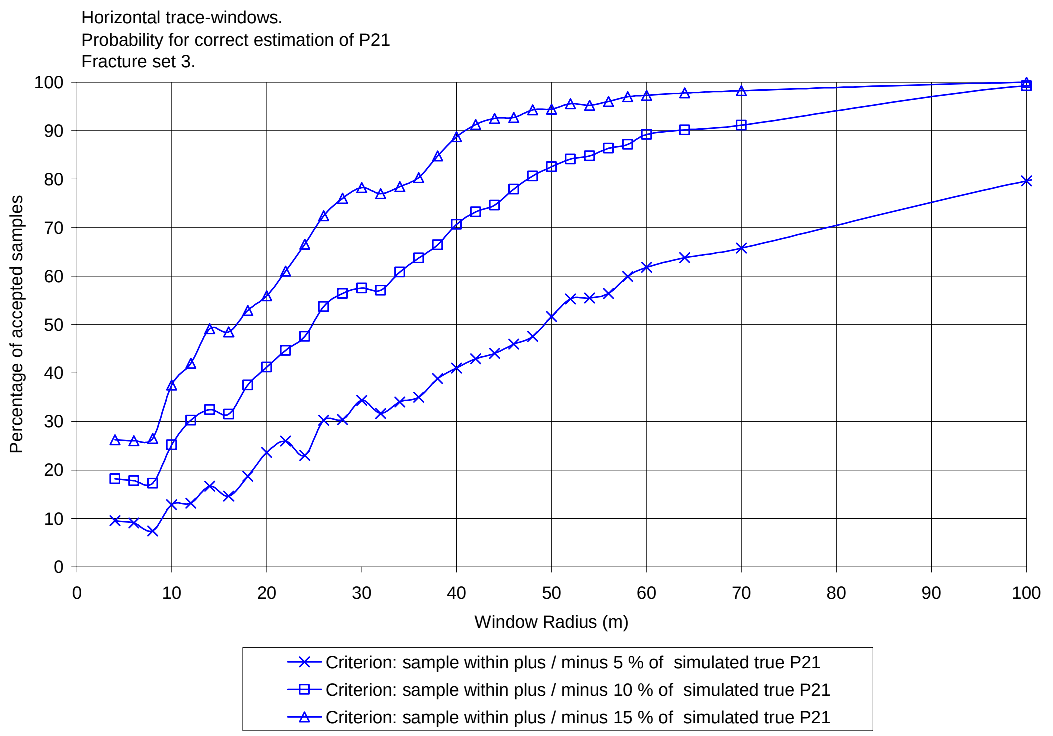


Figure 5-14. *Set 3. Horizontal circular window. Hypothesis testing for selected acceptable deviations in predicted P21 value. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

- Examples of results for Set 1 are as follows (see Figure 5-12): For a horizontal surface of radius larger than 24 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (P21\_\ \text{deviation} <=15\%)}$). If a horizontal surface has a radius larger than 24 metres, the probability is larger than 90 percent that the deviation in estimated *P21* value is within plus/minus 15 percent of the simulated true *P21* value of the population.

- Examples of results for Set 2 are as follows (see Figure 5-13): For a horizontal surface of radius larger than 22 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (P21\_\ \text{deviation} <=15\%)}$). If a horizontal surface has a radius larger than 22 metres, the probability is larger than 90 percent that the deviation in estimated *P21* value is within plus/minus 15 percent of the simulated true *P21* value of the population.

- Examples of results for Set 3 are as follows (see Figure 5-14): For a horizontal surface of radius larger than 40 metres, the probability is larger than 90 percent that a sample will not be rejected considering the first criterion ($H_{0\ (P21\_\ \text{deviation} <=15\%)}$). If a horizontal surface has a radius larger than 40 metres, the probability is larger than 90 percent that the deviation in estimated *P21* value is within plus/minus 15 percent of the simulated true *P21* value of the population.

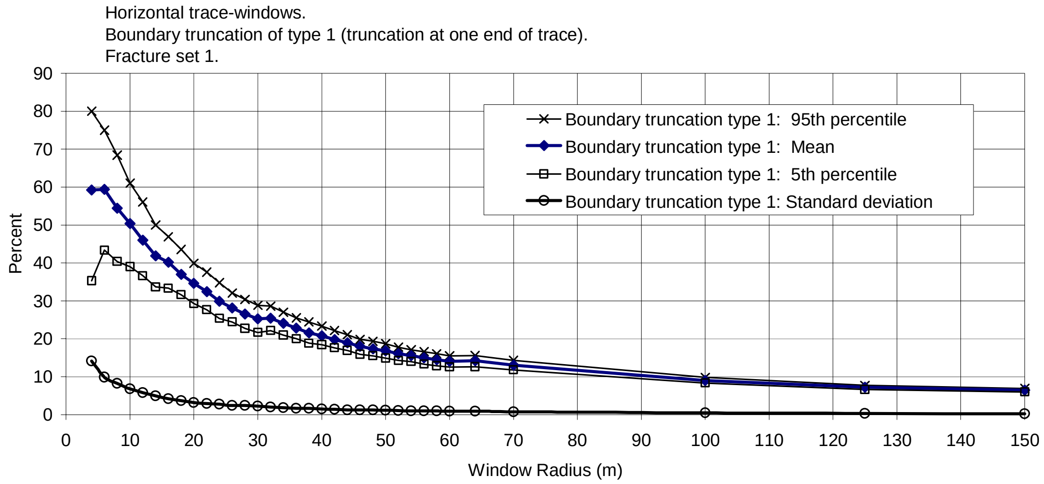
5.4.7 Proportion of boundary-truncated fractures and estimation of *P21*

Fracture traces that continue outside of the studied window are called boundary-truncated traces, because the observed (censored) lengths of these traces are truncated at the boundary of the studied window. It follows that the length of a boundary-truncated trace is not the correct length, but an observed (or censored) length that is shorter than the true trace-length. There are two different types of boundary-truncated traces.

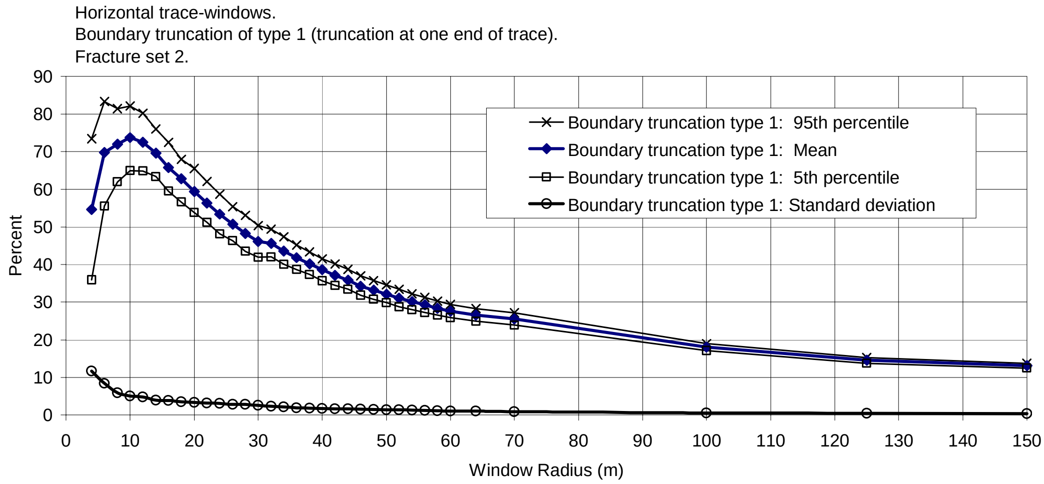
(i) Traces for which only one end terminates at the boundary of the window, these traces are called boundary-truncated traces of the first type

(ii) Traces for which both ends terminate at the boundary of the window, these traces are called boundary-truncated traces of the second type.

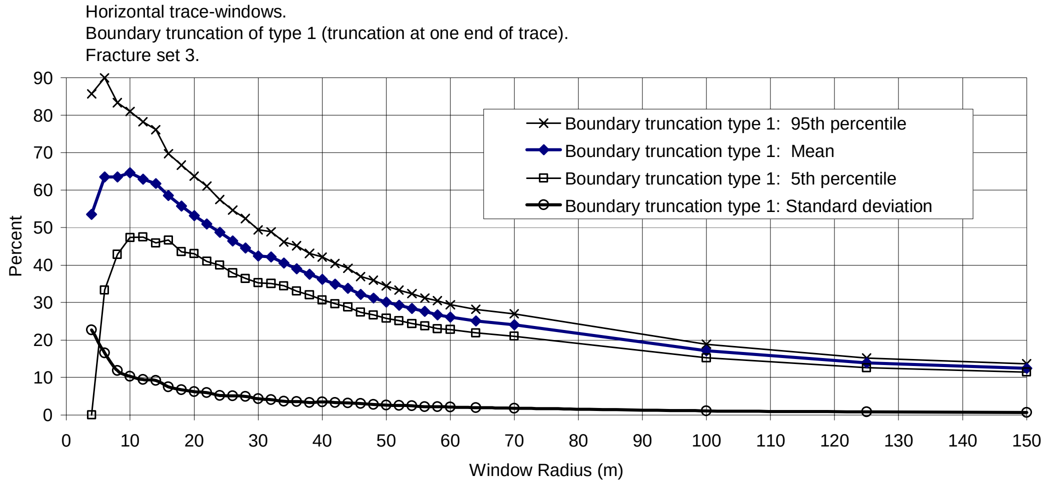
In this study both types of boundary-truncated traces are counted and the number of such traces are compared to the total number of observed traces, the result is given in Figure 5-15 and Figure 5-16 below.



SET 1

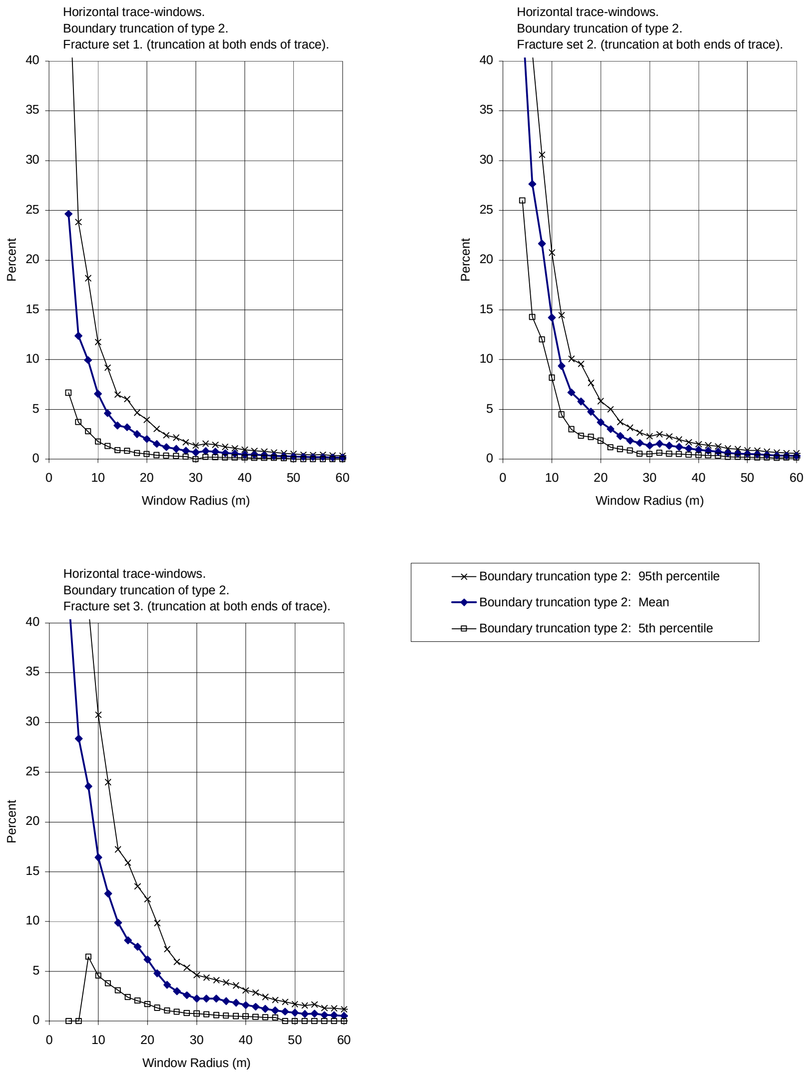


SET 2



SET 3

Figure 5-15. *Horizontal circular window. Percentage of boundary-truncated fracture traces (type 1), as a function of radius of window studied, for the three fracture sets studied. The figure gives the percentage of traces with one termination at the window boundary (termination of first type)*



SET 3

Figure 5-16. *Horizontal circular window. Percentage of boundary-truncated fracture traces (type 2), as a function of radius of window studied. The figure gives the percentage of traces with two termination at the window boundary (termination of second type).*

When studying a window of a certain size and when comparing the observed percentage of truncated fractures for different fracture sets, on the average the smaller the fractures of the sets, the smaller the percentage of truncated fractures. It follows that the smallest percentage of truncated fractures will on the average occur for Set 1, as this is the set with the smallest fractures.

However, on the average, there will always be boundary-truncated traces of the first type, regardless of window size, because a window is of finite size and as the traces are assumed to be distributed randomly with respect to the window.

For a window with a radius of 20 m, the mean amount of boundary-truncated traces of the first type is between 35 and 60 percent (dependent on set studied). For a window with a radius of 60 m, the mean amount of boundary-truncated traces of the first type is between 15 and 30 percent (dependent on set studied). If the fracture set studied contains few small fractures, the amount of boundary-truncated traces of the first type may increase with size of window, at small window sizes. This is because for such a fracture set many fractures are truncated at both ends (boundary-truncation of the second type) at small window sizes and these fractures will, up to a certain window size, change into boundary-truncated traces of the first type as the size of window is increased.

The results given in the figures above also demonstrate that for a circular window with radius larger than 22 m, the mean amount of boundary-truncated traces of the second type is less than five percent (for all three sets); and for windows with a radius larger than 40 m there are very few such traces, the mean amount for Set 1 is 0.5% and for Set 2 the mean amount is 0.9% and finally for Set 3 the mean amount is 1.6%

The efficiency of the point estimate of the *P21* parameter depends on size of window, because the average number of observed traces and average number of boundary-truncated traces depends on the window size. However, as the total length of all observed traces gives the *P21* value (including the observed length of the boundary-truncated traces), the *P21* value is not strongly dependent on the amount of boundary-truncated traces, except if the amount of boundary-truncated fractures are large. This is demonstrated by comparing (i) the hypothesis testing for selected acceptable deviations in predicted *P21* value and (ii) the percentage of truncated fractures. An obvious example is Set2, the estimate of the *P21* value is a good estimate for a window with a radius of 22 m, the expected deviation in estimation of *P21* is less than 15%, although the amount of boundary-truncated traces is about 60 percent.

When deriving a trace-length distribution, some methods uses the amount of boundary-truncated traces for correction of the sample distribution, such corrections are not applied in this study, however they are briefly mentioned in Chapter 6.

5.5 Point estimate and test considering *P32*

5.5.1 Introduction

The three-dimensional density is given by the *P32* parameter; it is equal to *fracture surface area per unit rock volume*, taken over a volume. It is more complicated to estimate the *P32* parameter than the *P10* and *P21* parameters, as normally no direct observations can be made of the complete extension of fracture surfaces inside a studied volume of rock. Consequently, when estimating the *P32* parameter it has to be calculated based on other measurable properties.

5.5.2 Methodology

As previously discussed, for a complete model of the fracture network we need to have knowledge about: (i) the distribution of fracture orientation and dispersion, (ii) the *P32* value and (iii) the fracture size distribution.

To estimate a *P32* value, the common method is a trial and error procedure based on a calibration of an assumed complete model of the fracture network. This procedure can be carried out in different ways, for example in the following way. The first step is a preliminary estimation of the distributions of fracture orientations, dispersion and density (e.g. based on borehole data) and a preliminary estimation of the fracture size distribution based on mapped fracture traces. The next step is to simulate a fracture network based on the established model. The fracture density of the network, the *P32* value, is varied until the established model simulates a *P21* value and/or a *P10* value, which is similar to derived (observed) values of *P21* and *P10.* The *P21* value as derived from mapped fracture traces and the *P10* value as derived from observations in boreholes. This procedure is also illustrated by Eq. 6-1, which can be rewritten as follows.

$$P32_{TRUE} = P32_{MODEL} \frac{P21_{TRUE}}{P21_{MODEL}} \qquad 5\text{-}3$$

or

$$P32_{TRUE} = P32_{MODEL} \frac{P10_{TRUE}}{P10_{MODEL}}$$

It follows from the equations above that the deviation in estimation of the true *P32* value will be directly proportional to the deviation in estimation of the true *P21* value and/or the true *P10* value. The deviation in estimation will however also be proportional to the accepted divergence between the simulated values (model) and the estimated true values (the convergence criteria). Furthermore, it follows from Eq. 6-3, that if the purpose is to estimate a *P32* value only, and not to establish a complete model of the fracture network, the fracture size distribution and the fracture orientation and dispersion do not need to be completely correct, as long as the estimated true *P21* or *P10* values are correct.

Hence, with the addition of the trial and error procedure and the convergence criteria of that method, the results presented in Section 5.4 regarding the estimation of the *P21* value is also directly applicable for the estimation of the *P32* value; and the same goes for the estimation of the *P10* value, as presented in Section 5.3.

However, the *P32* value can also be directly estimated from samples of rock mass. Samples can be produced by analyses of the properties of excavated rock inside a tunnel (mapping of fracture traces on tunnel walls, roof and floor), but also based on observed fractures inside a borehole, this will be discussed in Section 5.5.5.

5.5.3 Hypothesis testing and results considering *P32*, based on *P21*

The *P32* parameter was estimated with the method described above; this is estimation in parallel with the estimation of the *P21* parameter.

The null hypothesis (H_0) is that the property of a sample is a good representation of the true properties of the population. This hypothesis is rejected a significant deviation takes place in the *P21* value of the sample compared to the true *P21* value (simulated) of the population. The following criterions are used; the three criterions represent three different levels of significance.

First criterion: $H_{0\ (P32\_deviation\ <=15\%)}$ is rejected if:

$ABS[P21_{(sample)} – P21_{(simulated\ true)}] >= 0.15* P21_{(simulated\ true)}$

Second criterion: $H_{0\ (P32\_deviation\ <=10\%)}$ is rejected if:

$ABS[P21_{(sample)} – P21_{(simulated\ true)}] >= 0.10* P21_{(simulated\ true)}$

Third criterion: $H_{0\ (P32\_deviation\ <=5\%)}$ is rejected if:

$ABS[P21_{(sample)} – P21_{(simulated\ true)}] >= 0.05* P21_{(simulated\ true)}$

Considering the same values of window radius, the hypothesis testing considering the *P32* parameter yields the same results as the hypothesis testing considering the *P21* parameter; and these results are given in Section 5.4.5. Examples of results are as follows. Note that the results below are given without consideration of the efficiency and convergence criteria of the trial an error procedure.

- Considering Set 1 and a window with radius larger than 24 m, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_deviation\ <=15\%)}$). For a window with radius larger than 37 m, the probability is larger than 90 percent that a sample will fulfil the hypothesis at the second level of significance ($H_{0\ (P32\_deviation\ <=10\%)}$).
- Considering Set 2 and a window with radius larger than 22 m, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_deviation\ <=15\%)}$). For a window with radius larger than 32 m, the probability is larger than 90 percent that a sample will fulfil the hypothesis at the second level of significance ($H_{0\ (P32\_deviation\ <=10\%)}$).
- Considering Set 3 and a window with radius larger than 40 m, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_deviation\ <=15\%)}$). For a window with radius larger than 60 m, the probability is larger than 90 percent that a sample will fulfil the hypothesis at the second level of significance ($H_{0\ (P32\_deviation\ <=10\%)}$).

5.5.4 Hypothesis testing and results considering *P32*, based on *P10*

The *P32* parameter was estimated with the method described above; this is estimation in parallel with the estimation of the *P10* parameter.

The null hypothesis (H_0) is that the property of a sample is a good representation of the true properties of the population. This hypothesis is rejected if a significant deviation takes place in the *P10* value of the sample compared to the true *P10* value (simulated) of the population. The following criterions are used; the three criterions represent three different levels of significance.

First criterion: $H_{0\ (P32\_deviation\ <=15\%)}$ is rejected if:

$ABS[P10_{(sample)} – P10_{(mean\ sample\ at\ 1000m)}] >= 0.15* P10_{(mean\ sample\ at\ 1000m)}$

Second criterion: $H_{0\ (P32\_deviation\ <=10\%)}$ is rejected if:

ABS[$P10_{\text{(sample)}} - P10_{\text{(mean sample at 1000m)}}$] >= 0.10* $P10_{\text{(mean sample at 1000m)}}$

Third criterion: $H_{0\ (P32\_\text{deviation} <=5\%)}$ is rejected if:

ABS[$P10_{\text{(sample)}} - P10_{\text{(mean sample at 1000m)}}$] >= 0.05* $P10_{\text{(mean sample at 1000m)}}$

Considering the same borehole lengths, the hypothesis testing considering the *P32* parameter yields the same results as the hypothesis testing considering the *P10* parameter; and these results are given in Section 5.3.4. Examples of results are given below. Note that the results below are given without consideration of the efficiency and convergence criteria of the trial an error procedure.

<u>Vertical borehole (see Figure 5-2, Figure 5-3 and Figure 5-4)</u>

- Results for set 1: If the borehole has a length larger than 400 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population. The same goes for the *P32* value.
- Results for Set 2 : If the borehole has a length larger than 300 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population. The same goes for the *P32* value.
- Results for Set 3 : If the borehole has a length larger than 150 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population. The same goes for the *P32* value.

<u>Inclined borehole (see Figure 5-5, Figure 5-6, Figure 5-7)</u>

- Results for set 1: If the borehole has a length larger than 350 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population. The same goes for the *P32* value.
- Results for Set 2 : If the borehole has a length larger than 150 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population. The same goes for the *P32* value.
- Results for Set 3 : If the borehole has a length larger than 210 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the simulated true *P10* value of the population. The same goes for the *P32* value.

5.5.5 Direct estimation of *P32* considering the rock mass inside a borehole

Discussion of methodology

The number of fractures observed in a borehole is linked to the fracture density of the fracture population of the rock mass that surrounds the borehole. It is possible to make an estimate of the *P32* parameter of the rock mass, by studying the size of a borehole as well as the number and orientations of the fractures that intersect the borehole.

The first step is the calculation of the *P32* value of the rock mass inside the borehole. It is possible to calculate this value if we have knowledge of the acute angels between the borehole and the intersecting fractures, and if we assume that no fracture terminates in the borehole. To reach a good estimate of the *P32* value inside the borehole, the necessary length of borehole might be considerable; as the estimation is based on a small volume that is very elongated in one dimension (the direction of the borehole).

The next step is the assumption that the *P32* value of the rock mass that surrounds the borehole is well estimated by the calculated *P32* value of the rock mass inside the borehole. The applicability of this assumption varies with (i) the orientation of the borehole in relation to the mean orientation of the fracture planes of the rock mass that surrounds the borehole (the acute angles), as well as on (ii) the length of the borehole. If the studied fracture planes are at right angle to the borehole the assumption is applicable, even for short boreholes; but if the fracture planes of the rock mass that surrounds the borehole are along (parallel) to the borehole, the assumption is not correct. However, there is normally a certain spread of fracture orientations (dispersion) within a fracture set, it follows that fractures will intersect the borehole, even if the mean orientation of the fracture planes of the set is along the borehole, presuming that the borehole is long enough.

Consider a fracture that intersects a borehole. The fracture surface area inside the borehole depends on the acute angle between the borehole and the fracture plane. If the acute angel is small, the fracture area within the borehole will be large, and if the acute angel is large, the fracture area within the borehole will be small. It follows that if the mean orientation of the fracture planes is at right angel to the borehole, many fracture planes will intersect the borehole, but the fracture surface area inside the borehole is small for each intersection. On the other hand, if the mean orientation of the fracture planes is along (parallel) to the borehole, few fracture planes will intersect the borehole, but the fracture surface area inside the borehole is large for each intersection. In this way the fracture surface area inside the borehole will vary with direction of borehole, and consequently compensate for different directions of the borehole.

Thus, the assumption that the *P32* value of the rock mass is well estimated by the *P32* value of the rock mass inside a borehole is a correct assumption, presuming that (i) the borehole is long enough and (ii) the fracture set studied contains a certain spread in orientation (dispersion). However, the efficiency of the method (the necessary length of borehole) is proportional to the *P10* value of the fracture set studied, the larger the *P10* value the more efficient the method and the shorter the necessary length of borehole. The *P10* value will vary with direction of borehole; hence, the efficiency of the method depends on direction of borehole.

Standard Terzaghi correction should not be applied, as the above-discussed calculation of *P32* is based on fracture area inside a volume, even if is a very elongated volume (the method is not based on a geometrical line with zero volume). For fracture networks that are not extremely anisotropic, the above-discussed method can be more efficient than one may first perceive. The efficiency of the method will increase if the studied rock volume is larger than that of a borehole, for example if the method is applied on the volumes inside a tunnel system. On the other hand, if the volume studied is large and has a less elongated shape than a borehole, termination of fractures inside the volume studied has to be considered and included in the calculations.

Equations

The estimation of the *P32* parameter from borehole data is based on the following equation. The *P10C* is the *P32* value of the rock mass inside the borehole.

$$P10C = \frac{1}{L} \sum_{i=1}^{n} \frac{1}{\cos(\theta_i)} \qquad \text{5-4}$$

P10C = The P32 value of the rock mass inside a borehole

L = Length of studied section (borehole).

θ = Acute angle, the angle between the borehole and a normal to the fracture plane.

n = Number of fractures along the section studied.

The equation above can be derived in the following way. The fracture surface area inside a borehole is constrained by an ellipse (presuming that the fracture is a plane in space). The area (*A*) of an ellipse is given by the length of its semi-axis (*a*) and (*b*).

$$A = \pi \, a \, b$$

The lengths of the semi-axis (inside the borehole) are given by the radius (*r*) of the borehole and the acute angle, as follows:

$$a = \frac{1}{\cos(\theta_i)} r \qquad \text{and} \qquad b = r$$

It follows that the fracture surface area inside the boreholes is:

$$A = \pi \frac{1}{\cos(\theta_i)} r^2$$

Considering several fractures (*n*) intersecting the borehole, we will get the following expression.

$$A = \pi \, r^2 \sum_{i=1}^{n} \frac{1}{\cos(\theta_i)}$$

The volume (V) of the borehole along a given section of length (L) is:

$$V = \pi \ r^2 \ L$$

The total fracture area divided by the volume gives the *P10C* value.

$$P10C = \frac{\pi \ r^2 \sum_{i=1}^{n} \frac{1}{\cos(\theta_i)}}{\pi \ r^2 \ L}$$

Reducing the equation above for the constants (r) and (π) will produce Eq. 6-4, as given below.

$$P10C = \frac{1}{L} \sum_{i=1}^{n} \frac{1}{\cos(\theta_i)}$$

/Chilès and de Marsily, 1993/ proposed an equation similar to Eq. 6-4, but with the acute angel defined as the acute angle between the borehole and the fracture plane (and not as the acute angel between the borehole and a normal to the fracture plane).

Point estimate of the *P10*C and *P32* values

The fractures that intersect the borehole form samples of the fracture population. The properties of the sample are estimates of the properties of the population. The observed fractures are classified into three groups, one group for each theoretical fracture set. After the classification each fracture set is studied one by one, separate from the other sets. The test presented below is conducted for each fracture set separately.

The first step is calculation of *P10C* values, based on samples from boreholes and by use of the equation above (Eq. 6-4). The next step is the assumption that the *P32* value of the rock mass is well estimated by the calculated *P10C* values. Hence, we assume that for large samples (large lengths of borehole) the *P10C* value is an acceptable estimate of the *P32* value of the rock mass.

From a statistical point of view, the analysis is a point estimate of the variable *P10C* and this variable is a function of the properties of the samples. The efficiency of the point estimate increases with size of the sample (number of observed fractures) and the size of the sample increases with the length of the borehole. This is demonstrated in

Vertical borehole, results of point estimate, borehole length =1000 metres.

Set 1: True *P32* = 0.85

Set 1: *P10C*: Mean = 0.82 Standard dev. = 8.3% of Mean.

Deviation in estimation = 3.0%

Set 2: True *P32* = 1.59

Set 2: *P10C*: Mean = 1.53 Standard dev. = 6.8% of Mean.

Deviation in estimation = 3.6%

Set 3: True *P32* = 0.97

Set 3: *P10C*: Mean = 0.97006 Standard dev. = 3.5% of Mean.

Deviation in estimation = 0.006%

Inclined borehole, results of point estimate, borehole length =1000 metres.

Set 1: True *P32* = 0.85

Set 1: *P10C*: Mean = 0.84 Standard dev. = 6.4% of Mean.

Deviation in estimation = 0.7%

Set 2: True *P32* = 1.59

Set 2: *P10C*: Mean = 1.56 Standard dev. = 4.6% of Mean.

Deviation in estimation = 1.7%

Set 3: True *P32* = 0.97

Set 3: *P10C*: Mean = 0.966 Standard dev. = 5.0% of Mean.

Deviation in estimation = 0.5%

The change in mean *P10C* values with increasing borehole length is small, even for as short boreholes as 50 metres, but the variance in *P10C* is not insignificant for such short boreholes. It is possible to express the change in *P10C*, as the derivative of *P10C* with respect to borehole length (*L*). The derivative is approximated by a first order backward finite difference and calculated as follows.

$$\frac{\partial P10C}{\partial L} \approx \frac{\Delta P10C}{\Delta L} = \frac{P10C_{(L)} - P10C_{(L-\Delta L)}}{\Delta L}$$

For a vertical borehole, the average derivative in mean *P10C* with respect to borehole length, for the last five borehole lengths, is as follows

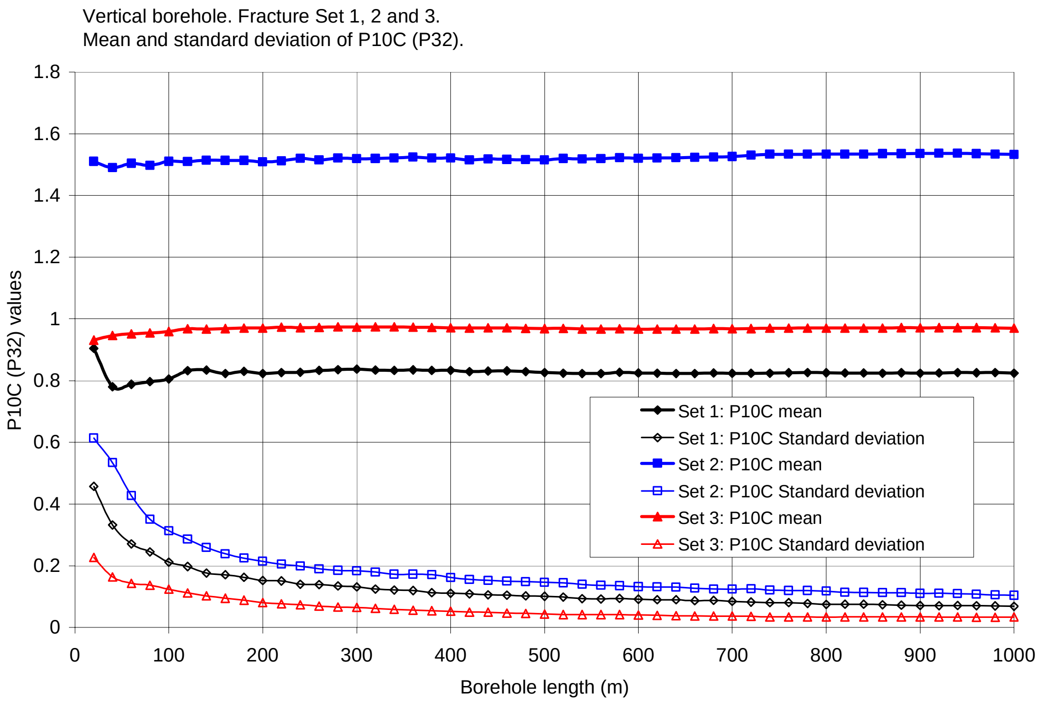
(i) Set 1: $5.3\text{x}10^{-7}$ (ii) Set 2: $-3.2\text{x}10^{-5}$ (iii) Set 3: $-7.4\text{x}10^{-6}$

For a vertical borehole, the average derivative in mean *P10C* with respect to borehole length, for the last five borehole lengths, as follows:

(i) Set 1: $-4.3\text{x}10^{-6}$ (ii) Set 2: $-2.8\text{x}10^{-5}$ (iii) Set 3: $-2.4\text{x}10^{-5}$

These results demonstrate that the mean *P10C* values at 1000 metre of borehole are very stable, and the change in *P10C* values that will come with longer boreholes are negligible.

The efficiencies of the point estimates of the *P10C* values are given in Figure 5-17, below.



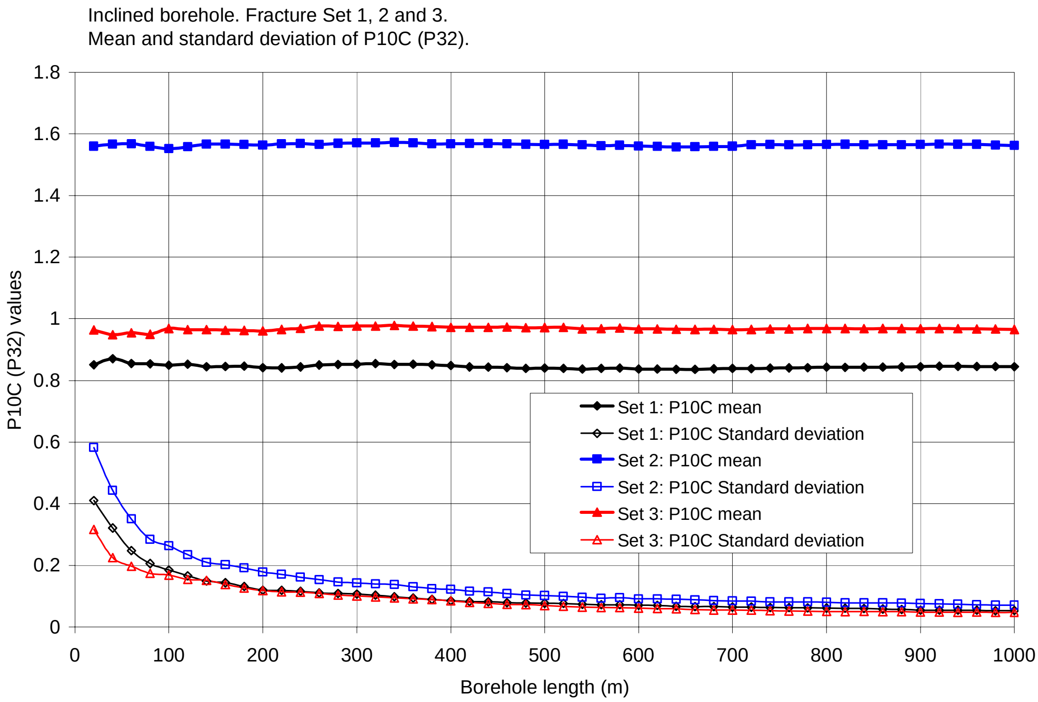


Figure 5-17. *Efficiency of the point estimate of mean and standard deviation of P10C values (P32 inside borehole). The upper figure gives results for a vertical borehole; the lower figure gives results for an inclined borehole.*

Hypothesis testing considering *P10*C (*P32*) and acceptable deviations

Purpose of test

The purpose of this test is to determine when the size of the sample is large enough to produce an acceptable estimate of the true properties with a certain probability.

Null hypothesis, acceptable deviations and criterion of significance

The samples were analysed by a statistical hypothesis testing. The hypothesis testing is based on the variable *P10C* and given criterions of significance. The given criterions of significance relate to the true *P32* value of the fracture population studied.

The null hypothesis (H_0) is that a sample is a good representation of the population. This hypothesis is rejected if a significant deviation takes place between the *P10C* value of the sample and the true *P32* value of the fracture population studied. The following criterions of significance are used. The three criterions represent three different levels of significance.

First criterion: $H_{0\ (P32\_\text{deviation} <=15\%)}$ is rejected if:

$ABS[P10C_{(\text{sample})} - P32_{(\text{population})}] >= 0.15 * P32_{(\text{population})}$

Second criterion: $H_{0\ (P32\_\text{deviation} <=10\%)}$ is rejected if:

$ABS[P10C_{(\text{sample})} - P32_{(\text{population})}] >= 0.10 * P32_{(\text{population})}$

Third criterion: $H_{0\ (P32\_\text{deviation} <=5\%)}$ is rejected if:

$ABS[P10C_{(\text{sample})} - P32_{(\text{population})}] >= 0.05 * P32_{(\text{population})}$

The results of the analysis are presented as the probability that a sample, at a certain borehole length, will fulfil the hypothesis considering three different criterions.

Results considering a vertical borehole

The results are given in the figures below, for the three fracture sets and for the three different levels of significance.

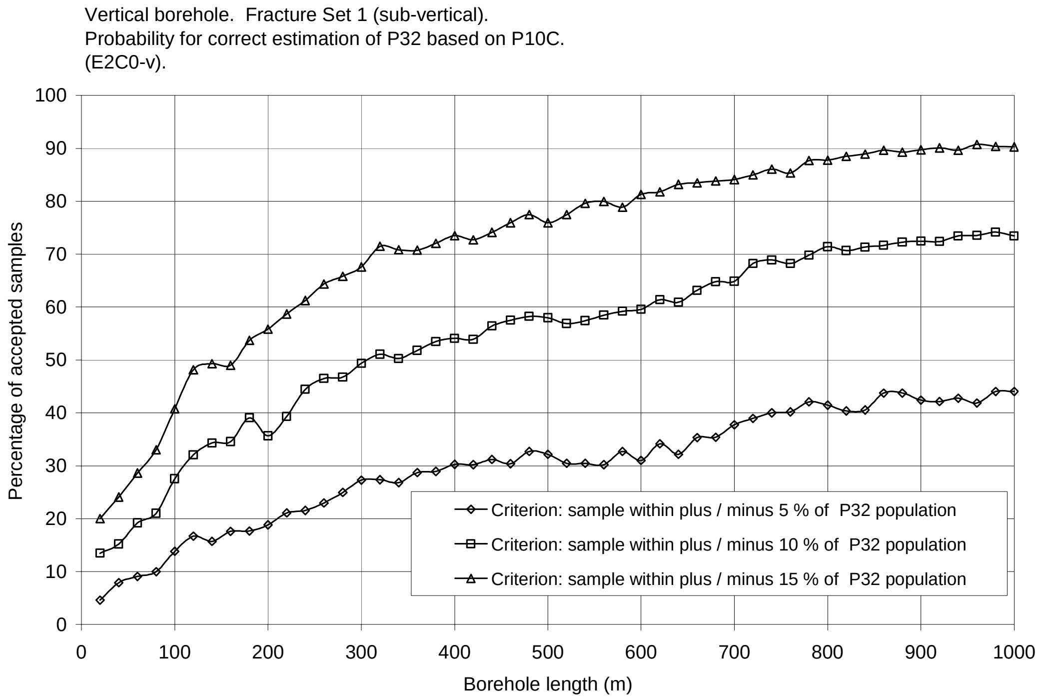


Figure 5-18. *Vertical borehole. Set 1. Hypothesis testing for selected acceptable deviations in predicted P32 value (based on P10C values). The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

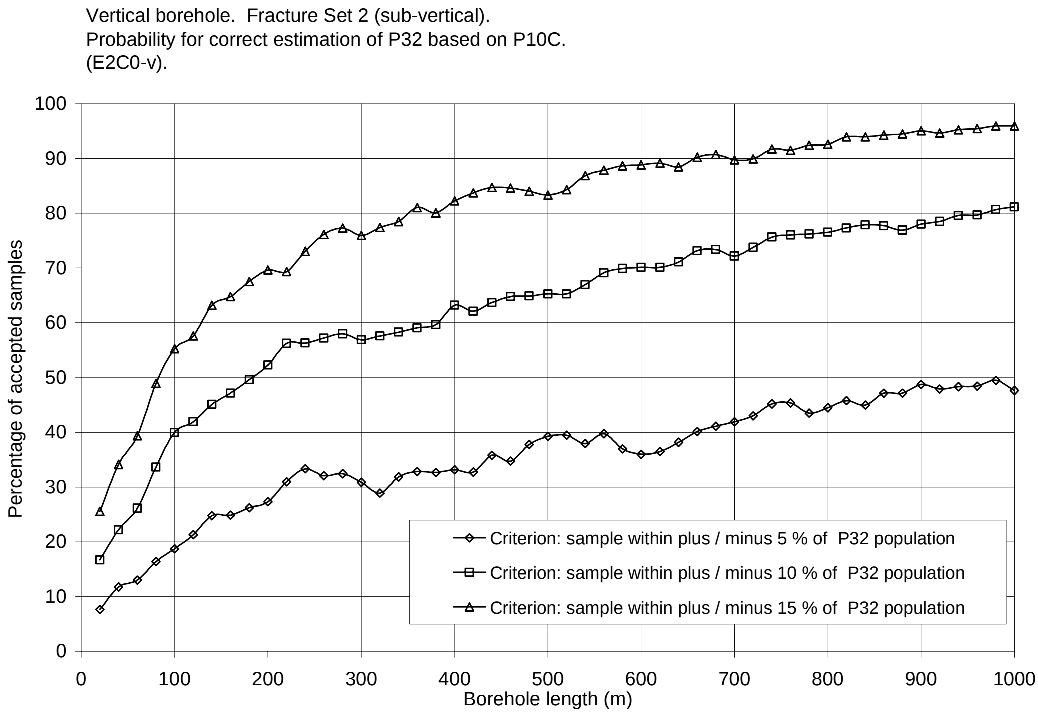


Figure 5-19. *Vertical borehole. Set 2. Hypothesis testing for selected acceptable deviations in predicted P32 value (based on P10C values). The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

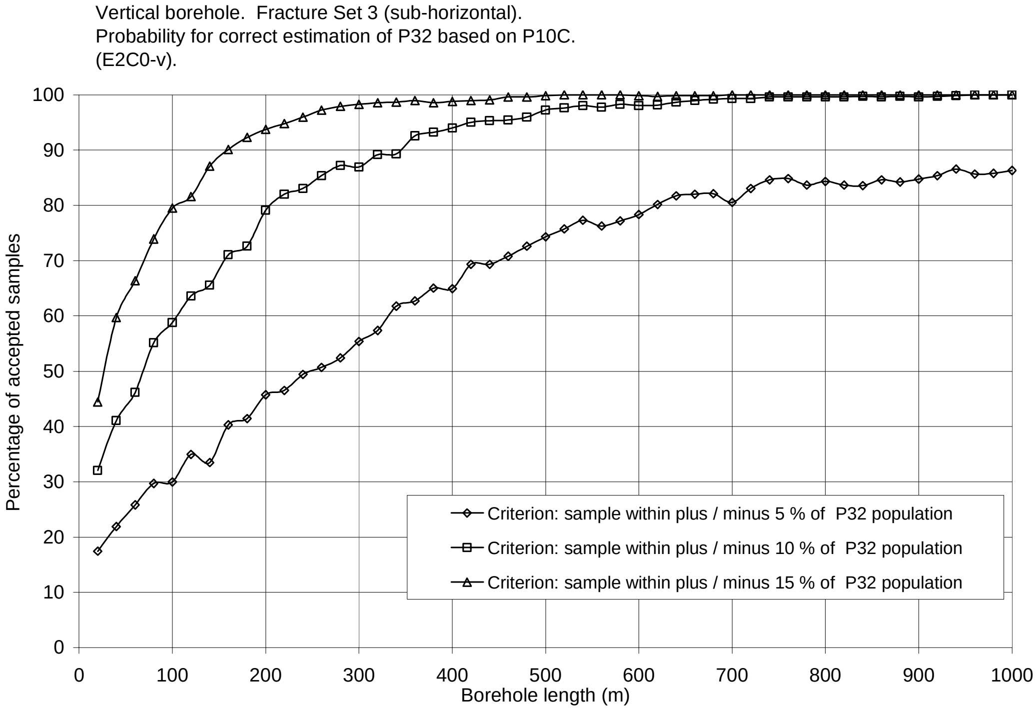


Figure 5-20. *Vertical borehole. Set 3. Hypothesis testing for selected acceptable deviations in predicted P32 value (based on P10C values). The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

- Examples of results for Set 1 are as follows (see Figure 5-18): For a vertical borehole with a length larger than 850 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_\text{deviation} <=15\%)}$). If the borehole has a length larger than 850 meters, the probability is larger than 90 percent that the deviation in estimated *P32* value is within plus/minus 15 percent of the true *P32* value of the population.

- Examples of results for Set 2 are as follows (see Figure 5-19): For a vertical borehole with a length larger than 650 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_\text{deviation} <=15\%)}$). If the borehole has a length larger than 650 meters, the probability is larger than 90 percent that the deviation in estimated *P32* value is within plus/minus 15 percent of the true *P32* value of the population.

- Examples of results for Set 3 are as follows (see Figure 5-20): For a vertical borehole with a length larger than 150 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_\text{deviation} <=15\%)}$). If the borehole has a length larger than 150 meters, the probability is larger than 90 percent that the deviation in estimated *P10* value is within plus/minus 15 percent of the true *P32* value of the population.

Results considering an inclined borehole

The results are given in the figures below, for the three fracture sets and for the three different levels of significance.

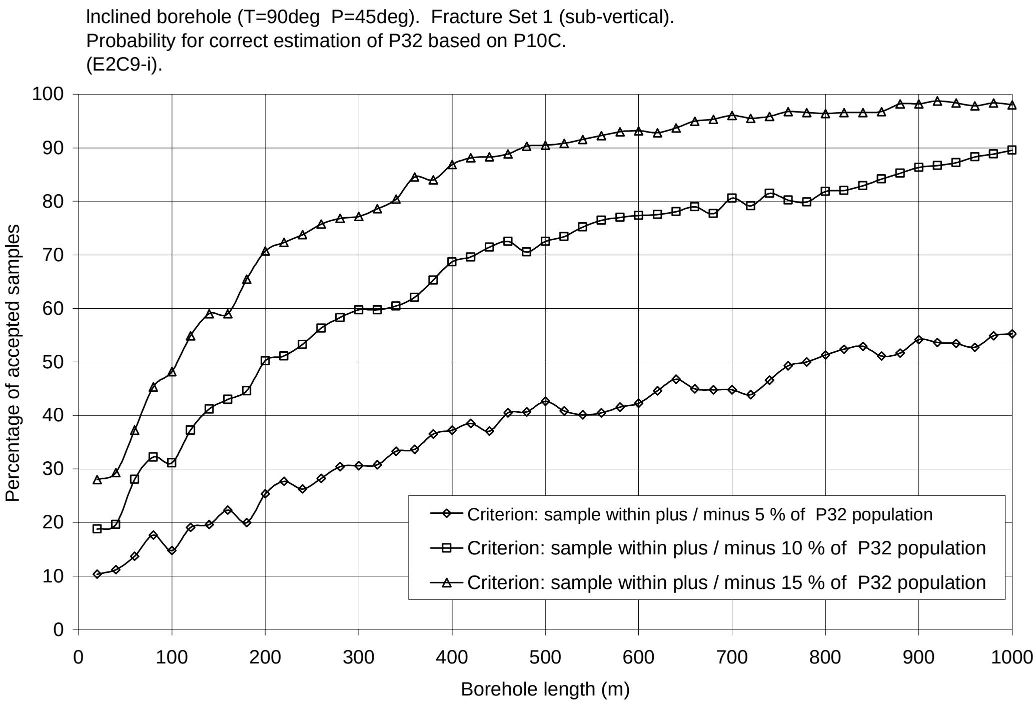


Figure 5-21. *Inclined borehole. Set 1. Hypothesis testing for selected acceptable deviations in predicted P32 value (based on P10C values). The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

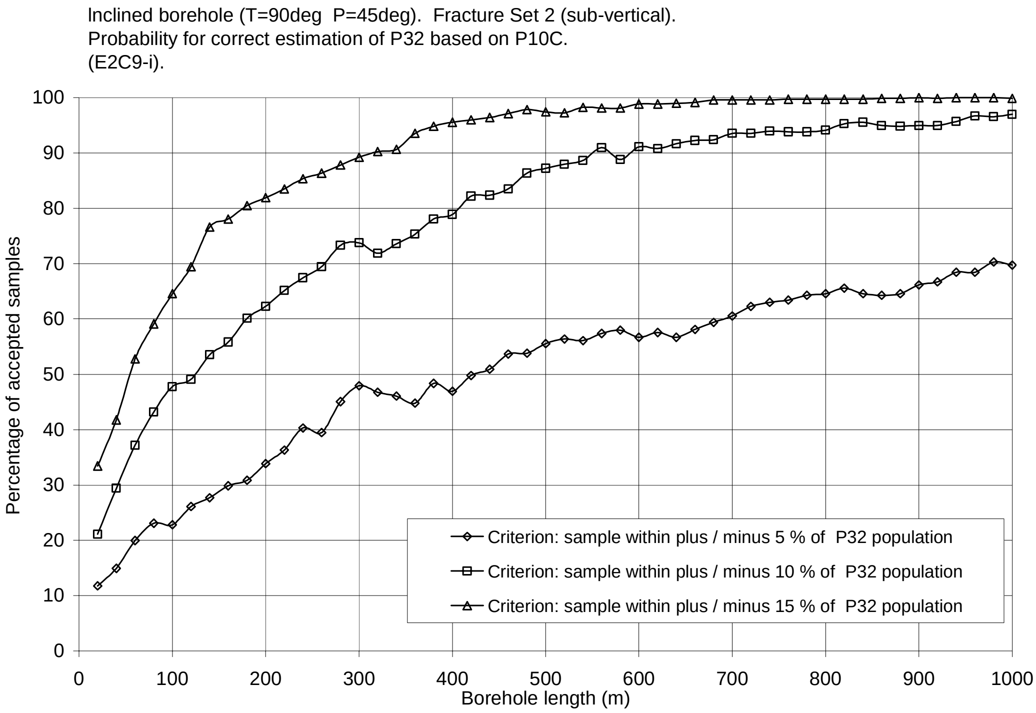


Figure 5-22. *Inclined borehole. Set 2. Hypothesis testing for selected acceptable deviations in predicted P32 value (based on P10C values). The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions.*

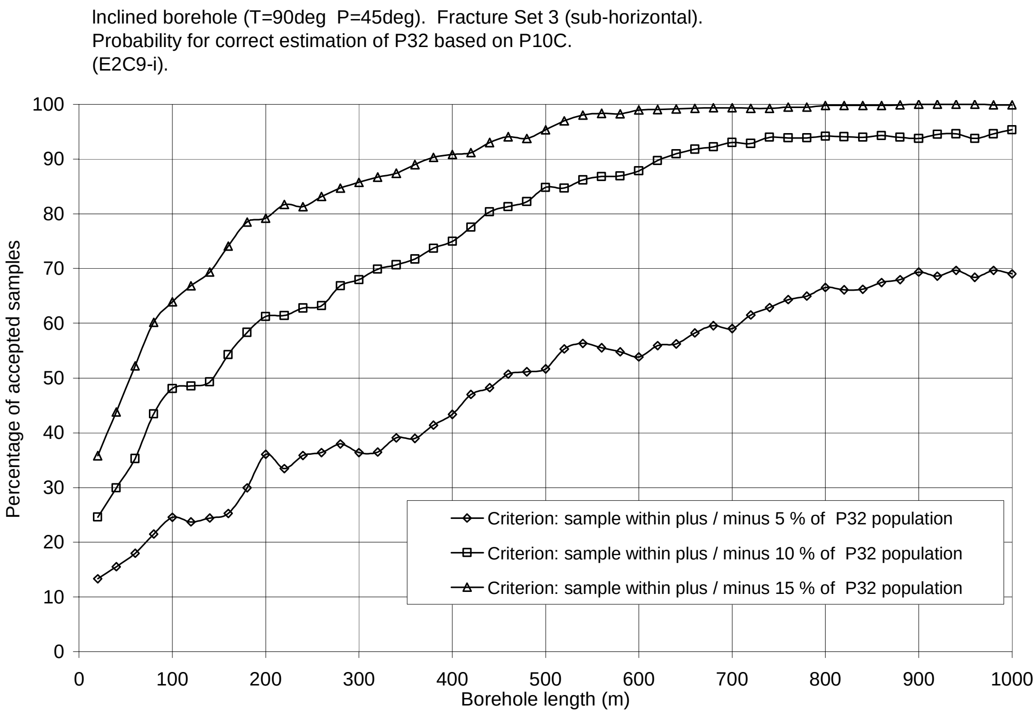


Figure 5-23. *Inclined borehole. Set 3. Hypothesis testing for selected acceptable deviations in predicted P32 value (based on P10C values). The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, for the different selected criterions*

- Examples of results for Set 1 are as follows (see Figure 5-21): For an inclined borehole with a length larger than 480 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_deviation\ <=15\%)}$). If the borehole has a length larger than 480 meters, the probability is larger than 90 percent that the deviation in estimated *P32* value is within plus/minus 15 percent of the true *P32* value of the population.

- Examples of results for Set 2 are as follows (see Figure 5-22): For an inclined borehole with a length larger than 350 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_deviation\ <=15\%)}$). If the borehole has a length larger than 350 meters, the probability is larger than 90 percent that the deviation in estimated *P32* value is within plus/minus 15 percent of the true *P32* value of the population.

- Examples of results for Set 3 are as follows (see Figure 5-23): For an inclined borehole with a length larger than 380 metres, the probability is larger than 90 percent that a sample will fulfil the hypothesis considering the first criterion ($H_{0\ (P32\_deviation\ <=15\%)}$). If the borehole has a length larger than 380 meters, the probability is larger than 90 percent that the deviation in estimated *P32* value is within plus/minus 15 percent of the true *P32* value of the population.

6 Estimation of trace-length distribution from rock surface data

6.1 Introduction

By measuring the lengths of fracture traces, as observed on surfaces, is possible to establish a trace-length distribution. In itself such a distribution is perhaps not very interesting, but the trace-length distribution is linked to the fracture size distribution and the fracture size distribution is a very important part of a description of a fracture network. The fracture size distribution is important because the distribution of fracture size largely determines the frequency of fracture intersections and hence the mechanical and hydraulic properties of the rock mass studied. However, as no direct observations can be made of the complete extension of fracture surfaces inside a studied volume of rock, the fracture size distribution is determined via the trace-length distribution, and that is why the trace-length distribution is important.

The DFN-model used in this study (as the base case) is the DFN 2 model presented in /Hermanson et al, 1999/. The main objective of the DFN 2 modelling was to establish a discrete fracture network model, representing the rock mass at the Prototype Repository, which could be used for simulation of groundwater flow. The DFN 2 model underestimates the total number of fractures in the rock mass at the Prototype Repository, as small fractures with minor or negligible hydraulic importance is not included in the model. We have therefore established an alternative DFN-model, which includes a larger number of small fractures, but has the same value of fracture density (*P32*-value). For this alternative DFN-model, different results are obtained considering the necessary sample sizes for reliable estimation of the fracture trace-length distribution. The results of the alternative model are presented in Chapter 8 (Limited Sensitivity Analysis).

6.2 Methodology

No direct observations can be made of the complete extension of fracture surfaces inside a studied volume of rock. Consequently, it is in practise not possible to directly observe the fracture size distribution. A two-dimensional survey of rock surfaces provides a distribution of the fracture trace-lengths, which is linked with the fracture size distribution; but the trace-length distribution is affected by several biases that have to be considered. Primarily the bias occurs because the three-dimensional fracture network is sampled by use of two-dimensional planes having a limited extension. There are five main causes for bias:

(i) *Boundary-truncated fracture traces*. Fracture traces that continues outside of the studied window -traces with terminations that are not observable- for such traces, the length observed (censored length) is shorter than the true length. In this study these fracture traces are included in the analysis and their observed (censored) length, within the window studied, gives their length.

(ii) *Size-truncated fracture traces.* In practise when performing mapping on a real rock surface, fracture traces that are smaller than a certain size will not be observed correctly. Both because these small traces are considered of secondary importance and are overlooked, or because it is difficult to discriminate between natural fracture traces and artificial fracture traces (e.g. caused by blasting). This bias will only have a minimal influence on the results of this study. In this study we have defined a size limit equal to 0.025 metre, only traces with a length smaller than this limit will be excluded from the analysis.

(iii) *Importance of fracture size.* The probability of observing a fracture trace on a two-dimensional surface is proportional to the size of the fracture that creates the trace. Hence, for a studied fracture size distribution it is more likely to observe the traces of the large fractures than the traces of the small fractures. /La Pointe and Hudson, 1985/ showed that, for the assumption that fractures are circular planar discs, the probability of a fracture intersecting a plane is linearly proportional to the fracture radius.

(iv) *Importance of geometrical shape and orientation of window studied.* The geometrical shape, size and orientation of the window studied will influence the observed trace-lengths. This bias will influence the trace-length distribution derived in this study. To minimise this bias we have in this study only used circular windows and in this study all windows studied are along the horizontal plane, e.g. corresponding to horizontal rock outcrops.

(v) *Importance of localisation of window studied.* In most cases, the localisation of the analysed two-dimensional surface (the window studied) is not picked at random, but given by circumstances that will influence the observed fracture size distribution. For example it is likely that naturally occurring rock outcrops corresponds to rock masses with a higher resistance to weathering etc. than the average rock mass. It is also likely that such rock masses (with a high resistance to weathering) also carries fewer fractures than the average rock mass. This bias will not influence the results of this study, because in this study the analysed windows are numerically generated and numerically analysed; and the observed properties are unbiased as regards the average quality of rock mass.

The above discussed causes for bias are well known, and different authors have proposed different methods, of varying efficiency and applicability, for deriving the true mean or underlying distribution /see Pahl 1981; Laslett, 1982/. In this study no correction of the observed distributions has been applied.

If all traces are put into one group, regardless of strike and dip of the traces etc, it is likely that a trace-length distribution, obtained in such a way, will demonstrate a complex shape with a tendency for a bi-modal or a multi-modal shape. Such a multi-modal tendency is the product of different fracture sets with different fracture diameter distributions together forming the common trace-length distribution, the different modes of the common trace-length distribution reflects the different mean-values of the fracture diameter distributions. For the studied rock mass such a distribution is given in, the distribution is derived from horizontal surfaces of radius 150 metres.

When fracture traces are observed on rock surfaces, it is often possible to separate the fracture traces into different sets, based on the observed strike of the fracture traces; and based on the dip of the fracture traces (assuming that it is possible to observe a dip). It follows that different trace-length distributions will be derived for different sets. In the analyses presented in this chapter, the fracture traces are divided into three different sets, based on the known Set identity of each fracture that creates a trace. The results of the analyses are given for each fracture sets separately. (In this study each fracture was marked with its proper set identity since this is known at the generation of the fracture. In a real situation, different methods and algorithms for identifying and delimiting sets will be necessary to ensure objective set identifications.)

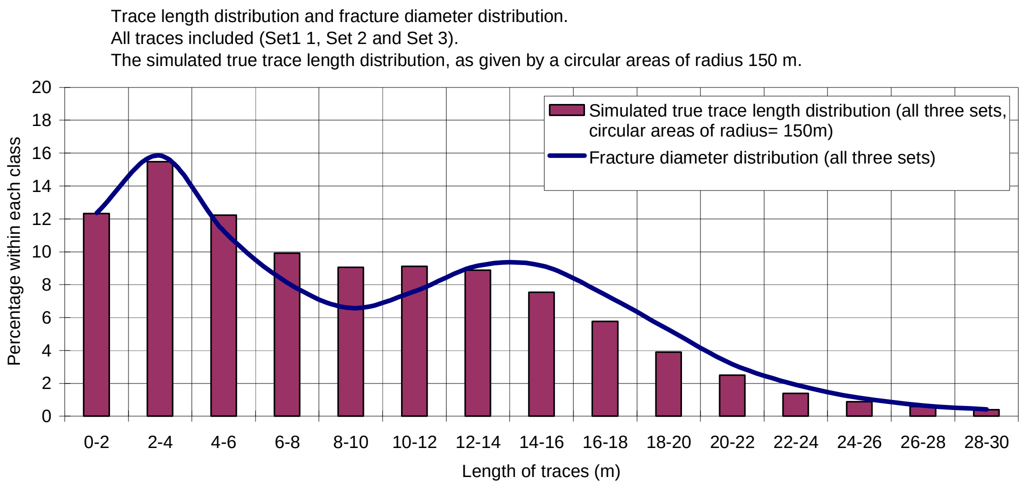


Figure 6-1. *Trace-length distribution and fracture diameter distribution, considering all traces observed on horizontal windows of radius 150 m. The somewhat complex shapes are the product of different fracture sets with different fracture diameter distributions, together forming the distributions. The different modes of the trace-length distribution reflects the different mean-values of the fracture diameter distributions.*

The characteristics of the trace-length distribution vary with the size of the window studied. For small windows, the lengths of the traces are limited by the size of the window; and in addition as the number of traces are small, the variation in distribution characteristics is large between different windows (different realisations). The larger the window the closer the characteristics of the sample distribution is to the unknown characteristics of the population studied, and the smaller the differences between different realisations.

Examples of fracture diameter distributions of the fracture sets of the population studied and the corresponding trace-length distributions for very large circular and horizontal windows (radius 150 metres) are given in Figure 6-1 (all sets together) and Figure 6-2 (set by set). The average trace-length distribution for windows of radius 150 metres is set as the true distribution, and it is called the simulated true distribution (this is further discussed below).

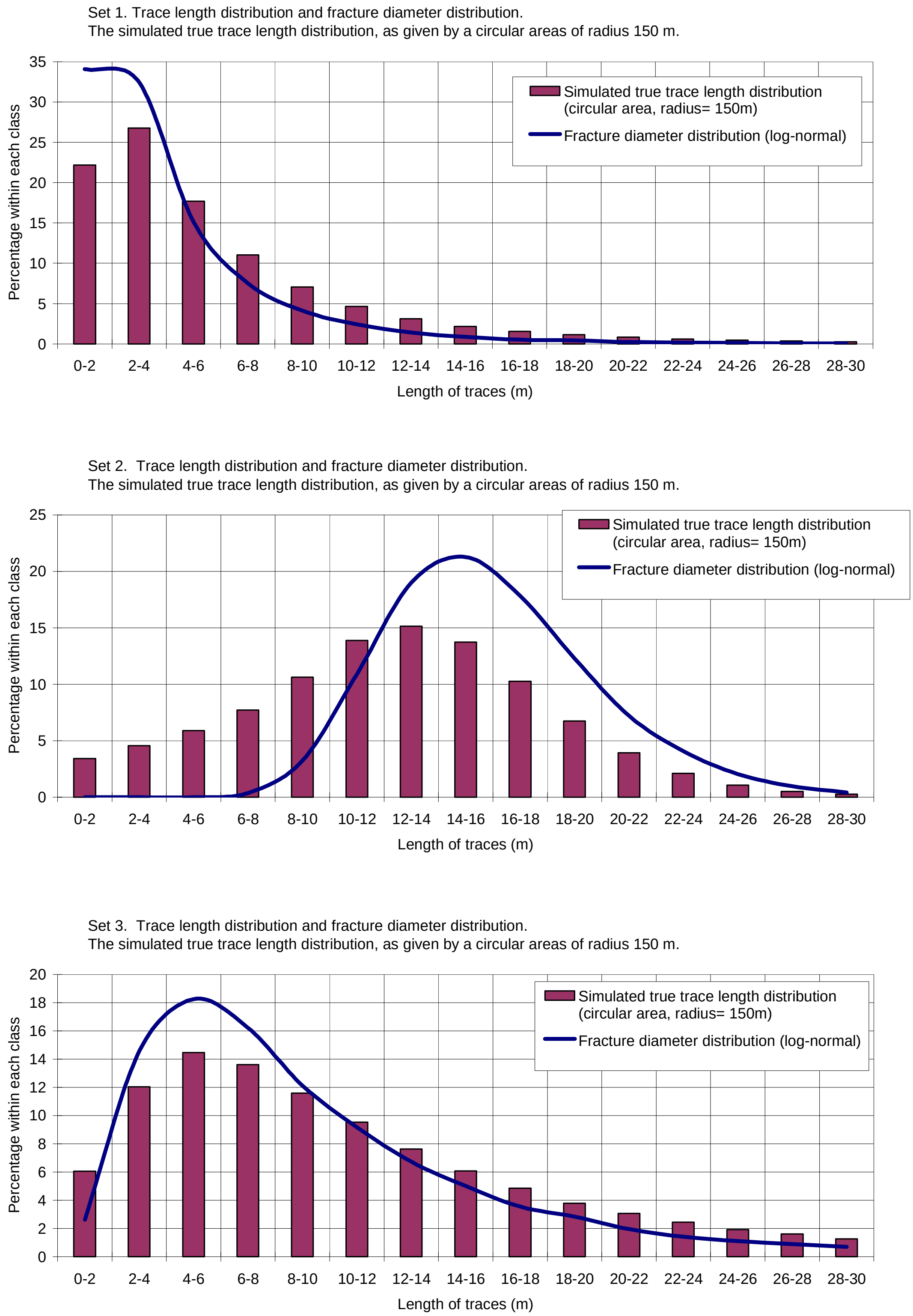


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Figure 6-2. *Comparison between: (i) the fracture diameter distributions and (ii) the corresponding trace-length distributions, considering the three fracture sets and a window of radius 150 m. The average trace-length distribution for windows of radius 150 m is set as the true trace-length distribution, and it is called the simulated true distribution.*

Among other things, Figure 6-2 demonstrates that the trace-length distributions will always include a tail of small traces even if the corresponding fracture diameter distribution demonstrates a small probability for small fractures (e.g. the results for Set 2 in Figure 6-2). The reason for this is that normally only a part of a fracture intersects a surface, it follows that on a surface there will be short traces that are created by large fractures. Hence, when comparing a trace-length distribution derived from a large window and the corresponding fracture diameter distribution, the probability for small traces is larger than the corresponding probability for fractures having the same diameter as the length of the small traces.

Even if the trace-length distributions are known for each fracture set, this is not enough to determine the fracture size distributions of the fractures of the rock mass, unless assumptions are made regarding the shape of fractures, and other properties of the fracture network. The observed trace-length distribution is linked to the fracture size distribution of the fractures that created the traces, and these fractures are a sample of the fractures of the rock mass. This is stated by /La Pointe et al, 2000/ in the following way: "The solution to the problem of how the scaling properties of trace-lengths relate to the scaling properties of the parent fracture distribution requires decomposition of the problem into two stages. (i) The relation between the radius distribution of the parent fracture population and the radius distribution of the fracture population intersecting the trace plane; and (ii) the relation between the radius distribution of fractures intersecting a trace plane and the observed trace-length distribution.".

A common assumption is that the fractures can be considered as discs. Even for this simple assumption, the derivation of the fracture size distribution from the trace-length distribution is not self-evident (except if the properties of the fracture network are very simple). Therefore, in practice when deriving a fracture size distribution one assumes a fracture shape and a distribution of the fracture sizes. The next step is the establishment of an assumed complete model of the fracture network, including fracture orientation and density. By use of this model and via a trial and error procedure considering different fracture size distributions, the observed trace-lengths are matched with simulated lengths and thereby the fracture size distribution is derived. The log-normal, the exponential or the power law distributions are commonly used for representing the fracture size distribution because "even if the disc-diameter distribution of the fractures is not log-normal, the trace-length distribution tends to look log-normal" /Chilès and de Marsily, 1993/. As revealed by Figure 6-2, the simulated true trace-length distributions for Set 1 and Set 3 have tendencies towards log-normal or an exponential shape, while the simulated true trace-length distribution of Set 2 has a more complicated shape.

We will in this study not derive a fracture size distribution, but only a trace-length distribution. We will analyse the derived trace-length distribution considering how its characteristic changes with size of window. If the trace-length distribution is well defined and stable, at a certain window size, theoretically also the fracture size distribution of the fractures that intersects the window should be well defined and stable at this size of window. Hence, at such a window size the fundamental data is available for a good estimation of the fracture size distribution. The actual calculation of the fracture size distribution based on a trace-length distribution includes assumptions regarding fracture shape etc, as discussed above, and is not a part of this study.

It should be noted that the three dimensional fracture density parameter, the *P32* parameter, is only a measure of the fracture density of the rock mass, it does not provide a description of the fracture sizes or the connectivity of the fracture network. For a given value of *P32* there is a better fracture connectivity with a small number of large fractures than with a large number of small fractures.

6.3 Point estimate of the moments of the trace-length distribution

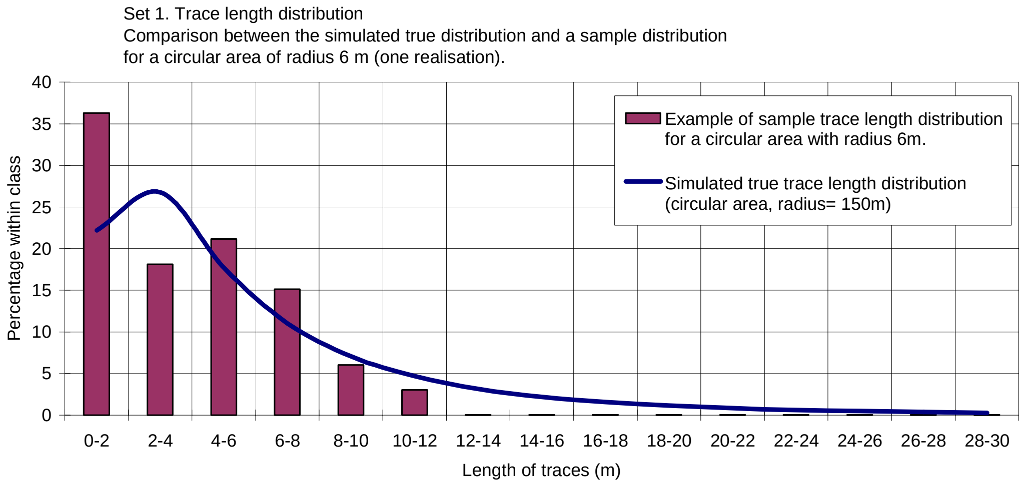
6.3.1 General

The fracture traces that take place on a windows studied are samples of the properties of the fracture population. The properties of the sample can be looked upon as an estimate of the properties of the population. From a statistical point of view, the analysis of the trace-length distribution, as given by windows of different sizes, is a point estimate of the properties (moments) of an unknown trace-length distribution.

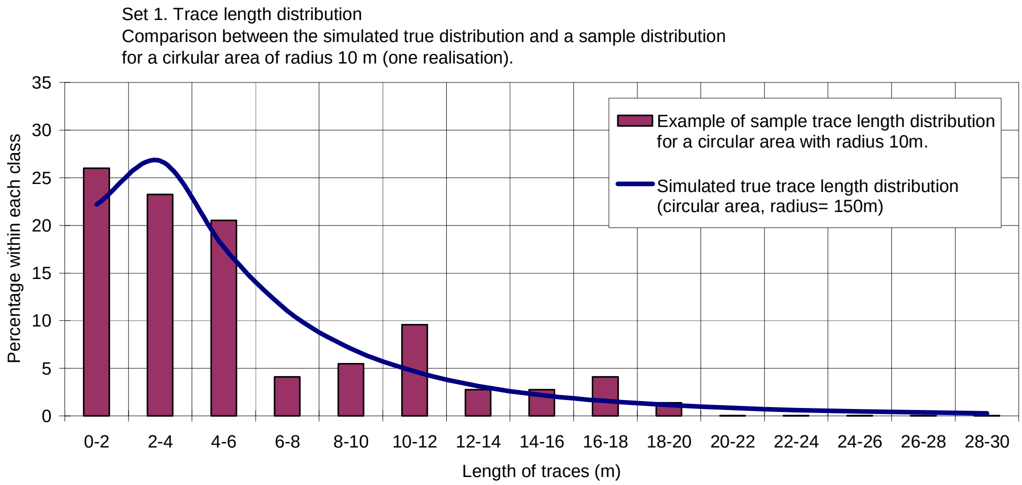
In the analyses presented below, the fracture traces are divided into three different sets, based on the known Set identity of each fracture that creates a trace. The results of the analyses are given for each fracture sets separately.

Examples of sample trace-length distributions are given below in Figure 6-3, Figure 6-4 and Figure 6-5. The characteristics of the trace-length distribution vary with the size of the window studied. For small windows, the lengths of the traces are limited by the size of the window; and in addition as the number of traces are small, the variation in distribution characteristics is large between different windows (different realisations). The larger the window the closer the characteristics of the sample distribution is to the unknown characteristics of the population studied, and the smaller the differences between different realisations; the rate of this progress towards the true characteristics are called the efficiency of the point estimate. Figure 6-2 presents the average trace-length distribution for a window of radius 150 metres. This distribution is set as the true distribution, and it is called the simulated true distribution.

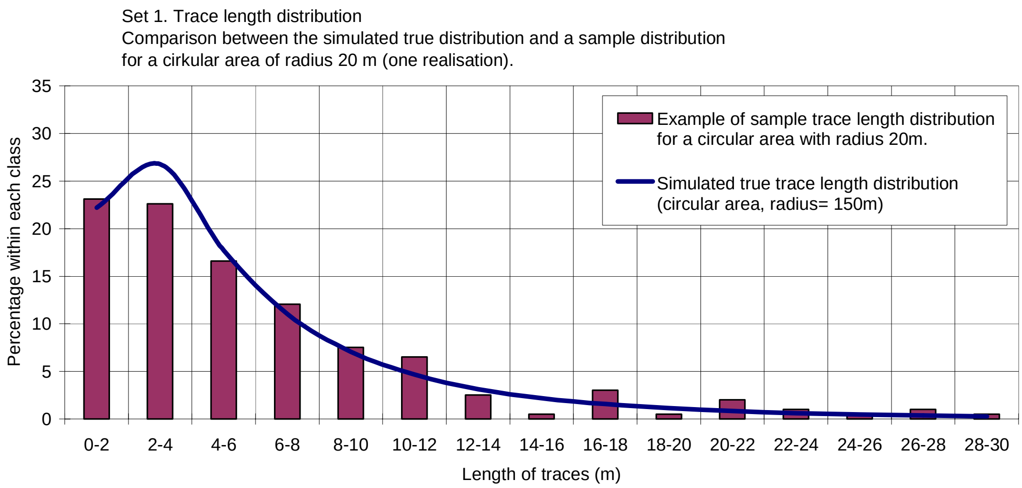
As regards a point estimate of a trace-length distribution based on samples taken from windows (rock surfaces) of different sizes, the following needs to be considered. The efficiency of a point estimate considering a set of traces will not be the same as the efficiency of a point estimate considering all traces put together in one group. The sample size (number of traces) will be larger if all traces are included in one group, compared to the sample size of different sets, but also the shape of the true trace-length distribution will be different. If all traces are put into one group, regardless of strike and dip of the traces etc, it is not unlikely that a trace-length distribution, obtained in this way, will demonstrate a tendency for a bi-modal shape. Such a bi-modal shape is the product of different fracture sets with different fracture diameter distributions, the different modes of the trace-length distribution reflects the different mean-values of the fracture diameter distributions.



SET 1: Window of radius 6 m (diameter 12 m), one realisation.

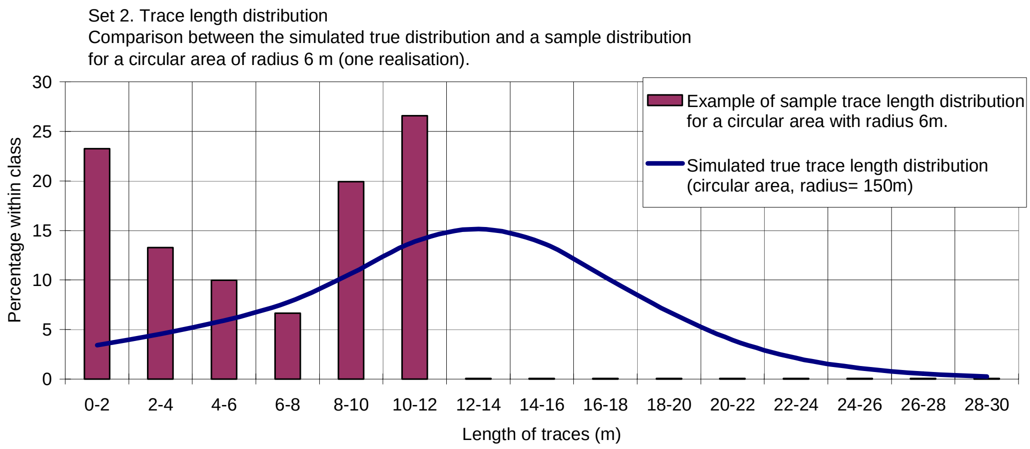


SET 1: Window of radius 10 m (diameter 20 m), one realisation.

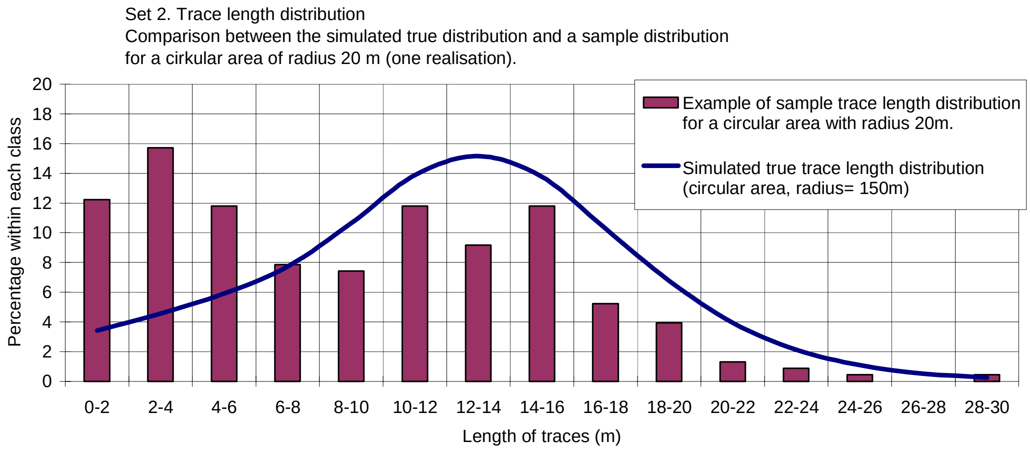


SET 1: Window of radius 20 m (diameter 40 m), one realisation.

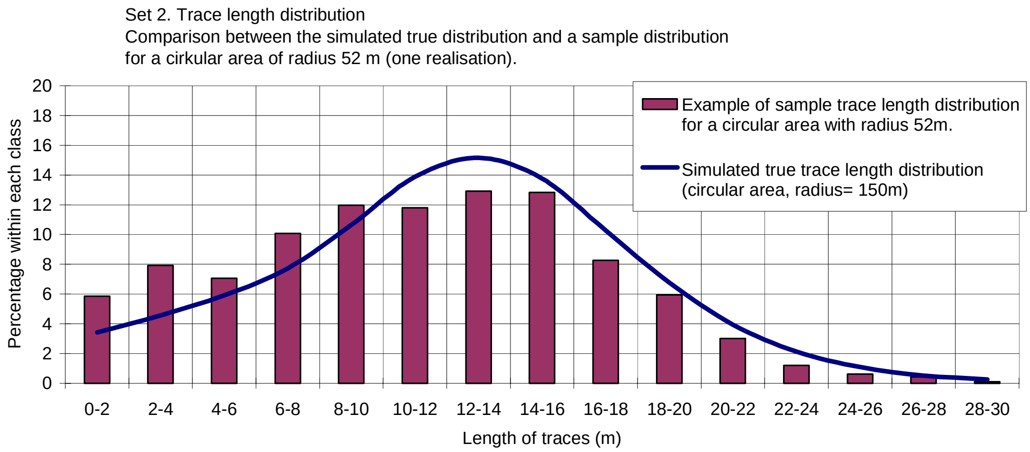
Figure 6-3. *SET 1: Comparison between the simulated true trace-length distribution and examples of sample trace-length distributions for windows of different radii.*



SET 2: Window of radius 6 m (diameter 12 m), one realisation.

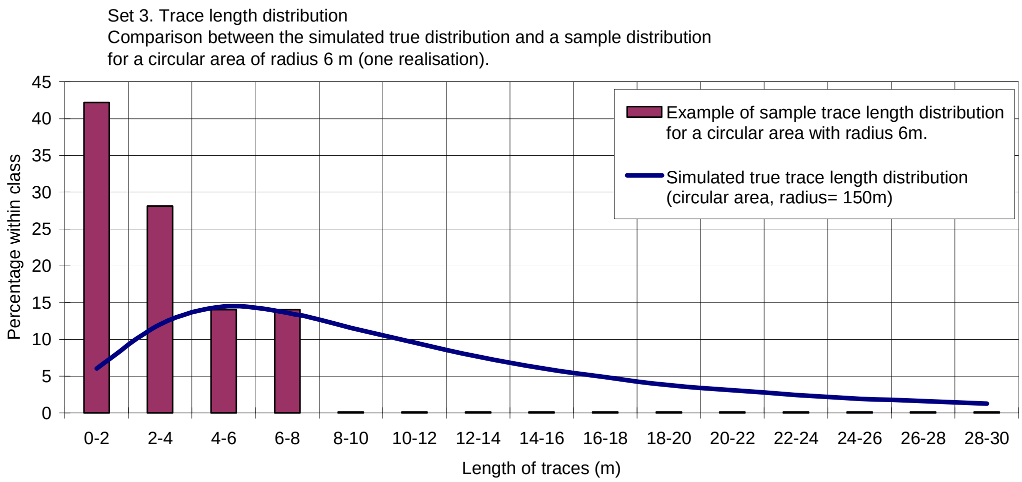


SET 2: Window of radius 20 m (diameter 40 m), one realisation.

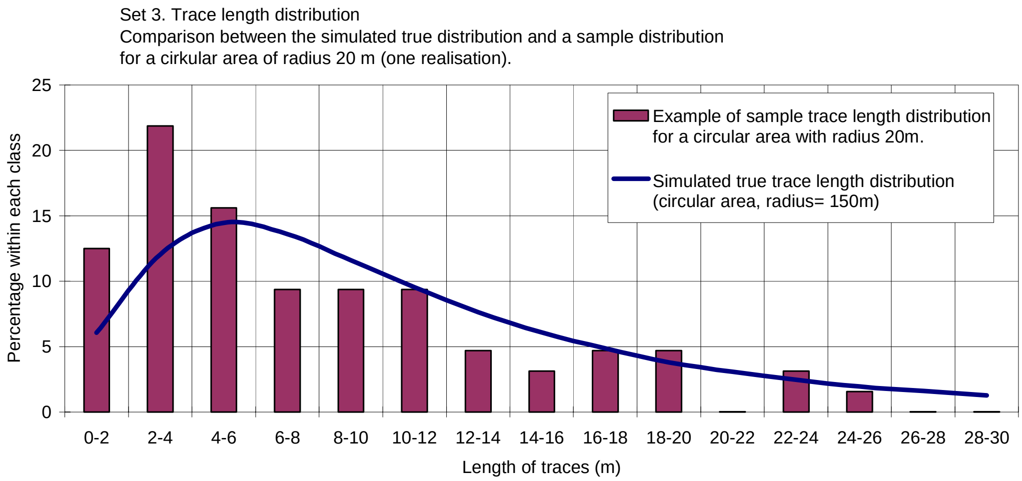


SET 2: Window of radius 52 m (diameter 104 m), one realisation.

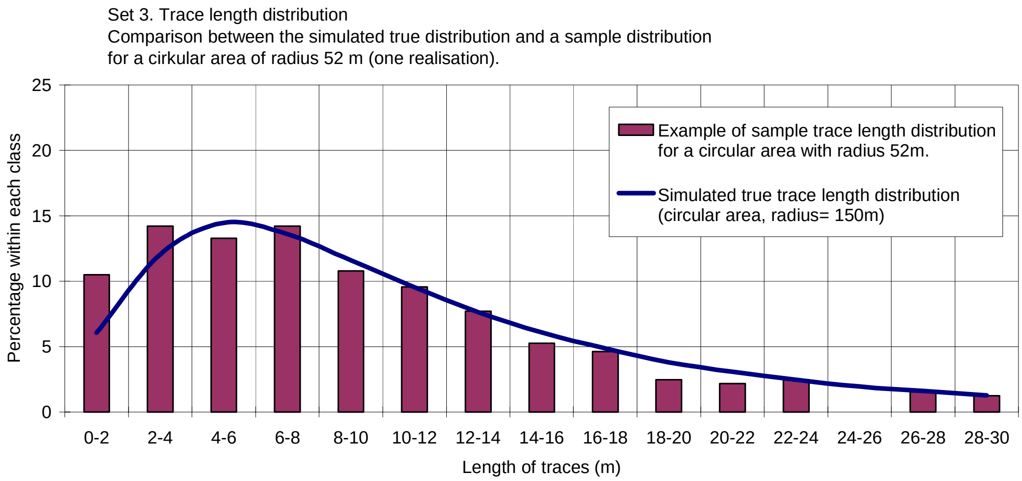
Figure 6-4. *SET 2: Comparison between the simulated true trace-length distribution and examples of sample trace-length distributions for windows of different radii.*



SET 3: Window of radius 6 m (diameter 12 m), one realisation.



SET 3: Window of radius 20 m (diameter 40 m), one realisation.



SET 3: Window of radius 52 m (diameter 104 m), one realisation.

Figure 6-5. *SET 3: Comparison between the simulated true trace-length distribution and examples of sample trace-length distributions for windows of different radii.*

The efficiency of a point estimate increases with sample size, however for the sampling of traces also the size of the studied window is important. The observations are made on windows that have a limited size, and the upper tail of the trace-length distribution (traces with a large length) can only be directly observed on windows of a size (radius) comparable to length of the large traces. Hence, for small windows there will be a systematic bias in the estimate of the trace-length distribution, due to boundary truncation, even if the sample size is large. (Small window sizes could be sufficient if it is possible to fit a mathematical distribution to the observed truncated trace-length distributions, even if such a curve fitting procedure will introduce uncertainty regarding the ability of such a distribution to represent the part of the true distribution that is unknown at small window sizes.)

It is not a purpose of this study to derive the fracture diameter distributions of the different fracture sets. The purpose is to study the trace-length distributions of the three fracture sets, considering how the characteristic of these trace-length distributions changes with size of window, and to find at what window sizes the distributions studied are well defined and stable.

We have divided the traces into different sets; therefore we will be able to compare the efficiency of the point estimates for the different sets. It is possible that a small window is sufficient for deriving a trace-length distribution of a set that mainly includes small fractures, but for sets that include large fractures, large window sizes are necessary. Considering different theoretical fracture sets, the necessary window sizes for deriving reliable trace-length distributions for such sets depend on (i) the properties of these fracture sets (orientation, fracture size, fracture density etc) and (ii) how the different observed fracture traces are classified into the different fracture sets.

6.3.2 Point estimate of the moments of the observed distribution

The efficiency of the point estimate of the mean and standard deviation of the trace-length distribution is given in Figure 6-6 and Figure 6-7, below. Considering a circular window of radius 150 m, the point estimate produces the following results:

<u>Set 1</u>

Mean values of trace-length distribution, window radius = 150m.

Mean of mean values = 6.09 m

Standard deviation of mean values = 1.9% of mean of mean values.

Standard deviation of trace-length distribution, window radius = 150m.

Mean of standard deviation values = 6.51 m

Standard deviation of standard deviation values = 0.3% of mean of stand.dev.values.

Set 2

Mean values of trace-length distribution, window radius = 150m.

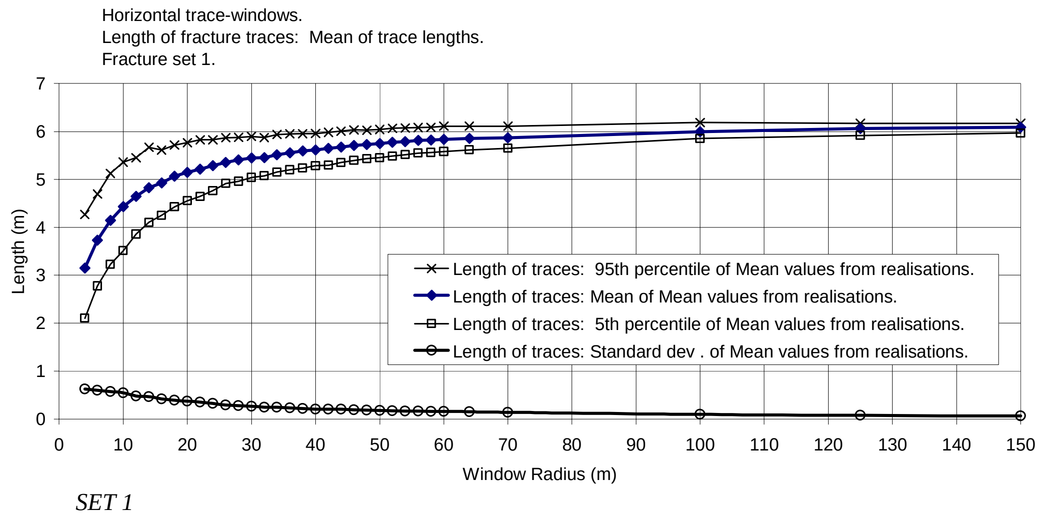
Mean of mean values = 12.39 m

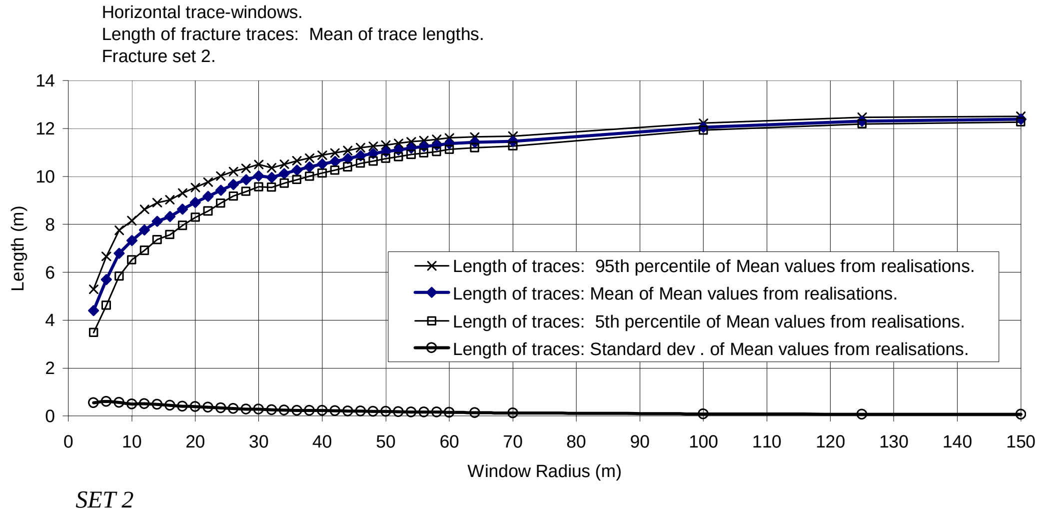
Standard deviation of mean values = 0.2% of mean of mean values.

Standard deviation of trace-length distribution, window radius = 150m.

Mean of standard deviation values = 5.56 m

Standard deviation of standard deviation values = 1.3% of mean of stand.dev.values.





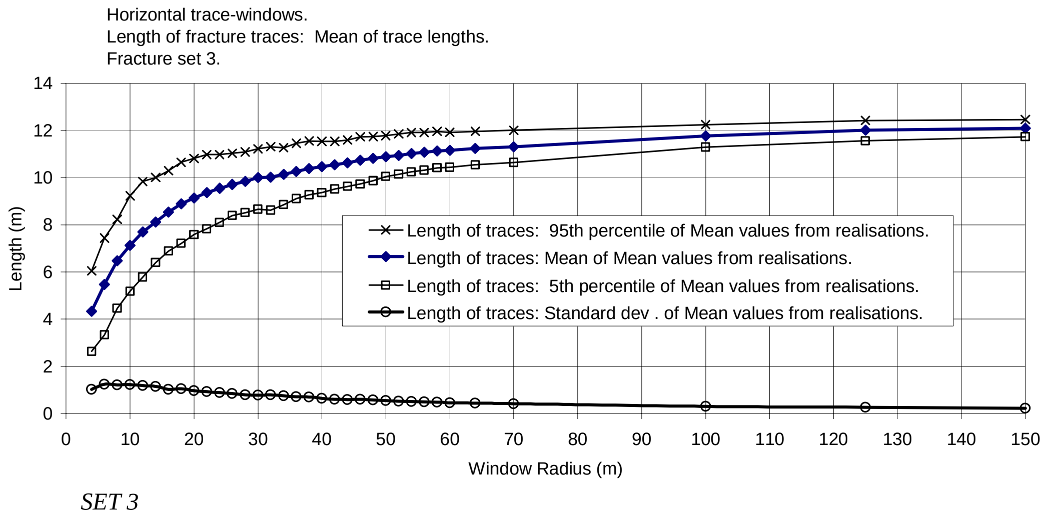
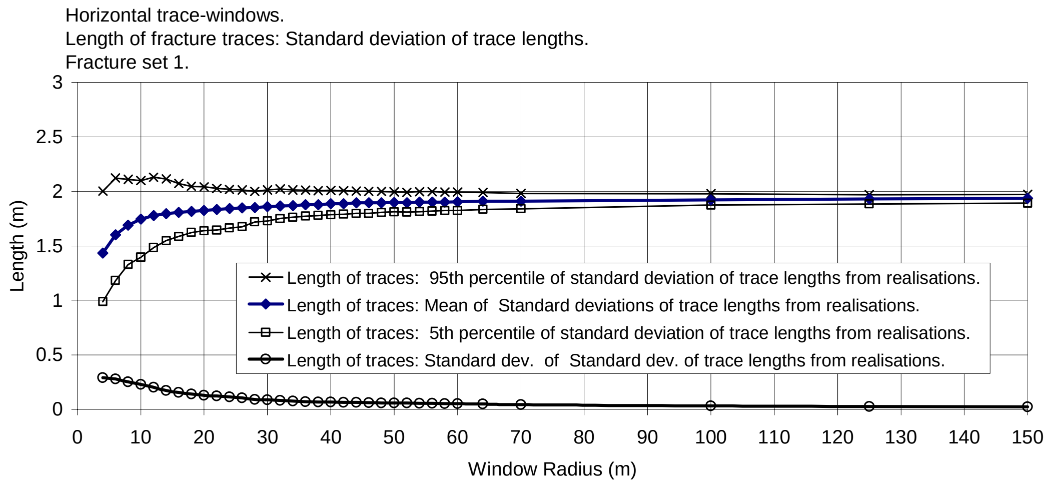
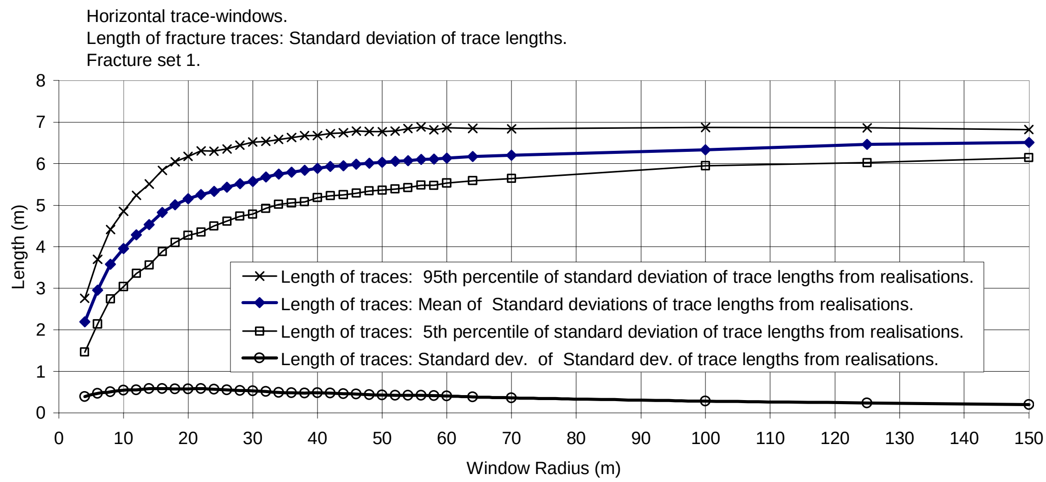


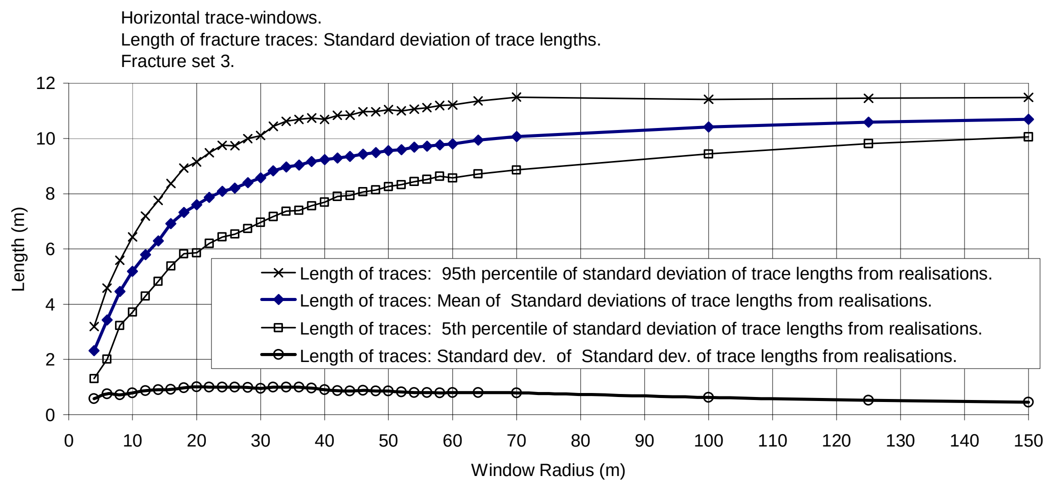
Figure 6-6. *Efficiency of the point estimate of the mean value of the observed trace-length distribution, considering windows of different sizes.*



SET 1



SET 2



SET 3

Figure 6-7. *Efficiency of the point estimate of the standard deviation of the observed trace-length distribution, considering windows of different sizes*

Set 3

Mean values of trace-length distribution, window radius = 150m.

Mean of mean values = 12.10 m

Standard deviation of mean values = 0.6% of mean of mean values.

Standard deviation of trace-length distribution, window radius = 150m.

Mean of standard deviation values = 10.70 m

Standard deviation of standard deviation values = 0.2% of mean of stand.dev.values.

Analysing the figures that presents the efficiency of the point estimate of the mean and standard deviation of the trace-length distribution (Figure 6-6 and Figure 6-7). It is concluded that the change in mean and standard deviation of the distribution is small for windows with a radius larger than 150 m. This is of interest, as we have no knowledge of the true characteristics of the distribution studied (as given by the population). The sample trace-length distribution for a window of radius 150 m is set as the true distribution, and it is called the simulated true distribution.

6.3.3 Point estimate of the moments of a log normal distribution fitted to the observed distribution

As previously discussed, for Set 1 and Set 3 the simulated true trace-length distribution has tendency towards a log-normal or an exponential shape. It follows that it is possible to fit log-normal curves (or exponential curves) to the observed trace-length distributions. An advantage that comes with fitting a mathematical probability distribution to the observed trace-length distributions, is that such distributions will include an upper tail of the trace-length distribution, which otherwise is truncated at small window sizes. (As previously discussed there are other methods available, of varying efficiency and applicability, for correcting the sample distributing as regards boundary-truncation etc). Another advantage that comes with fitting a mathematical function (probability distribution) to an observed trace-length distribution is that such a function can be mathematically analysed and developed; for example if one tries to derive an analytical relationship between trace-length and a fracture size distributions.

An observed variable which has a distribution that resembles a log-normal distribution can have either (i) a normal curve fitted to a histogram of the logarithms of the values or (ii) a log-normal curve fitted to the actual values. Opinions differ as to which method of representation is more effective. In this study we have fitted a normal curve to the logarithms of the observed trace-lengths by use of a simple and robust approach; the mean and standard deviation of the logarithms of the observed trace-lengths were calculated, and these two moments were then used for definition of the log-normal distribution. Logarithms to any base can be employed to "correct" the skew of distributions that demonstrate a tendency to a log-normal distribution, and render a normal distribution, but logarithms to bas e (natural logarithms) are mathematically most convenient.

The underlying data-the trace-length distributions-demonstrate non-symmetric (skewed) shapes, but they are not distributed as perfect log-normal distributions, therefore it is necessary to be careful when comparing log-normal distributions derived in different ways from the underlying data. In addition there is variance between different realisations of the trace-length distributions. Even when studying the simulated true distribution (windows of radius 150 m) there is a variance between different realisations, but it is very small at a such a large window. It follows that, for a window of a given size, different distributions (modes) will be derived for the following two alternatives: (i) Fitting one log-normal distribution to all trace-lengths that are obtained from a large number of realisations and calculate the modes of this distribution. (ii) Fitting log-normal distributions to the trace-lengths of each realisations and calculate mean values of the obtained modes of the different distributions. However, for the large window (radius 150m) used for deriving the true trace-length distributions, the results obtained from the two methods are very similar; in this study we have used method (i) when deriving the log-normal distribution representing the simulated true trace-length distribution.

Considering the simulated true trace-lengths (radius of windows = 150 m), the following moments were obtained by use of natural logarithms (eLog[trace_length]):

Set 1: Mean, μ_{eLog} = 1.38 m , Standard deviation, σ_{eLog} = 0.96 m.

Set 3: Mean, μ_{eLog} = 2.15 m , Standard deviation, σ_{eLog} = 0.90 m.

The corresponding values for Set 2 are given below, but as stated above and demonstrated in Figure 6-10 (below), the trace-length distribution of Set 2 is not well represented by a log-normal distribution. Set2: Mean, μ_{eLog} = 2.35 m , Standard deviation, σ_{eLog} = 0.70 m.

Based on the values above, the moments of the corresponding log-normal distribution can be calculated as follows, see /Dudewicz and Mishra, 1988; Williams, 1984/:

$$Mean,\ \mu = e^{(\mu_{eLog} + 0.5\sigma^2_{eLog})} \qquad 6\text{-}1$$

$$STD,\ \sigma = \sqrt{e^{(2\mu_{eLog} + \sigma^2_{eLog})}\left(e^{\sigma^2_{eLog}} - 1\right)}$$

$$Median = e^{\mu_{eLog}}$$

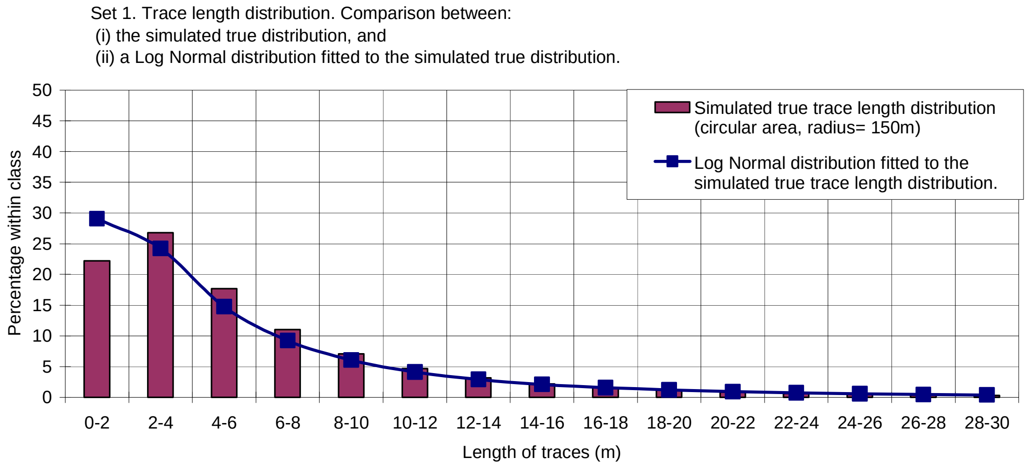
The log normal distributions, representing the simulated true trace-length distribution, will have the following moments:

Set 1: Mean, μ = 6.32 m , Standard deviation (STD), σ = 7.75 m.

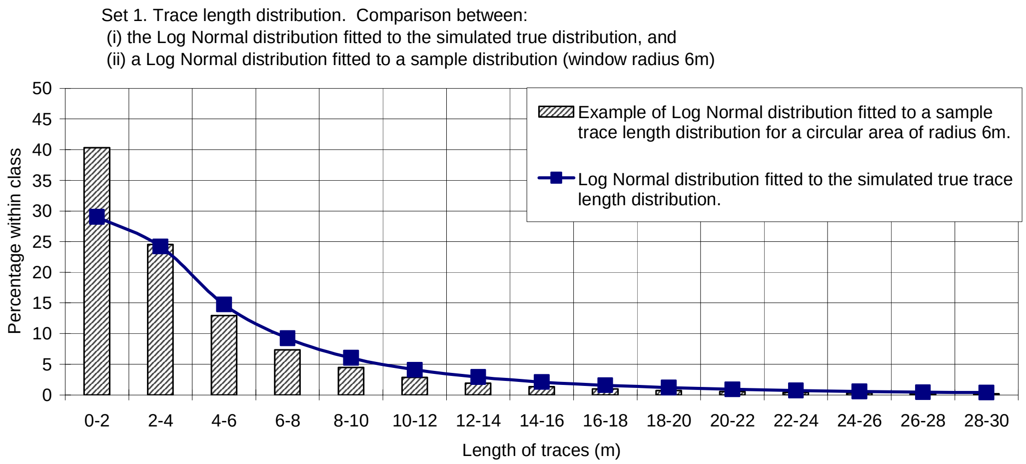
Set 3: Mean, μ = 12.85 m , Standard deviation (STD), σ = 14.38 m.

The resulting distributions are given in Figure 6-8(i),

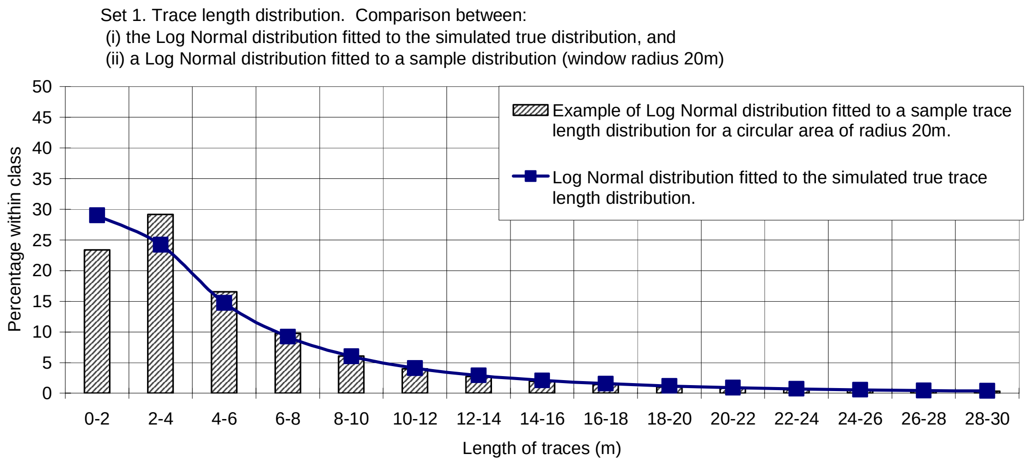
Figure 6-9(i) and Figure 6-10(i). Note that the trace-length distribution of Set 2 is not well represented by a log-normal distribution. Examples of a log-normal curve fitting is given below in Figure 6-8 and Figure 6-9. The log-normal curves were fitted to the sample trace-length distribution by use of the following method, the mean and standard deviation of the natural logarithms of the observed trace-lengths were calculated, and these two moments were then used for definition of the log-normal distribution. This is an efficient method, if the observed values are distributed according to a perfect log-normal distribution, the method will produce the correct moments for such a distribution (by use of Eq. 7-1).



(i) SET 1. Simulated true trace-length distribution and fitted Log-Normal distribution

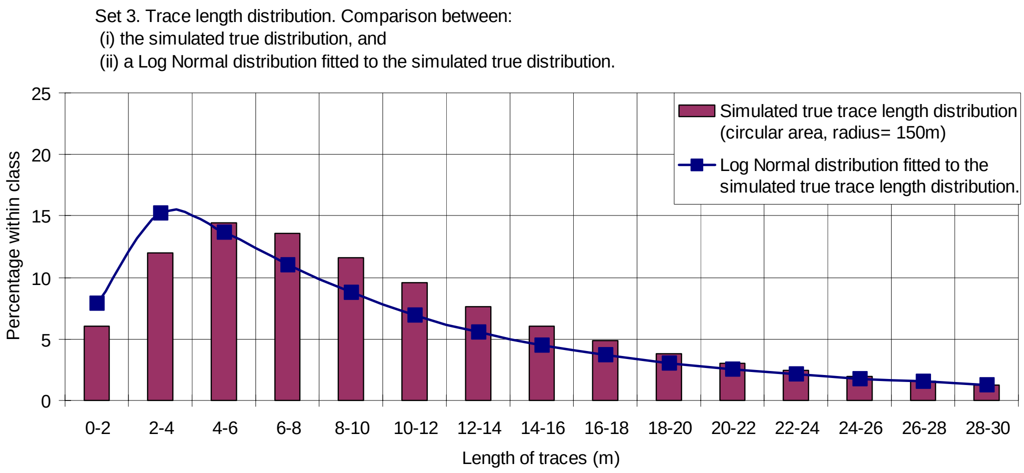


(ii) SET 1. Log-Normal distribution fitted to the true trace-length distribution and a log normal distribution fitted to a sample distribution taken from a window of radius 6m.

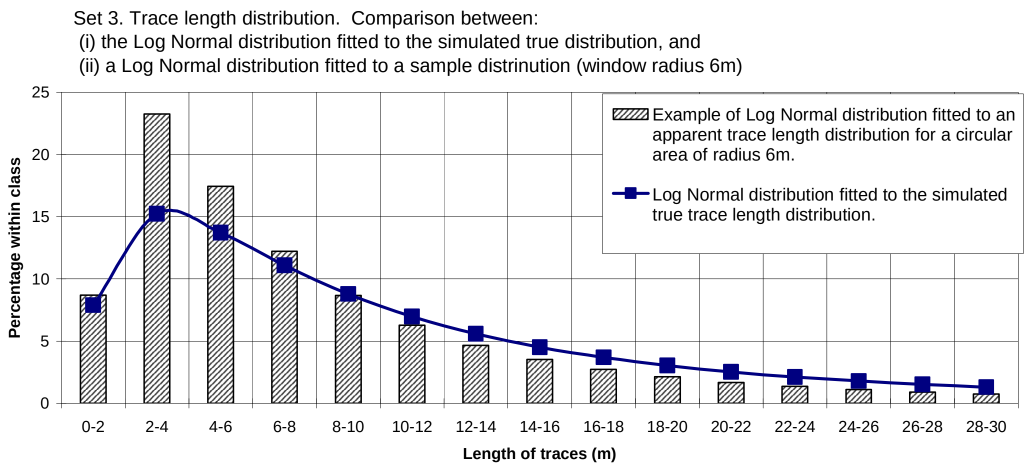


(iii) SET 1. Log-Normal distribution fitted to the true trace-length distribution and a log normal distribution fitted to a sample distribution taken from a window of radius 20m.

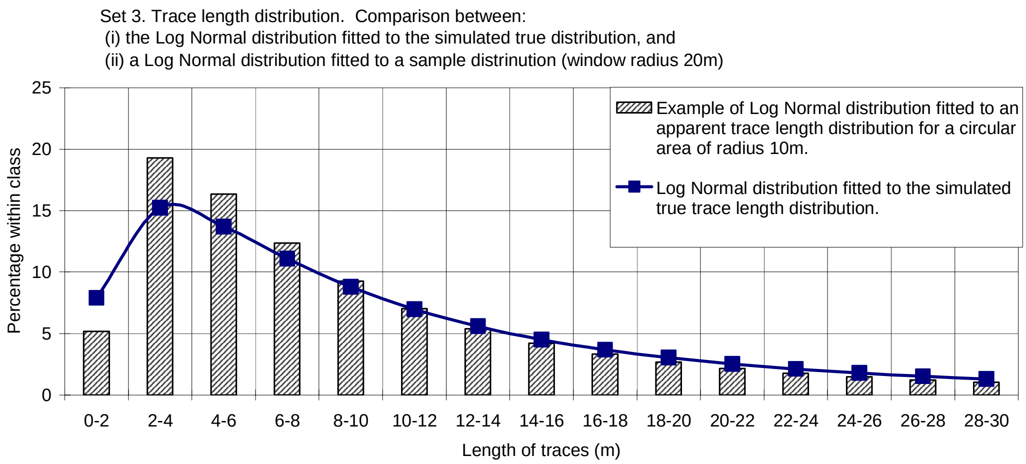
Figure 6-8. *SET 1: Comparison between the simulated true trace-length distribution and a log-normal distribution fitted to the simulated true distribution, and comparisons between fitted Log-Normal distributions at different window sizes. Note that the first part of the Log-Normal distributions (with increasing values) are inside the first class of trace-lengths.*



(i) SET 3. Simulated true trace-length distribution and fitted Log-Normal distribution



(ii) SET 3. Log-Normal distribution fitted to the true trace-length distribution and a log normal distribution fitted to a sample distribution taken from a window of radius 6m.



(iii) SET 3. Log-Normal distribution fitted to the true trace-length distribution and a log normal distribution fitted to a sample distribution taken from a window of radius 20m.

Figure 6-9. *SET 3: Comparison between the simulated true trace-length distribution and a log-normal distribution fitted to the simulated true distribution, and comparisons between fitted Log-Normal distributions at different window sizes.*

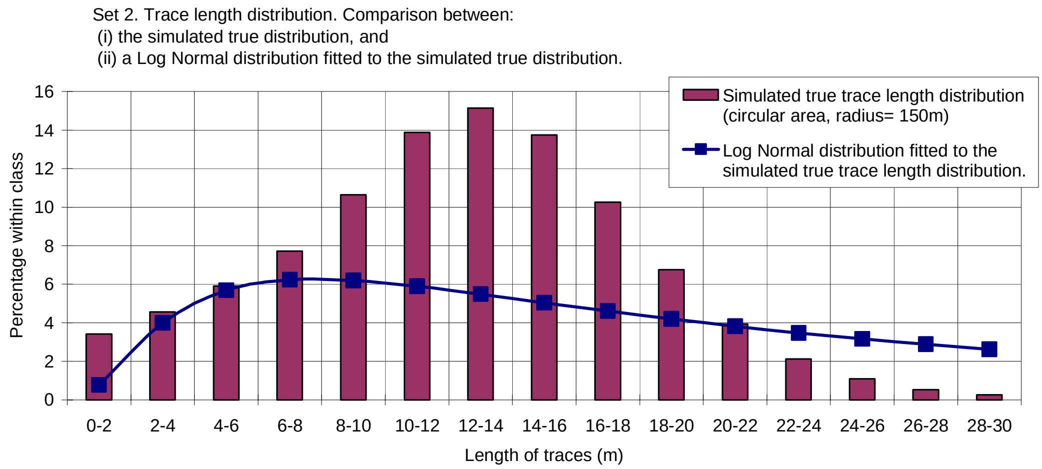
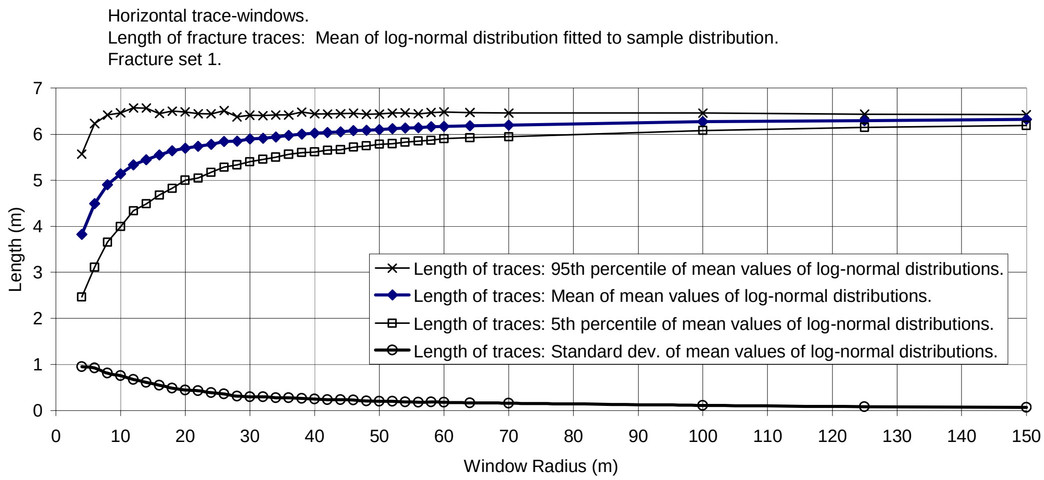


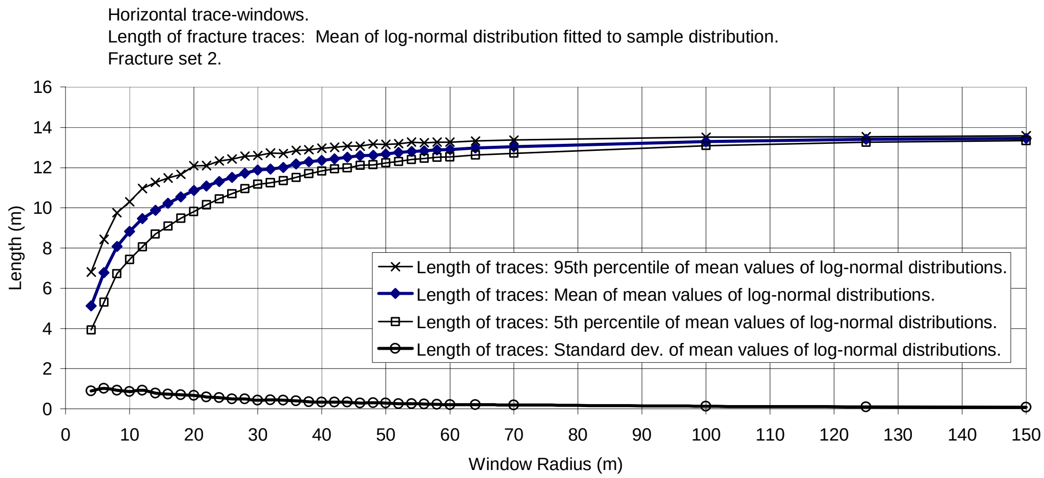
Figure 6-10. *SET 2: Comparison between the simulated true trace-length distribution and a log-normal distribution fitted to the simulated true distribution. As seen in the figure, the trace-length distribution of Set 2 is not well represented by a log-normal distribution.*

When studying the figures above it is obvious that the trace-length distribution of Set 2 is not well represented by a log-normal distribution; but also considering the trace-length distributions of Set 1 and Set 3, the log-normal representation is not a very good representation. The log-normal distributions tend to overestimate the number of very small and very large traces, and underestimate the number of traces having a length close to the mean of the trace-length distributions.

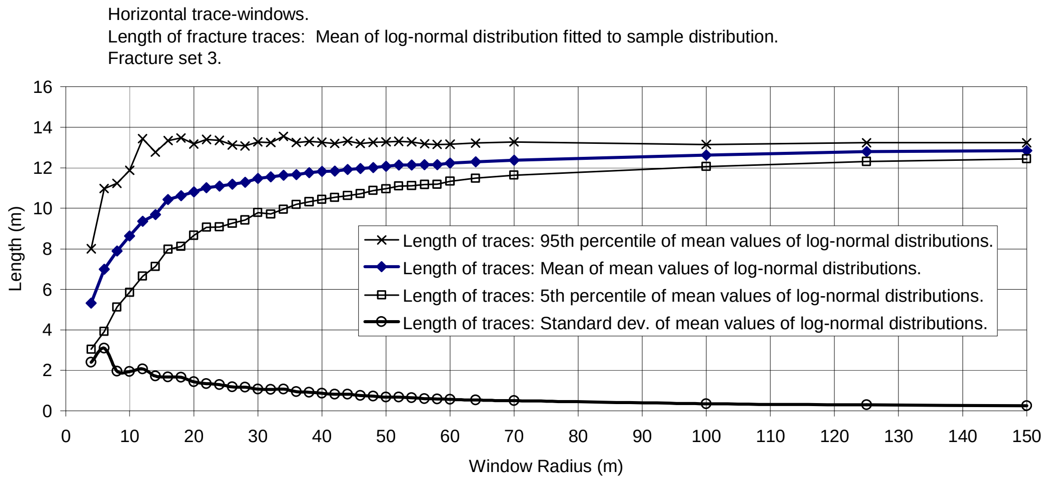
The efficiency of the point estimate of the mean and standard deviation of the log-normal distributions representing the observed trace-length distributions are not the same as the efficiency of the point estimate of the observed distributions. The efficiency of the point estimate as regards the log-normal distributions are given in Figure 6-11 and Figure 6-12.



SET 1

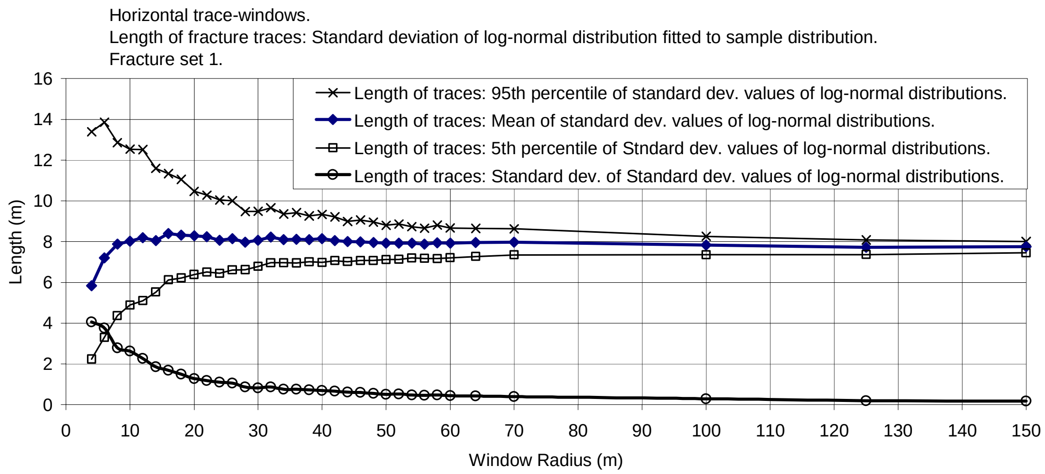


SET 2

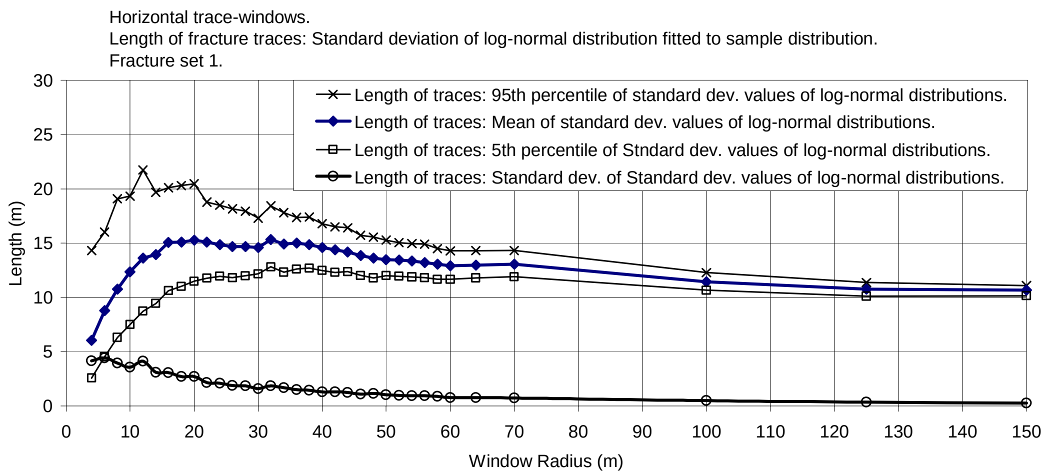


SET 3

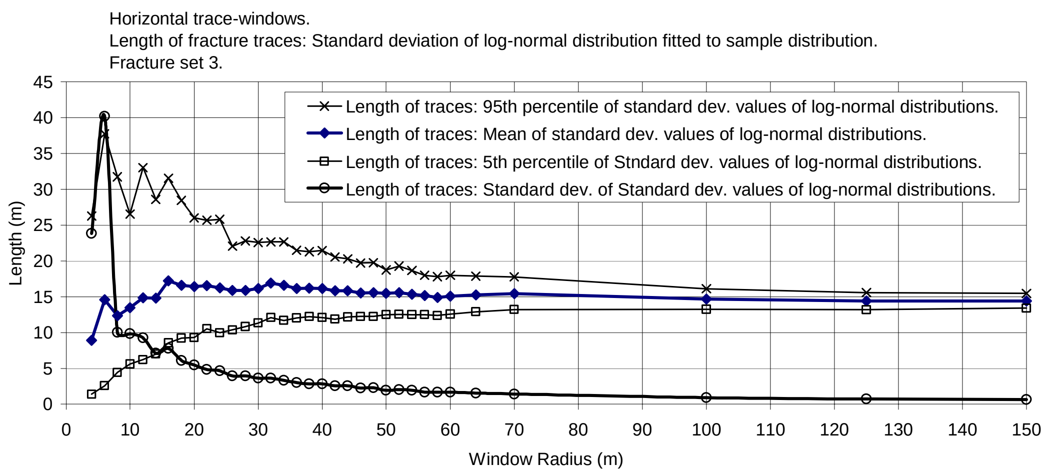
Figure 6-11. *Efficiency of the point estimate of the mean value of the log-normal distributions fitted to the observed trace-length distribution, considering windows of different sizes.*



SET 1



SET 2



SET 3

Figure 6-12. *Efficiency of the point estimate of the standard deviation of the log-normal distributions fitted to the observed trace-length distribution, considering windows of different sizes.*

6.4 Hypothesis testing considering the moments of the trace-length distribution and acceptable deviations

6.4.1 Purpose of tests

The purpose of these tests are to determine when the size of the sample is large enough to produce acceptable estimates of the properties of the simulated true trace-length distributions, with a certain probability. The hypothesis testing of this section is based on the moments (mean and standard deviation) of the studied distributions of trace-lengths. Considering the somewhat complex shape of the simulated true distribution, one may wonder why we are interested in the moments and not of the actual shape of the distribution. The answer is that we are interested in both – this section presents a test of the moments and the next section presents a test of the shape. The moments are of interest, because when the samples produce a good estimate of the true moments, with a large probability, this is an indication that the shape of the distribution is stable and we have found a sample size large enough for prediction of the true properties.

6.4.2 Test for the sample distributions

Null hypothesis, acceptable deviations and criterion of significance

The samples were analysed by a statistical hypothesis testing. The hypothesis testing of this section is based on the moments (mean and standard deviation) of the studied distributions of trace-lengths and given criterions of significance. A difficulty is that we do not know the true mean and standard deviation of the population studied. The established criterions of significance will therefore correspond to the mean values derived from a very large sample (the simulated true distribution). This is an acceptable method as the mean values are stable at such a large sample.

The null hypothesis (H_0) is that a sample is a good representation of the true properties of the population. This hypothesis is rejected if the deviation between the values of the sample and the true values of the population (simulated) is large. The following criterions of significance are used: three criterions for the mean of the distribution and three criterions for standard deviation of the distribution. The criterions represent different aspects of the distribution and different levels of significance.

<u>Criterions for mean of distribution:</u>

First criterion: $H_{0\ (\text{Mean\_deviation} <=15\%)}$ is rejected if:

$$\text{ABS}[\text{Mean}_{(\text{sample})} - \text{Mean}_{(\text{simulated true})}] >= 0.15^* \,\text{Mean}_{(\text{simulated true})}$$

Second criterion: $H_{0\ (\text{Mean\_deviation} <=10\%)}$ is rejected if:

$$\text{ABS}[\text{Mean}_{(\text{sample})} - \text{Mean}_{(\text{simulated true})}] >= 0.10^* \,\text{Mean}_{(\text{simulated true})}$$

Third criterion: $H_{0\ (\text{Mean\_deviation} <=5\%)}$ is rejected if:

$$\text{ABS}[\text{Mean}_{(\text{sample})} - \text{Mean}_{(\text{simulated true})}] >= 0.05^* \,\text{Mean}_{(\text{simulated true})}$$

Criterions for standard deviation (STD) of distribution:

First criterion: $H_{0\ (STD\_deviation <=15\%)}$ is rejected if:

$ABS[STD_{(sample)} - STD_{(simulated\ true)}] >= 0.15 * STD_{(simulated\ true)}$

Second criterion: $H_{0\ (STD\_deviation <=10\%)}$ is rejected if:

$ABS[STD_{(sample)} - STD_{(simulated\ true)}] >= 0.10 * STD_{(simulated\ true)}$

Third criterion: $H_{0\ (STD\_deviation <=5\%)}$ is rejected if:

$ABS[STD_{(sample)} - STD_{(simulated\ true)}] >= 0.05 * STD_{(simulated\ true)}$

These tests are carried out separately, considering the traces of each fracture set. The results of the analysis are presented as the probability that a sample, at a certain window size, will not be rejected, considering the criterions above.

Results considering mean and standard deviation of trace-lengths

Results are given for two different moments, mean and standard deviation. The results are given in Figure 6-13 (below). Generally the following is demonstrated. To estimate the mean of the trace-length distribution, with a small uncertainty, the radius of the window studied needs to be much larger (e.g. more than two times larger) than the mean of the fracture diameter distribution. If the standard deviation of the trace-length distribution is much smaller than the mean of the distribution, the standard deviation might be estimated (with a small uncertainty) using much smaller windows than the windows necessary for producing a good estimate of the mean of the distribution. We conclude the following results.

SET 1:

If the radius of the window studied is larger than 32 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the simulated true value of the population.

If the radius of the window studied is larger than 52 m, the probability is larger than 90 percent that the deviation in estimated standard deviation value is within plus/minus 15 percent of the simulated true value of the population.

SET 2:

If the radius of the window studied is larger than 45 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the simulated true mean value of the population

If the radius of the window studied is larger than 12 m, the probability is larger than 90 percent that the deviation in estimated standard deviation value is within plus/minus 15 percent of the simulated true value of the population

SET 3:

If the radius of the window studied is larger than 52 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the simulated true mean value of the population

If the radius of the window studied is larger than 70 m, the probability is larger than 90 percent that the deviation in estimated standard deviation value is within plus/minus 15 percent of the simulated true value of the population

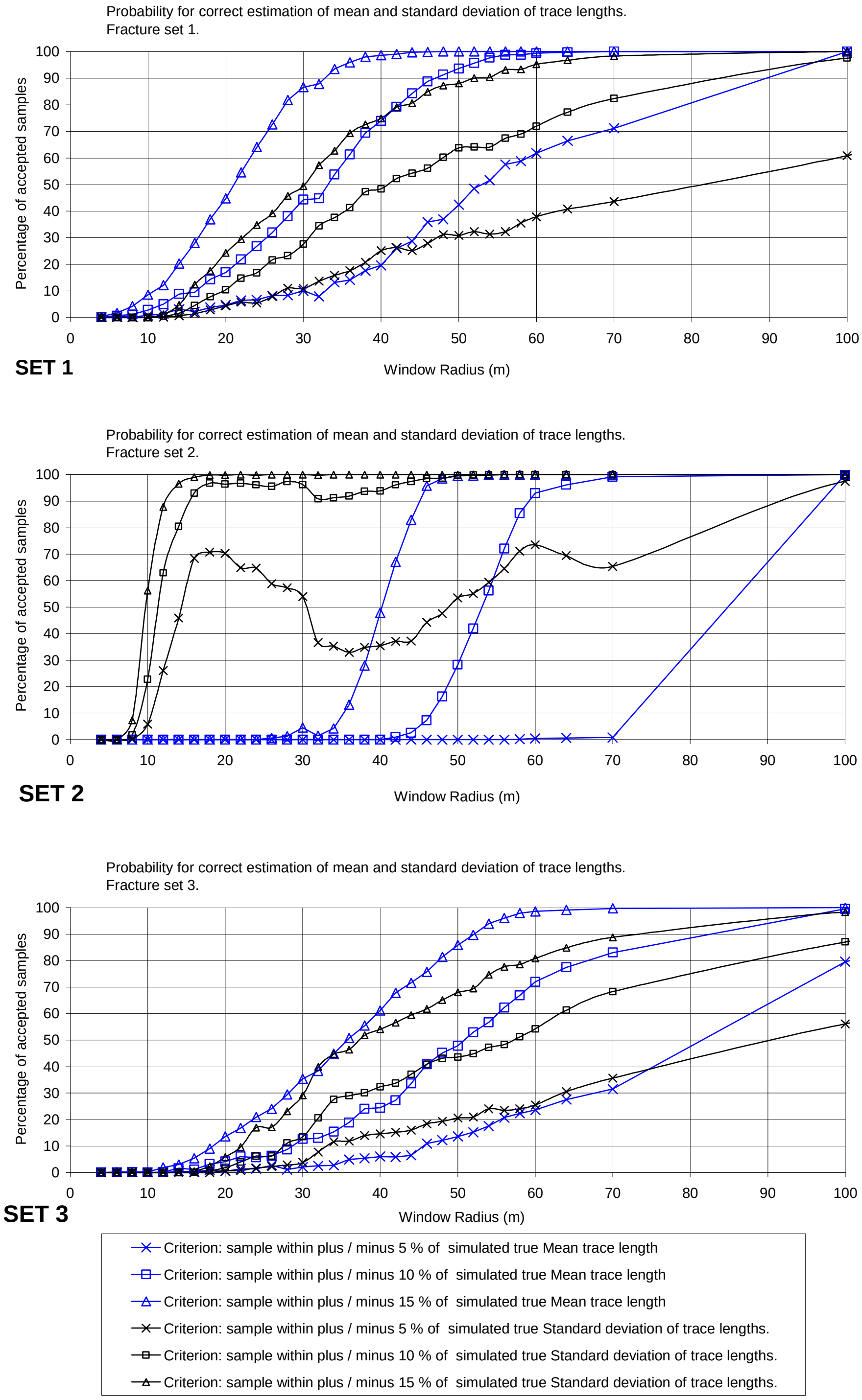


Figure 6-13. *SET 1, SET 2 and SET 3: Hypothesis testing for selected acceptable deviations in predicted mean and standard deviation of trace-length distribution. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, considering the different selected criterions.*

6.4.3 Test for the log-normal distributions fitted to the sample distributions

Null hypothesis, acceptable deviations and criterion of significance

The samples were analysed by a statistical hypothesis testing. The hypothesis testing of this section is based on (i) the moments of the log-normal distributions fitted to the sample distributions, as discussed in Section 6.3.3; and (ii) given criterions of significance. The established criterions of significance will correspond to the log-normal distribution fitted to the simulated true distribution (see Figure 6-8 and Figure 6-9). The criterions refer to the mean and standard deviation of the distribution, in normal space, and not in log-space. The trace-length distribution of Set 2 will not be included in these tests, as this distribution is not well represented by a log-normal distribution.

The null hypothesis (H_0) is that a sample is a good representation of the true properties of the population. This hypothesis is rejected if the deviation is large between the values of the sample and the true values of the population (simulated). The following criterions of significance are used: three criterions for the mean of the distribution and three criterions for standard deviation of the distribution. The criterions represent different aspects of the distribution and different levels of significance.

The criterions below refer to properties of the log-normal distributions fitted to the data, and not directly to the properties of the sample data. It follows that the ranges of accepted samples, considering the log-normal distributions, are not the same as the ranges of accepted samples when considering the sample distribution

<u>Criterions for mean of fitted log-normal distribution:</u>

First criterion: $H_{0\ (\text{Mean\_deviation} <=15\%)}$ is rejected if:

$\text{ABS}[\text{Mean}_{(\text{sample log-normal})} - \text{Mean}_{(\text{simulated true log-normal})}] >= 0.15 * \text{Mean}_{(\text{simulated true log-normal})}$

Second criterion: $H_{0\ (\text{Mean\_deviation} <=10\%)}$ is rejected if:

$\text{ABS}[\text{Mean}_{(\text{sample log-normal})} - \text{Mean}_{(\text{simulated true log-normal})}] >= 0.10 * \text{Mean}_{(\text{simulated true log-normal})}$

Third criterion: $H_{0\ (\text{Mean\_deviation} <=5\%)}$ is rejected if:

$\text{ABS}[\text{Mean}_{(\text{sample log-normal})} - \text{Mean}_{(\text{simulated true log-normal})}] >= 0.05 * \text{Mean}_{(\text{simulated true log-normal})}$

<u>Criterions for standard deviation (STD) of fitted log-normal distribution</u>:

First criterion: $H_{0\ (\text{STD\_deviation} <=15\%)}$ is rejected if:

$\text{ABS}[\text{STD}_{(\text{sample log-normal})} - \text{STD}_{(\text{simulated true log-normal})}] >= 0.15 * \text{STD}_{(\text{simulated true log-normal})}$

Second criterion: $H_{0\ (\text{STD\_deviation} <=10\%)}$ is rejected if:

$\text{ABS}[\text{STD}_{(\text{sample log-normal})} - \text{STD}_{(\text{simulated true log-normal})}] >= 0.10 * \text{STD}_{(\text{simulated true log-normal})}$

Third criterion: $H_{0\ (\text{STD\_deviation} <=5\%)}$ is rejected if:

$\text{ABS}[\text{STD}_{(\text{sample log-normal})} - \text{STD}_{(\text{simulated true log-normal})}] >= 0.05 * \text{STD}_{(\text{simulated true log-normal})}$

These tests are carried out separately, considering the traces of each fracture set (Set 1 and Set 3). The results of the analysis are presented as the probability that a sample, at a certain window size, will not be rejected, considering the criterions above.

Results considering mean and standard dev of log-normal distributions

Results are given for two different moments, mean and standard deviation, see Figure 6-14 below. (The trace-length distribution of Set 2 will not be included in these tests, as this distribution is not well represented by a log-normal distribution.)

The efficiency of a point estimate refers to the rate with which the estimates converge towards the true values, considering sample size. When comparing two point estimates, the most efficient point estimate is the one that for the smallest samples produces the estimates closest to the true values.

The efficiency of the point estimates considering the moments of the log-normal distributions is not the same as the efficiency of the point estimates considering the moments of the sample distributions. For Set 1 and Set 3, the point estimate considering the log-normal distributions is more efficient than the estimates concerning the sample distribution. This is because Set 1 and Set 3 are well represented by a log-normal distributions.

We conclude the following the following results.

SET 1:

If the radius of the window studied is larger than 25 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the simulated true mean value of the population

If the radius of the window studied is larger than 42 m, the probability is larger than 90 percent that the deviation in estimated standard deviation value is within plus/minus 15 percent of the simulated true value of the population

SET 3:

If the radius of the window studied is larger than 45 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the simulated true mean value of the population

If the radius of the window studied is larger than 65 m, the probability is larger than 90 percent that the deviation in estimated standard deviation value is within plus/minus 15 percent of the simulated true value of the population

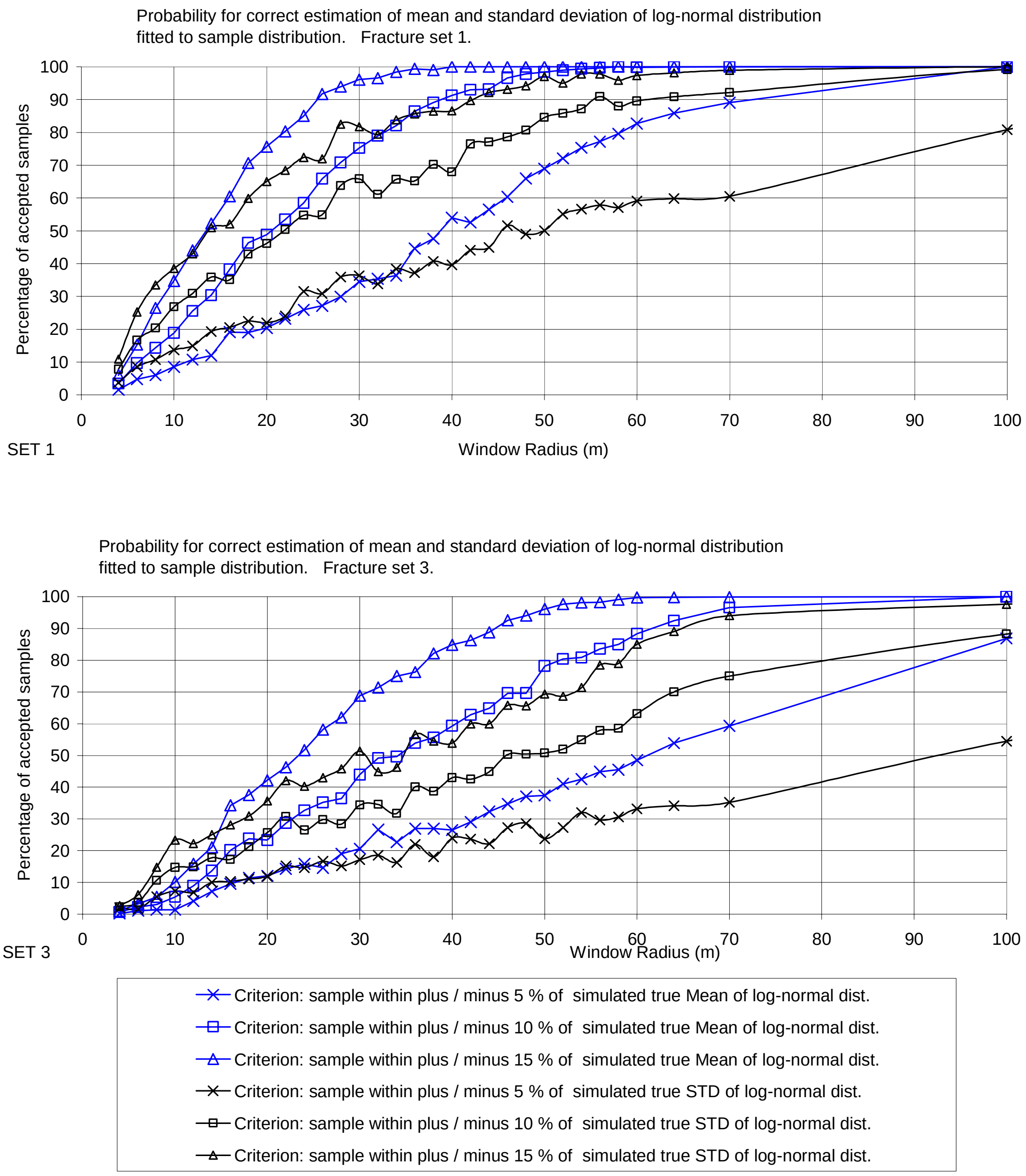


Figure 6-14. *SET 1 and SET 3: Hypothesis testing for selected acceptable deviations in predicted mean and standard deviation of log-normal distributions fitted to the trace-length distributions. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, considering the different selected criterions.*

6.5 Hypothesis testing considering the shape of the trace-length distribution and given confidence levels

6.5.1 Purpose of tests

The purpose of this test is to determine when the size of the sample is large enough to produce an acceptable estimate of the true properties of the trace-length distribution, at a certain given level of confidence. The hypothesis testing of this section are based on the shapes of the distributions studied (tests as regard the moments of the studied distributions are given in the previous section). Distribution shape is of interest as it characterises the distribution studied, and when samples produce a good estimate of the shape of the true distribution, with a large probability, the samples are large enough for prediction of the true properties.

6.5.2 Methodology of the chi-square test

Tests of the shape of the trace-length distributions were carried out as chi-square tests of "goodness-of-fit". The observed trace-lengths are grouped into classes and the frequencies are compared with the expected frequencies, as given by the simulated true trace-length distributions.

Null hypothesis and confidence levels

The samples were analysed by use of statistical hypothesis testing. The hypothesis testing were based on the shape of the trace-length distributions and given confidence levels. Examples of trace-length distributions from samples are given in Figure 6-3, Figure 6-4 and Figure 6-5. The sample distributions are compared to the simulated true distribution, this distribution is presented in Figure 6-2.

The null hypothesis (H_0) is that a sample is a good representation of the simulated true distribution. The hypothesis is rejected if, when comparing the sample and the true distribution, it is found that the deviations between the two distributions are large. The confidence level gives the size of deviation that is acceptable, a deviation larger than this is considered as a significant deviation.

The confidence level should be selected in a way that the probability for rejection of the hypothesis is small if the hypothesis is true. We have studied three different levels of confidence: 90, 95 and 99 percent. The hypothesis tests are as follows:

- First confidence level 99%: The hypothesis, $H_{0\ (C=99\%)}$ is rejected if the sample deviates significantly at this level of confidence.
- Second confidence level 95%: The hypothesis, $H_{0\ (C=95\%)}$ is rejected if the sample deviates significantly at this level of confidence.
- Third confidence level 90%: The hypothesis, $H_{0\ (C=90\%)}$ is rejected if the sample deviates significantly at this level of confidence.

The tests are carried out separately, considering the traces-length distribution of each fracture set. For each confidence level, the result of the analysis is presented as the percentage of accepted samples at different window sizes.

Results – chi-square test of goodness-of-fit

The results of the chi-square tests are given in Figure 6-15, below. The figure presents the results for three different confidence levels. In a goodness-of-fit test, the shape of the distributions are tested, and the deviations in mean and spread may balance each other in a way that the shape is accepted although the moments are not well predicted.

We conclude the following results.

SET 1:

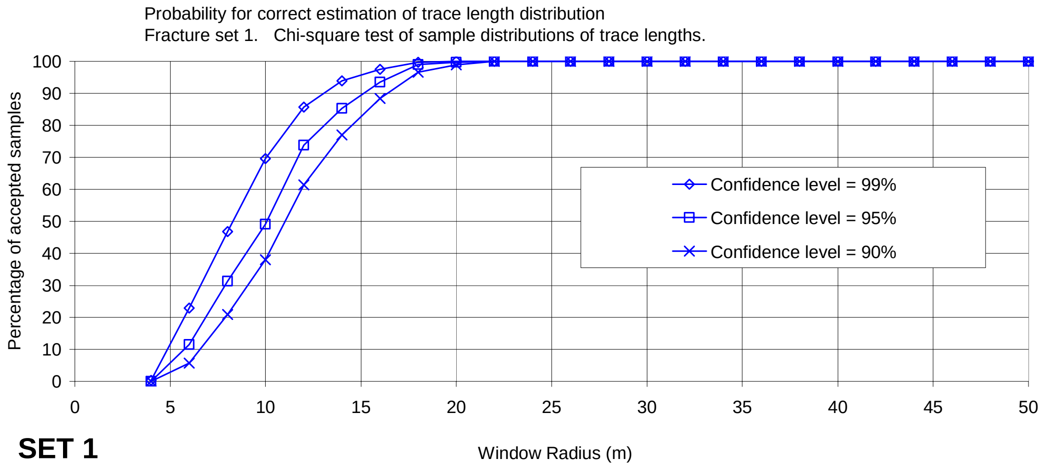
For a window with radius larger than 13 m, the probability is larger than 90 percent that a sample will not be rejected at the first level of confidence ($H_{0\ (C=99\%\ )}$). Or with other words. If the radius of the window studied is larger than 13 m, the probability is larger than 90 percent that the shape of the trace-length distribution derived from a sample is a good representation of the simulated true distribution, at a confidence level of 99 percent.

SET 2:

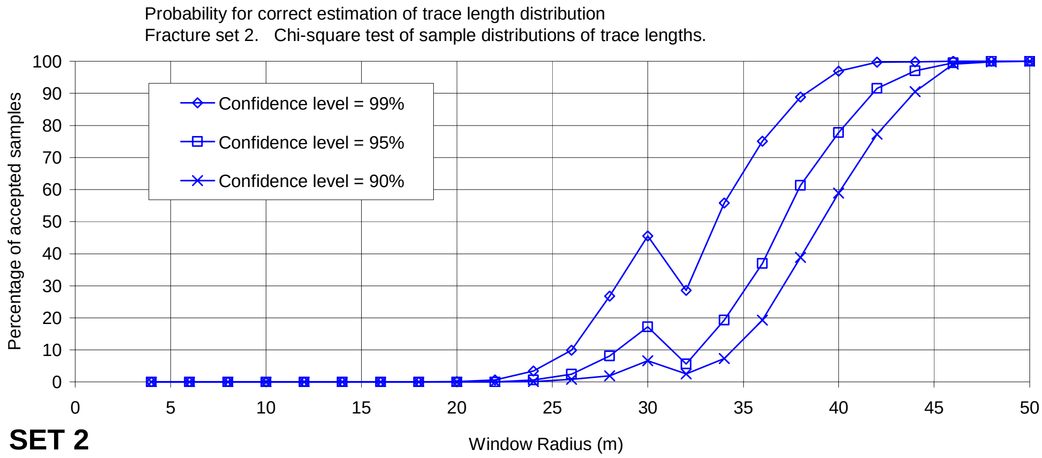
For a window with radius larger than 38 m, the probability is larger than 90 percent that a sample will not be rejected at the first level of confidence ($H_{0\ (C=99\%\ )}$). Or with other words. If the radius of the window studied is larger than 38 m, the probability is larger than 90 percent that the shape of the trace-length distribution derived from a sample is a good representation of the simulated true distribution, at a confidence level of 99 percent.

SET 3:

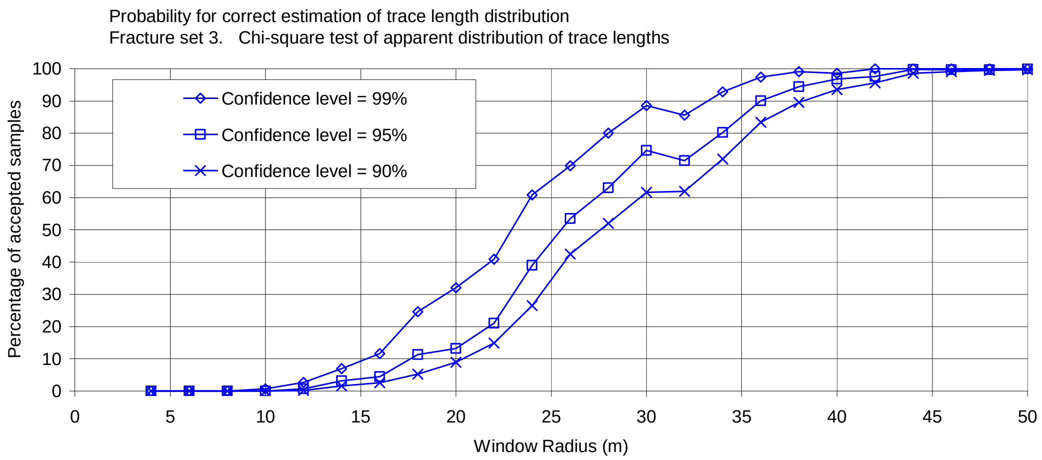
For a window with radius larger than 30–33 m, the probability is larger than 90 percent that a sample will not be rejected at the first level of confidence ($H_{0\ (C=99\%\ )}$). Or with other words. If the radius of the window studied is larger than 30–33 m, the probability is larger than 90 percent that the shape of the trace-length distribution derived from a sample is a good representation of the simulated true distribution, at a confidence level of 99 percent.



SET 1



SET 2



SET 3

Figure 6-15. *SET 1, SET 2 and SET 3:Hypothesis testing for shape of trace-length distribution. A chi-square goodness-of-fit comparison between sample distributions and the simulated true distribution, at different window sizes and for three different confidence levels. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, considering the different selected confidence levels.*

7 Estimation of fracture – set orientation from fracture traces on rock surfaces

7.1 Introduction

Fracture traces, observed on a surface, have a direction; this direction is often given as the Strike; if it is possible to identify a trace it also possible to measure the strike of the trace. The strike is the bearing or direction of a horizontal line in the horizontal plane, and it is normally measured in an easterly sense from north. In the following chapter, we will present the efficiency of estimating the mean strike of the different fracture sets, based on fracture traces as seen on rock surfaces.

In addition to strike also other properties of fracture orientation could be measured (e.g. fracture set dispersion), presuming that it is possible to measure (or calculate) the dip of the fracture that created the trace, because strike and dip can be recalculated to pole trend and pole plunge. It is however not always possible to measure or calculate the dip. In the following chapter we will also present fracture set orientation data (mean direction and dispersion), based on pole trend and pole plunge. For these calculations we have assumed that it is possible measure (or calculate) the trend and plunge of the fracture that created the observed fracture traces.

7.2 Estimation of direction of fracture traces

7.2.1 Methodology

In classical geology, the orientation of a planar feature (e.g. a fracture trace) is defined by its strike and dip. The strike is the bearing or direction of a horizontal line in the plane. The dip is the angle of the inclination of the planar feature. The strike and the dip should be defined in a way that a consistent definition is obtained of the orientation in space of the planar feature studied (see Section 2.2.2).

The definition of strike and dip corresponds to a planar feature, but a fracture trace is a linear feature. A fracture trace points in two different directions (a trace is not vector but an axis) therefore, unless we have information of the fracture dip, two different direction of strike are possible for every trace. It follows that for estimates of the mean strike, without knowledge of the dip, all strike values have to be transformed to a range within 180 degrees (for example as bearings between North and South). If the dip is known only one direction of strike is possible, for such a situation values of strike are often defined between 0 and 360 degrees.

In this chapter, when estimating the mean strike, we have assumed that there is enough information available regarding the dip of the fracture traces observed, that for each fracture trace observed it is possible to select the appropriate main direction of strike (from the two directions that are possible for each trace). The problem with the unknown main direction of a fracture trace is not necessarily a large problem, except if the sample dispersion of strike values of the fracture traces is large. In this study the dispersion of Set 1 is large by definition. In addition we are in this chapter analysing fracture traces on horizontal surfaces. It follows that a sample (as seen on a horizontal surface) of the strike values of the sub-horizontal fracture set 3 will demonstrate a very large dispersion, even if the true dispersion (in three-dimensions) of Set 3 is not very large. When plotting distributions of strike values, we have defined the strike as the bearing of a horizontal line in the plane measured in an easterly sense from north. It follows that the strike distribution contains values from zero degrees, which represents north, and up to 180 degrees, which represents south.

As for the trace-length distribution, the observed strike distribution might be biased. Such bias occurs because the three-dimensional fracture network is sampled by use of two-dimensional planes having a limited extension. If the fractures are of different sizes and the orientation of a fracture is related to the fracture size, all the biases discussed in the previous chapter, except boundary truncation, will influence the strike distribution. We will not discuss the causes for bias in this section, for such a discussion we refer to Section 6.2.

As stated above, the strikes are given as values between 0 and 180 degrees and correspond to different directions of the fracture planes studied. Together they form a strike distribution. In this chapter, when we refer to the concept of mean strike, we are actually discussing the dominating mode of such a strike distribution. It should be noted that to calculate the mean strike direction, which corresponds to the mode of the strike distribution, it is necessary to treat the strike values as vectors, or to analyse the shape of the distribution. It is not correct to calculate a mean value by using the strike values as scalars. In this study we have used vector algebra. No Tersaghi correction was included in the calculations presented in this chapter.

One should note the following when comparing:

(i) deviation in estimated mean strike of a fracture set (in degrees), and

(ii) deviation in estimated mean direction of a fracture set (as an acute angel in degrees on the unit sphere).

The estimate of mean strike of a fracture set, based on strike values only, could be a more uncertain estimate than estimates of the mean orientation (three-dimensional) of the same fracture set based on consistent values of strike and dip (or trend and plunge). Because at small sample sizes it is uncertain where the dominating mode of the strike-distribution is located (especially if the dispersion of the fracture set is large). More information is available if both strike and dip (or trend and plunge) is used; therefore (at small sample sizes) it is possible that a less uncertain estimate can be derived of the mean direction of the fracture set than of the mean strike.

The characteristics of the strike distributions vary with the size of the window studied. For small windows, the number of traces are small, and consequently the variation in distribution characteristics is large between different windows (different realisations). The larger the window the closer the characteristics of the sample distribution is to the unknown characteristics of the population studied, and the smaller the differences between different realisations.

The strike distributions of the fracture sets of the population studied, based on direction of fracture traces as seen on very large circular and horizontal windows (radius 150 metres) are given in Figure 7-1, Figure 7-2, Figure 7-3 and Figure 7-4. The average strike distribution for windows of radius 150 metres is set as the true distribution, and it is called the simulated true distribution (this is further discussed below).

The strike is correlated to the pole trend in the following way (in degrees):

$$\text{Strike} = \text{PoleTrend} + 90$$

Based on the given values of pole trend (see Table 2-2) we have calculated the true mean strike, which is:

- Set 1. Mean strike = 39.0 deg.
- Set 2. Mean strike = 127.0 deg.
- Set 3. Mean strike = 20.6 deg.

If all strike values (derived from fracture traces), observed on a large number of horizontal windows of radius 150 metres, are put together in one group, the corresponding strike distribution is given in Figure 7-1. The demonstrated strike distribution has a bi-modal shape. The first mode corresponds to the mean strike of fracture set 1, and the second mode corresponds to the mean strike of fracture set 2. Fracture set 3 is sub-horizontal and on a horizontal surface it has no well-developed mean trend direction, at least not when covered by the large number of traces that corresponds to Set 1 and Set 2. (It is however possible to estimate the mean direction of Set 3, if the set-identities of the traces are known, this is discussed in more detail below.) Considering a circular window of radius 150 metres the average number of fractures per set is: Set 1= 35% , Set 2= 51% , Set 3= 14%

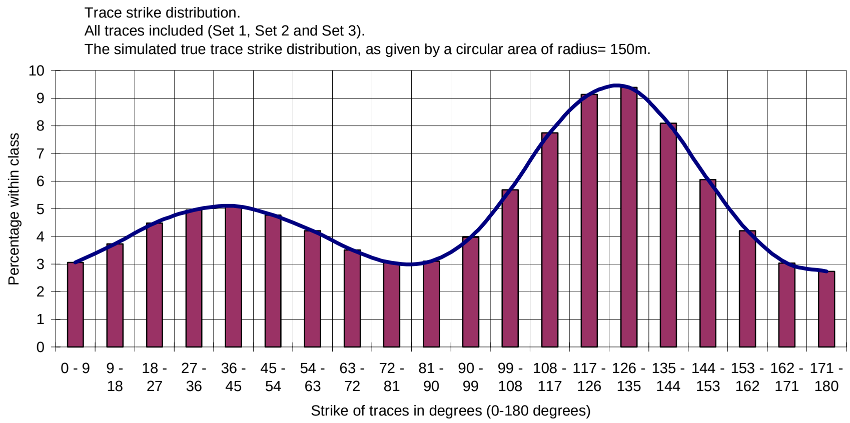


Figure 7-1. *Strike distribution of all traces (Set 1, Set 2 and Set 3) based on the directions of all fracture trace, as seen on horizontal windows of radius 150 m (simulated true distribution).*

The first mode, which represents Set 1, is actually a very good estimate of the mean strike of Set 1, the difference is 0.2 degrees (true value= 39.0 deg., estimate= 39.2 deg.). The second mode, which represents Set 2, is a very good estimate of the mean strike of Set 2, the difference is 0.5 degrees (true value= 127.0 deg., estimate= 126.5 deg.). These results are not surprising, because the distribution studied is based on a huge number of fractures (about 1000 000 observed fractures). The purpose is to demonstrate that as the number of tracer increases, the strike distribution observed converges towards the true distribution of strike values; and for large sample sizes, the sample estimates are very close to the true values.

However, in the remaining parts of this chapter we will study the strikes of each fracture set separately. When fracture traces are observed on rock surfaces, it is often possible to separate the fracture traces into different sets, based on the observed strike of the fracture traces; and based on the dip of the fracture traces (assuming that it is possible to observe a dip). It follows that different strike distributions will be derived for different sets. In the analyses presented below, the fracture traces are divided into three different sets, based on the known set-identity of each fracture that creates a trace. The results of the analyses are given for each fracture sets separately. (In this study each fracture was marked with its proper set identity since this is known at the generation of the fracture. In a real situation, different methods and algorithms for identifying and delimiting sets will be necessary to ensure objective set identifications.)

Considering Set 1 and windows of radius 150 metres, the corresponding simulated true strike distribution (based on fracture traces) is given below (Figure 7-2). The mode of the distribution is at 39.03 degrees, which is a very good estimate of the true mean strike (39.00 deg). Again it should be noted that the distribution below is a simulated true distribution and it is based on a huge number of fractures (about 1000 000 observed fractures).

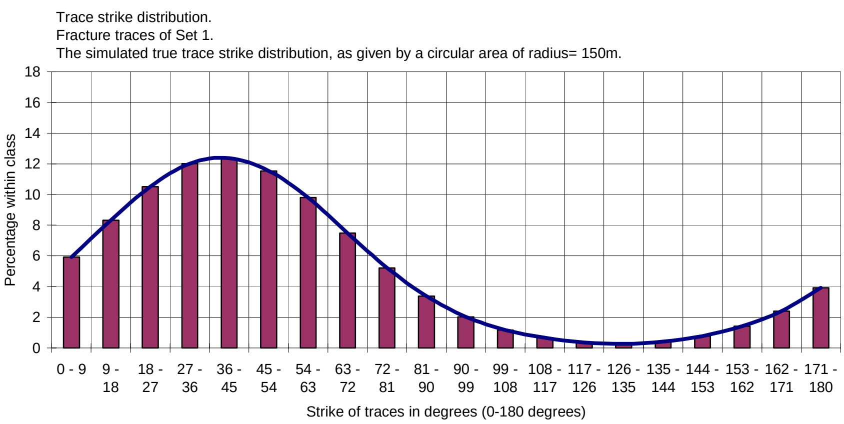


Figure 7-2. *Strikes distribution of Set 1,based on the directions of fracture traces of Set 1, as seen on horizontal windows of radius 150 m (simulated true distribution).*

Considering Set 2 and windows of radius 150 metres, the corresponding strike distribution is given below (Figure 7-3). The mode of the distribution is at 126.97 degrees, which is a very good estimate of the true mean strike (127.00 deg). And again it should be noted that the distribution below is a simulated true distribution and it is based on a huge number of fractures (c:a 1000 000 observed fractures).

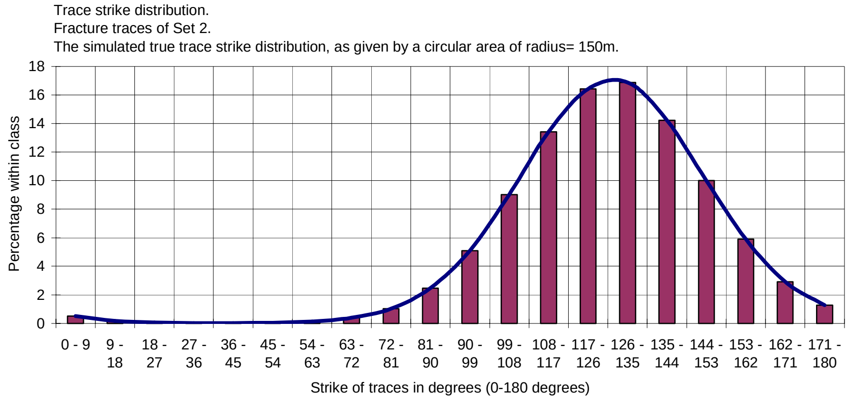


Figure 7-3. *Strikes distribution of Set 2, based on the direction of the fracture traces of Set 2, as seen on horizontal windows of radius 150 m (simulated true distribution).*

Considering Set 3 and windows of radius 150 metres, the corresponding strike distribution is given below (Figure 7-4). Fracture set 3 is sub-horizontal and on a horizontal surface it has no well-developed mean trend direction. Nevertheless, if the window is large enough, the mode of the distribution is a very good estimate of the true mean strike. The mode of the distribution is at 20.61 degrees, which is a very good estimate of the true mean strike (20.60 deg). It should however be noted that the distribution below is a simulated true distribution and it is based on a huge number of fractures (about 1000 000 observed fractures).

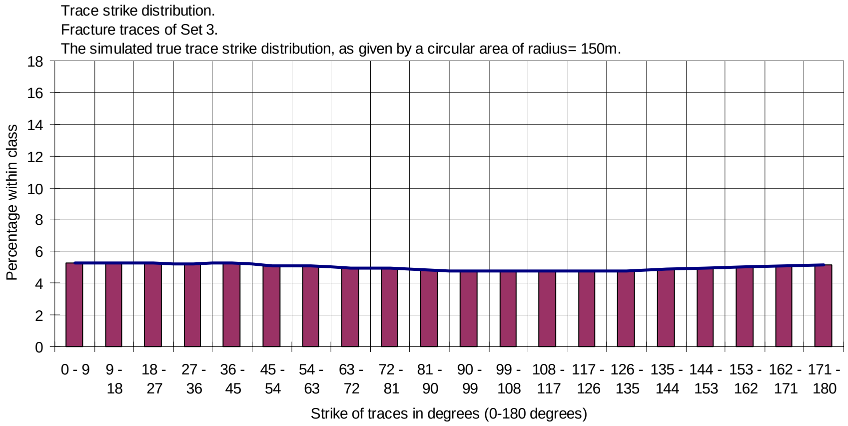


Figure 7-4. *Strike distribution of Set 3, based on the direction of the fracture traces of Set 3, as seen on windows of radius 150 m (simulated true distribution).*

7.2.2 Point estimate of strike distribution based on fracture traces

General

The directions of the fracture traces that are observed on a window studied are samples of the properties of the fracture population. The properties of the sample can be looked upon as an estimate of the properties of the population. From a statistical point of view, the analysis of the strike distribution, as given by fracture traces on windows of different sizes, is a point estimate of the properties of an unknown strike distribution.

The characteristics of the strike distribution vary with the size of the window studied. For small windows, the number of traces are small and the variation in distribution characteristics is large between different windows (different realisations of the distribution). The larger the window the closer the characteristics of the sample distribution is to the unknown characteristics of the population studied, and the smaller the differences between different realisations; the rate of this progress towards the true characteristics are called the efficiency of the point estimate.

The strike distributions of the fracture sets of the population studied, considering fracture traces on very large circular and horizontal windows (radius 150 metres) are given in Figure 7-2, Figure 7-3 and Figure 7-4.

Examples of sample strike distributions, derived from fracture traces observed on horizontal windows with a radius smaller than 150 metres, are given below in Figure 7-5, Figure 7-6 and Figure 7-7.

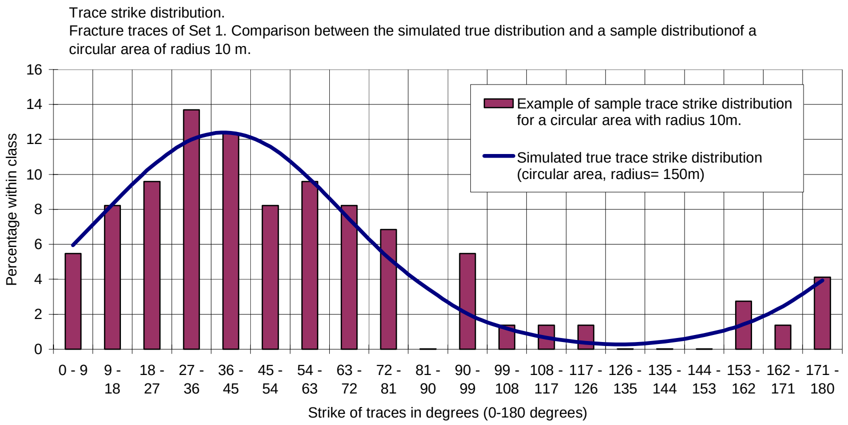


Figure 7-5. *SET 1. Comparison between the simulated true strike distribution and an example of a strike distribution based on fracture traces seen on a window of radius 10 m.*

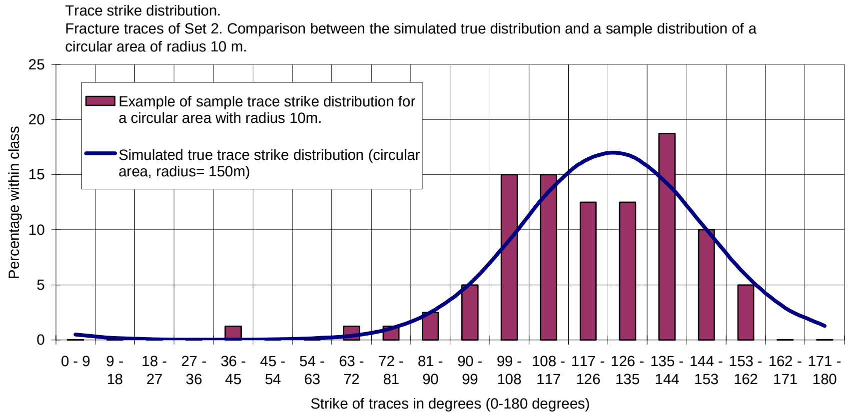


Figure 7-6. *SET 2. Comparison between the simulated true strike distribution and an example of a strike distribution based on fracture traces seen on a window of radius 10 m.*

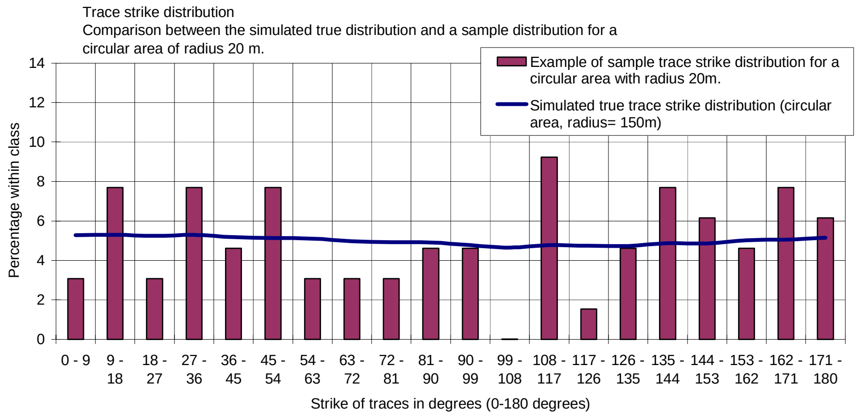


Figure 7-7. *SET 3. Comparison between the simulated true strike distribution and an example of a strike distribution based on fracture traces seen on a window of radius 20 m.*

Point estimate of the moments of the observed distribution

The efficiency of the point estimate of the mean and standard deviation of the strike distribution (from fracture traces as seen on horizontal windows) is given in Figure 7-8, below. Considering a circular window of radius 150 m, the point estimate produces the following results:

Set 1

Mean values of strike distribution, window radius = 150m.

Mean of mean values = 38.9 degrees (True value= 39.00 degrees)

Standard deviation of mean values = 0.2% of mean of mean values.

Set 2

Mean values of strike distribution, window radius = 150m.

Mean of mean values = 126.7 degrees (True value= 127.00 degrees)

Standard deviation of mean values = 1.1% of mean of mean values.

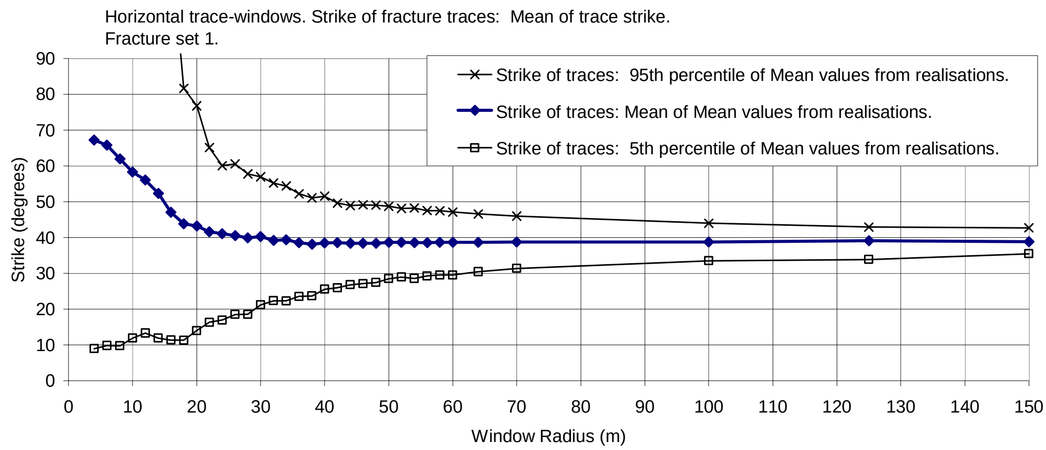
Set 3

Mean values of strike distribution, window radius = 150m.

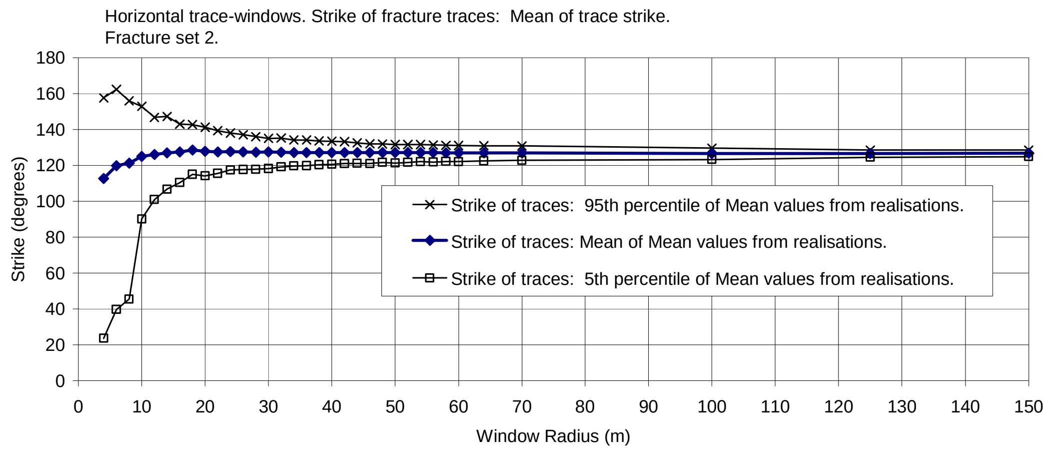
Mean of mean values = 20.6 degrees (True value= 20.6 degrees)

Standard deviation of mean values = 0.1% of mean of mean values.

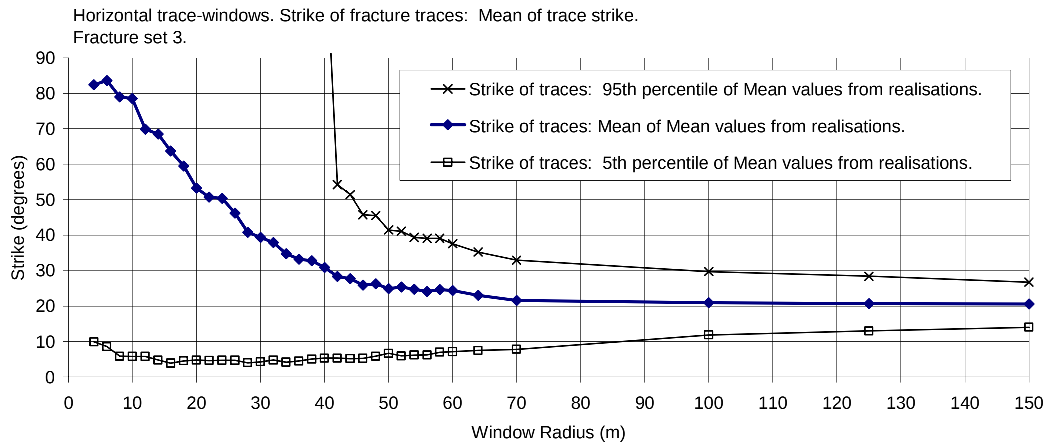
Analysing the figures that presents the efficiency of the point estimate of the mean of the strike distributions (Figure 7-8). It is concluded that the changes in mean and variance of the distributions are small for windows with a radius larger than 150 m. The sample strike distribution for a window of radius 150 m is set as the true distribution, and it is called the simulated true distribution.



SET 1.



SET 2.



SET 3.

Figure 7-8. *Efficiency of the point estimate of the mean value of the observed strike distribution. The strike distributions are based on fracture traces observed on windows of different sizes.*

7.2.3 Hypothesis testing considering mean of strike distribution and acceptable deviations

Purpose of tests

The purpose of this test is to determine when the size of sample of fracture traces (number of traces) is large enough to produce an acceptable estimate of the true properties of the strike distribution, with a certain probability. The hypothesis testing of this section is based on the mean values of the strike distribution. Considering the somewhat complex shape of the simulated strike distribution, one may wonder why we are interested in the moments and not of the actual shape of the distribution. The answer is that we are interested in both – this section presents a test of the mean values and the next section presents a test of the shape. The mean values are of interest, because when the samples produce a good estimate of the true mean values, with a large probability, this is an indication that the shape of the distribution is stable and we have found a sample size large enough for prediction of the true properties.

Null hypothesis, acceptable deviations and criterion of significance

The samples were analysed by a statistical hypothesis testing. The hypothesis testing of this section is based on the mean values of the studied distributions and given criterions of significance. The established criterions of significance correspond to the known true mean strikes of the fracture sets studied

The null hypothesis (H_0) is that a sample is **not** a good representation of the true properties of the population. This hypothesis is rejected if a small deviation takes place between the values of the sample and the true values of the population. The following criterions of significance are used: the sample is rejected if the deviation from the simulated true value is smaller than 15 degrees (first level) or 10 degrees (second level) or 5 degrees (third level). These criterions are applied to the mean value (or mode) of the distributions studied. The criterions represent different aspects of the distribution studied and different levels of significance.

First criterion: $H_{0\ \text{(deviation in mean value >=15 deg)}}$ is rejected if:

$$\text{ABS}[\text{Mean}_{(\text{sample})} - \text{Mean}_{(\text{simulated true})}] <= 15 \text{ degrees}$$

Second criterion: $H_{0\ \text{(deviation in mean value >=10 deg)}}$ is rejected if:

$$\text{ABS}[\text{Mean}_{(\text{sample})} - \text{Mean}_{(\text{simulated true})}] <= 10 \text{ degrees}$$

Third criterion: $H_{0\ \text{(deviation in mean value >=5 deg)}}$ is rejected if:

$$\text{ABS}[\text{Mean}_{(\text{sample})} - \text{Mean}_{(\text{simulated true})}] <= 5 \text{ degrees}$$

The results of the analysis are presented as the probability that a sample, at a certain window size, will fulfil the hypothesis considering the criterions above.

Results considering mean values

As previously stated, when comparing the efficiencies of the different point estimates for the different sets, it is important to note the following. In this study the number of observed fractures on a studied area (with a given radius), gives the sample size and these sample sizes are different for different sets. For example on a very large horizontal circular area and considering the different sets, on the average the amounts of fracture traces are: Set1=35%, Set2= 51% and Set3=14%. Considering a situation with two fracture sets with approximately the same dispersion, the different amounts of fractures observed for each set will influence the efficiency of the point estimates; on the average the large the number of observed fractures the less uncertain is the estimate.

The results are given in below. We conclude the following results.

SET 1:

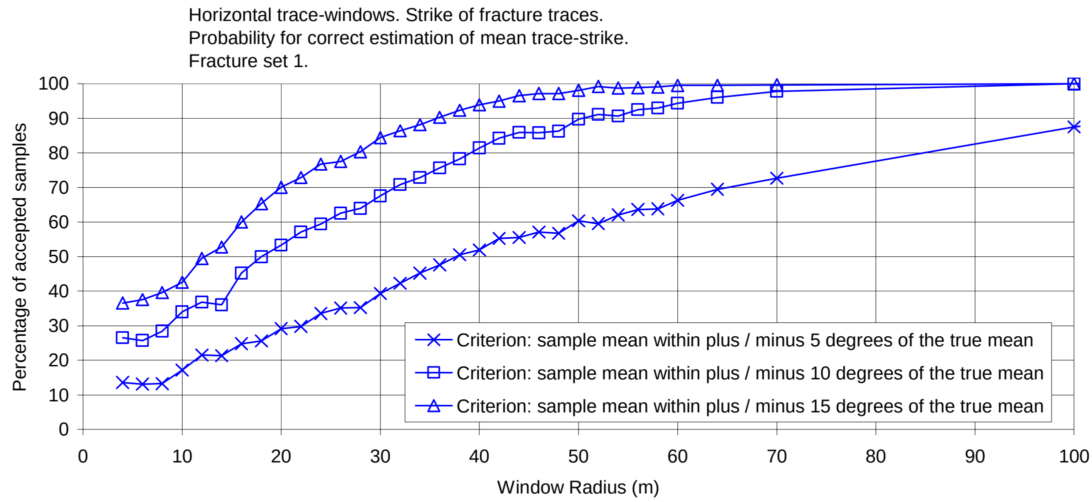
If the radius of the window studied is larger than 35 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the true value of the population.

SET 2:

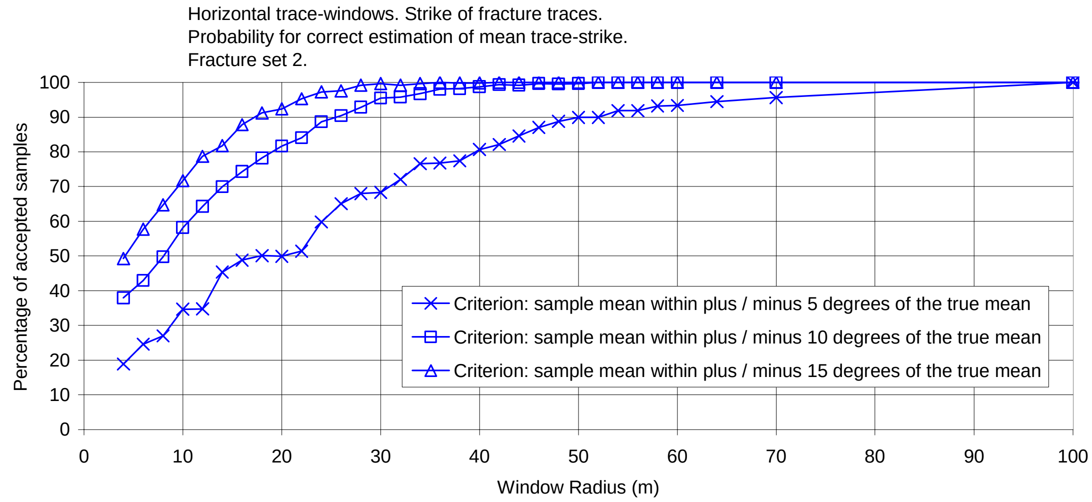
If the radius of the window studied is larger than 18 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the true mean value of the population

SET 3:

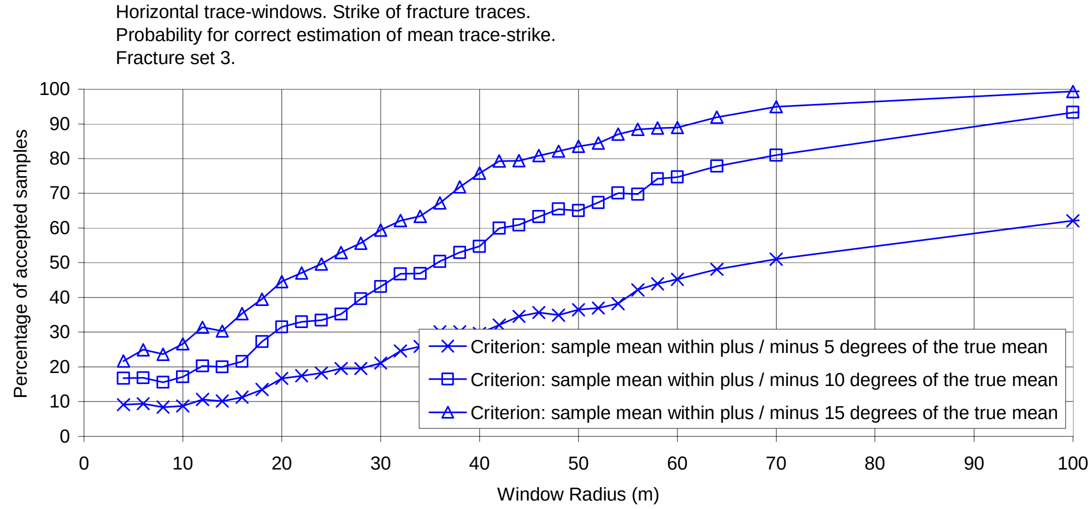
If the radius of the window studied is larger than 60 m, the probability is larger than 90 percent that the deviation in estimated mean value is within plus/minus 15 percent of the true mean value of the population



SET 1.



SET 2.



SET 3.

Figure 7-9. *Hypothesis testing for selected acceptable deviations, considering mean of predicted strike distribution. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, considering the different selected criterions.*

7.2.4 Hypothesis testing considering the shape of the strike distribution and given confidence levels

Purpose of tests

The purpose of this test is to determine when the size of the sample is large enough to produce an acceptable estimate of the true properties of the strike distributions, at a certain given level of confidence. The hypothesis testing of this section is based on the shapes of the distributions studied (tests as regard the moments of the studied distributions are given in the previous section). Distribution shape is of interest as it characterises the distribution studied, and when samples produce a good estimate of the shape of the true distribution with a large probability, the samples are large enough for prediction of the true properties.

Methodology of the chi-square test

Tests of the shape of the strike distribution were carried out as chi-square tests of "goodness-of-fit". The observed strikes are grouped into classes and the frequencies are compared with the expected frequencies, as given by the simulated true strike distributions (see Figure 7-2, Figure 7-3 and Figure 7-4).

Null hypothesis and confidence levels

The samples were analysed by use of statistical hypothesis testing. The hypothesis testing were based on the shape of the strike distributions and given confidence levels. Examples of strike distributions from samples are given in Figure 7-5, Figure 7-6 and Figure 7-7. The sample distributions are compared to the simulated true distribution (see Figure 7-2, Figure 7-3 and Figure 7-4).

The null hypothesis (H_0) is that a sample is a good representation of the simulated true distribution. The hypothesis is rejected if, when comparing the sample and the true distribution, it is found that the deviations between the two distributions are large. The confidence level gives the size of deviation that is acceptable, a deviation larger than this is considered as a significant deviation.

The confidence level should be selected in a way that the probability for rejection of the hypothesis is small if the hypothesis is true. We have studied three different levels of confidence: 90, 95 and 99 percent. The hypothesis tests are as follows:

- First confidence level 99%: The hypothesis, $H_{0\,(C=99\%)}$ is rejected if the sample deviates significantly at this level of confidence.
- Second confidence level 95%: The hypothesis, $H_{0\,(C=95\%)}$ is rejected if the sample deviates significantly at this level of confidence.
- Third confidence level 90%: The hypothesis, $H_{0\,(C=90\%)}$ is rejected if the sample deviates significantly at this level of confidence.

For each confidence level, the result of the analysis is presented as the percentage of accepted samples at different window sizes.

Results – chi-square test of goodness-of-fit

The results of the chi-square tests are given in Figure 7-10, below. The figure presents the results for three different confidence levels. In a goodness-of-fit test, the shape of the distributions are tested, and the deviations in mean and spread may balance each other in a way that the shape is accepted although the moments are not well predicted.

We conclude the following results.

SET 1:

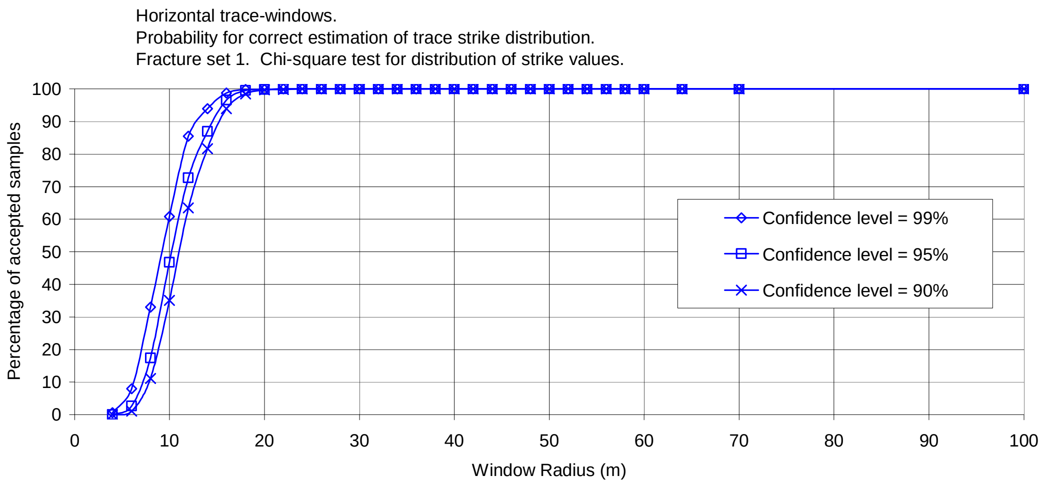
If the radius of the window studied is larger than 13 m, the probability is larger than 90 percent that the shape of the strike distribution derived from a sample is a good representation of the simulated true distribution. This conclusion is based on a chi-square goodness-of-fit test with a confidence level of 99 percent.

SET 2:

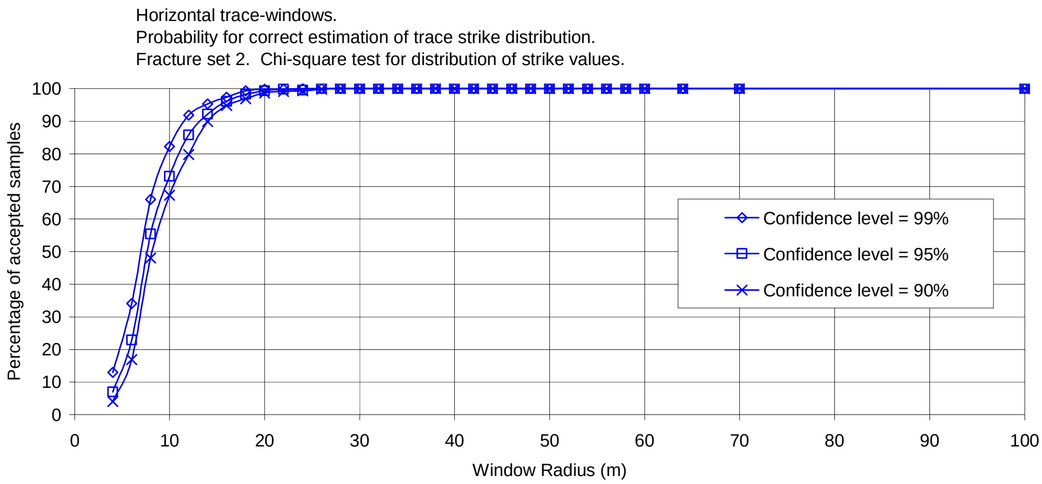
If the radius of the window studied is larger than 11 m, the probability is larger than 90 percent that the shape of the strike distribution derived from a sample is a good representation of the simulated true distribution. This conclusion is based on a chi-square goodness-of-fit test with a confidence level of 99 percent.

SET 3:

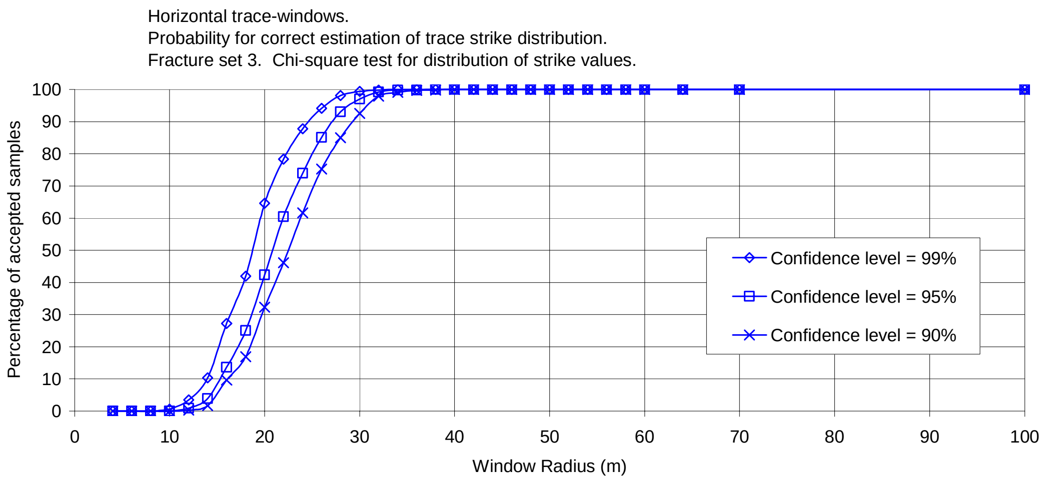
If the radius of the window studied is larger than 24 m, the probability is larger than 90 percent that the shape of the strike distribution derived from a sample is a good representation of the simulated true distribution. This conclusion is based on a chi-square goodness-of-fit test with a confidence level of 99 percent.



SET 1



SET 2



SET 3

Figure 7-10. *SET 1, SET 2 and SET 3:Hypothesis testing for shape of strike distribution. A chi-square goodness-of-fit comparison between sample distributions and the simulated true distribution, at different window sizes and for three different confidence levels. The figure gives the percentage of accepted samples, which is approximately the same thing as the probability for correct estimation, considering the different selected confidence levels.*

7.3 Estimation of fracture set mean direction, from fracture measurements on rock surfaces

7.3.1 Introduction

In addition to strike also other properties of fracture orientation could be estimated (e.g. fracture set dispersion), presuming that it is possible to measure (or calculate) both the strike and the dip of the fracture that created the trace. Because strike and dip can be recalculated to trend and plunge, and based on trend and plunge it is possible to calculate mean direction and dispersion of identified fracture sets. In addition to the direction (trend or strike) of a fracture trace, which is normally easily observed, it is also possible to measure or calculate the dip (or plunge) of the fracture that created the trace, if the necessary resources are made available. In the following chapter we have assumed that measurements (and/or calculations) are carried out of both trend and plunge of the fractures that created the observed fracture traces, and we also assume that all measurements are carried out without any measurement errors.

7.3.2 Methodology

If strike and dip of the fractures that creates the fracture traces are known, and recalculated to pole trend and pole plunge, it is possible to calculate the mean direction and dispersion of the fracture sets. Actually, all the tests and analyses described and presented in Chapter 3 and 4, regarding fracture set orientation, can equally well be performed with the data from a rock surface (presuming that trend and plunge values are known of the fractures that created the traces). In this Chapter we will not discuss details of tests or the methods, for such a discussion we refer to the previous chapters and applicable Appendices. We have, with use of data from rock surfaces, performed all the tests and calculations presented in Chapter 3 and 4. The results of those calculations are presented below, but the results are not presented to the same level of detail as in the previous chapters.

A sampling bias will occur when a three-dimensional fracture system is sampled by use of two-dimensional surfaces. It is possible to use an areal correction for this sampling bias, a correction very similar to the correction used when analysing borehole data (the Terzaghi correction). However, no correction for sampling bias (Terzaghi correction) was included in the calculations and tests of this chapter. If we had included such a correction in the analyses, the necessary sample sizes would have been smaller. This especially the case for the sub-horizontal fracture set (Set 3), as this set is not well represented on horizontal surfaces.

7.3.3 Fracture set orientation – acute angle – results

Based on the methods discussed in Chapter 3, the mean direction of the fracture sets were calculated and compared to the known true direction (as discussed in the above-mentioned chapter). The results are given in the Table 7-1 below.

When comparing the results given below to the results given in the previous section regarding the strike, we note that the point estimate of mean direction of a fracture set, is more efficient than the point estimate of mean strike, this is also discussed in

Section 7.2.1. The high efficiency of the point estimate of mean direction of fracture set, assumes that it is possible to measure (without any errors) strike (trend) and dip (plunge) of all fractures that intersects the surface studied.

Table 7-1. Estimated Mean direction of fracture sets based on data from horizontal circular surfaces. Presuming that trend and plunge is measured without any errors and that Set ID is known for all traces studied. Correction for sampling bias was not included.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	HORIZONTAL CIRCULAR SURFACE	RADIUS OF SURFACE (1) *(Sample size)*
ORIENTATION **MEAN DIRECTION** The deviation in degrees corresponds to the acute angle between the true mean direction and that of a sample.	Deviation < = 15 deg (2)	>= 90%	Set 1 Set 2 Set 3	>= 6 m >= 4 m >= 11 m
	Deviation < = 10 deg (2)	>= 90%	Set 1 Set 2 Set 3	>= 8 m >= 5 m >= 18 m
	Deviation < = 5 deg (2)	>= 90%	Set 1 Set 2 Set 3	>= 21 m >= 11 m >= 88 m

(1) Results and conclusions given in this study are only directly applicable to the fracture network studied
(2) Samples are within a range of plus or minus 5, 10 or 15 degrees from the True value, considering a range centred on the true value

7.3.4 Fracture set orientation – dispersion – results

Based on the methods discussed in Chapter 4, the dispersion of the fracture sets were calculated, both considering the Fisher-Kappa parameter and the SR1 parameter, and the values obtained from samples were compared to the known true values (as discussed in the above-mentioned chapter). The results are given in the table below,

Table 7-2. Fracture orientation from horizontal circular surfaces. Presuming that trend and plunge is measured without any errors and that Set ID is known for all traces studied. Correction for sampling bias was not included.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	HORIZONTAL CIRCULAR SURFACE	RADIUS OF SURFACE (1) *(Sample size)*
ORIENTATION Dispersion **SR1**	Deviation < = +/–15% of true value (2)	>= 90%	Set 1 Set 2 Set 3	>= 15 m >= 8 m Not possible
	Deviation < = +/–10% of true value (2)	>= 90%	Set 1 Set 2 Set 3	>= 24 m >= 14 m Not possible
	Deviation < = +/–5% of true value (2)	>= 90%	Set 1 Set 2 Set 3	>= 60 m >= 33 m Not possible
ORIENTATION Dispersion **KAPPA**	Deviation < = +/–15% of true value (2)	>= 90%	Set 1 Set 2 Set 3	>= 38 m >= 26 m Not possible
	Deviation < = +/–10% of true value (2)	>= 90%	Set 1 Set 2 Set 3	Not possible >= 55 m Not possible
	Deviation < = +/–5% of true value (2)	>= 90%	Set 1 Set 2 Set 3	Not possible Not possible Not possible

(1) Results and conclusions given in this study are only directly applicable to the fracture network studied
(2) Samples are within a range of plus or minus 5, 10 or 15 percent of the True value, considering a range centred on the true value, i.e. within: 0.85*TV–1.15*TV (TV=TrueValue)

8 Limited sensitivity analysis

8.1 Methodology

Results and conclusions given in the previous chapters are only directly applicable to the fracture network studied (the DFN-model), which properties are defined in Section 2.3 (see Table 2-1, Table 2-2 and Table 2-3). It is of course interesting to know how small changes in the properties of the DFN-model studied will influence the results given in previous chapters. To investigate this a limited sensitivity analysis have been carried out, which investigate how small changes in the properties of the DFN-model will change the results presented in previous chapters.

In theory the number of possible sensitivity cases is infinite, it is therefore necessary to follow a selected strategy when performing a sensitivity analysis. The sensitivity analysis of this study is based on the fracture density of the rock mass, as defined by the *P32*-parameter and the fracture size distribution. The *P32*-parameter is defined as the fracture area per unit volume of rock mass. Presuming that the fractures are circular planar discs, the *P32*-value depends on number of fractures and radius of fractures. The principles of how the *P32*-value varies with fracture size and number of fractures in a volume of rock is demonstrated in Figure 8-1, below.

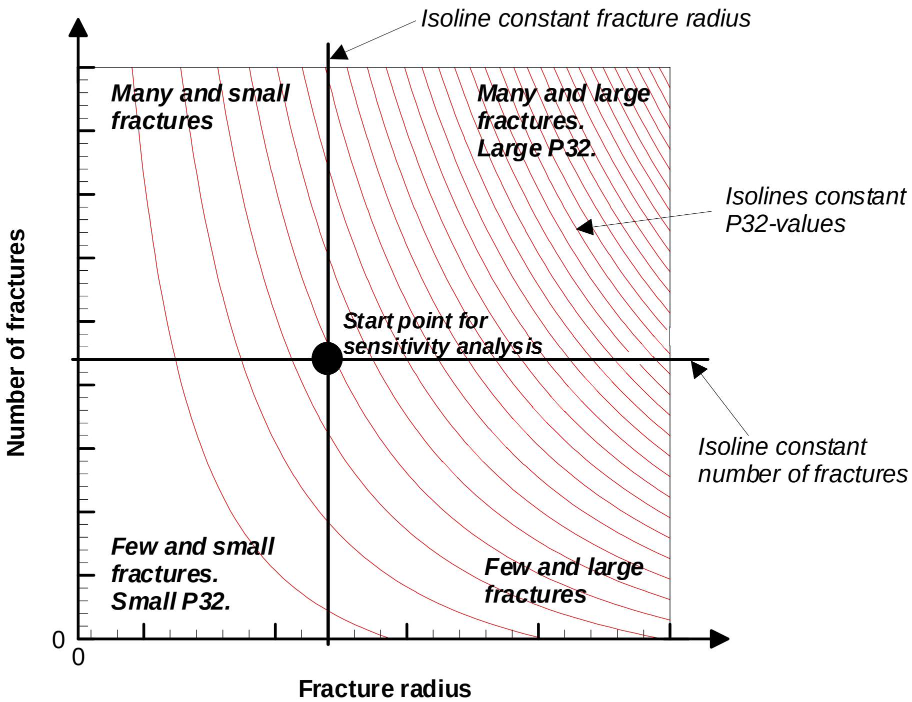


Figure 8-1. *The principles of how P32 (fracture surface area per unit rock volume) varies with fracture size and number of fractures in a studied volume of rock. The figure also indicates the principles on which the sensitivity analysis is based. (i) P32 is constant and fracture radius is varied (movement along a P32 isoline). (ii) Number of fractures is constant and radius of fractures is varied (varying P32-values). (iii) Fracture radius is constant and number of fractures is varied (varying P32-values).*

This study contains a limited sensitivity analysis; therefore only two sensitivity cases are included.

Case 1.

P32 is not changed but fracture radii is varied. This is movement along an isoline with constant *P32*.

Case 2.

P32 is defined as two times the values of the base-case. The fracture radius distributions are not changed. This is movement along an isoline representing constant fracture radius.

The results of the sensitivity cases are compared the results of the DFN-model analysed in the previous chapters, this DFN-model is called the "base-case".

Base-case

This DFN-model is defined in Section 2.3 (see Table 2-1, Table 2-2 and Table 2-3)

8.2 Case 1 – Same *P32* value but different fracture radii

8.2.1 Definition of Sensitivity Case 1

The DFN-model used in this study (as the base-case) is the DFN 2 model presented in /Hermanson et al, 1999/. The main objective of the DFN 2 modelling was to establish a discrete fracture network model, representing the rock mass at the Prototype Repository, which could be used for simulation of groundwater flow. The DFN 2 model underestimates the total number of fractures in the rock mass at the Prototype Repository, as small fractures with minor or negligible hydraulic importance is not included in the model. We have therefore established an alternative DFN-model (Case 1), that includes a larger number of small fractures, but has the same value of fracture density.

Case 1 is identical to the base-case of this study (see Table 2-1, Table 2-2 and Table 2-3), except that the fracture radius distributions are defined as different to those of the base-case. The *P32* value of Case 1 is identical to that of the base-case. It follows that, as the fracture radius distribution is different and the *P32*-value is the same, the number of fractures will be different in Case 1 compared to the base-case.

The new fracture radius distribution (for Case 1) were defined as log-normal distributions as for the base-case, but with smaller values of mean and standard deviation than in the base-case. When analysing log-normal distributions it is often convenient to analyse the data in eLog-space, because in eLog-space the log-normal distribution becomes a normal-distribution (this is also discussed in Section 6.3.3). To establish the new distribution the following method was applied:

- The mean values of the log-normal distributions in elog-space (Mean, μ_{eLog}) was defined as: Case 1 μ_{eLog} = 0.43 BaseCase μ_{eLog}

- The standard deviation values of the log-normal distributions in elog-space (Mean, σ_{eLog}) was defined as: Case 1 σ_{eLog} = 0.66 BaseCase σ_{eLog}

Hence, in eLog-space, the new mean is 43% of the value of the base-case, and the new standard deviation is 66% of the value of the base-case. It follows that the new distributions (Case 1) will consist of smaller fractures. In a DFN-model with a constant *P32* value, reducing the radii of the fractures will be compensated by a larger number of fractures. Compared to the base-case, the DFN-network of Case 1 will contain smaller fractures, but more of them.

The properties of the fracture radius distributions of Case 1 are given in Table 8-1 (below).

Fracture traces, created by the fractures of Case 1, as seen on circular horizontal windows are given in Figure 8-2 (compare Figure 8-2 and Figure 2-5).

A comparison of the shape of the fracture diameter distributions of Case 1 and the corresponding trace-length distributions are given in Figure 8-3 (compare Figure 8-3 and Figure 6-2).

Table 8-1. Size of fractures in Sensitivity Case 1.

Set No.	**1**	**2**	**3**
Fracture shape	Planar discs	Planar discs	Planar discs
Distribution	TlogNormal (1)	TLogNormal (1)	TlogNormal (1)
Mean radius [m] (2)	1.64	2.47	2.01
Mean of LN(radius) (3)	0.1505	0.8899	0.5915
Stdv radius [m] (4)	1.64	0.40	0.98
Stdv of LN(radius) [m] (5)	0.5487	0.1623	0.4635
Termination % (6)	0	0	0
Lower bound [m] (7)	0.0025	0.0025	0.0025
Upper bound [m] (7)	10000	10000	10000

(1) A Log-Normal distribution which is truncated at lower and upper bounds.

(2) Mean of distribution.

(3) Mean of the natural logarithms of the values of radius

(4) Standard deviation of distribution.

(5) Standard deviation of the natural logarithms of the values of radius

(6) Amount of fractures that terminate at other fractures.

(7) Upper and lower boundaries for the truncated Log-Normal distribution

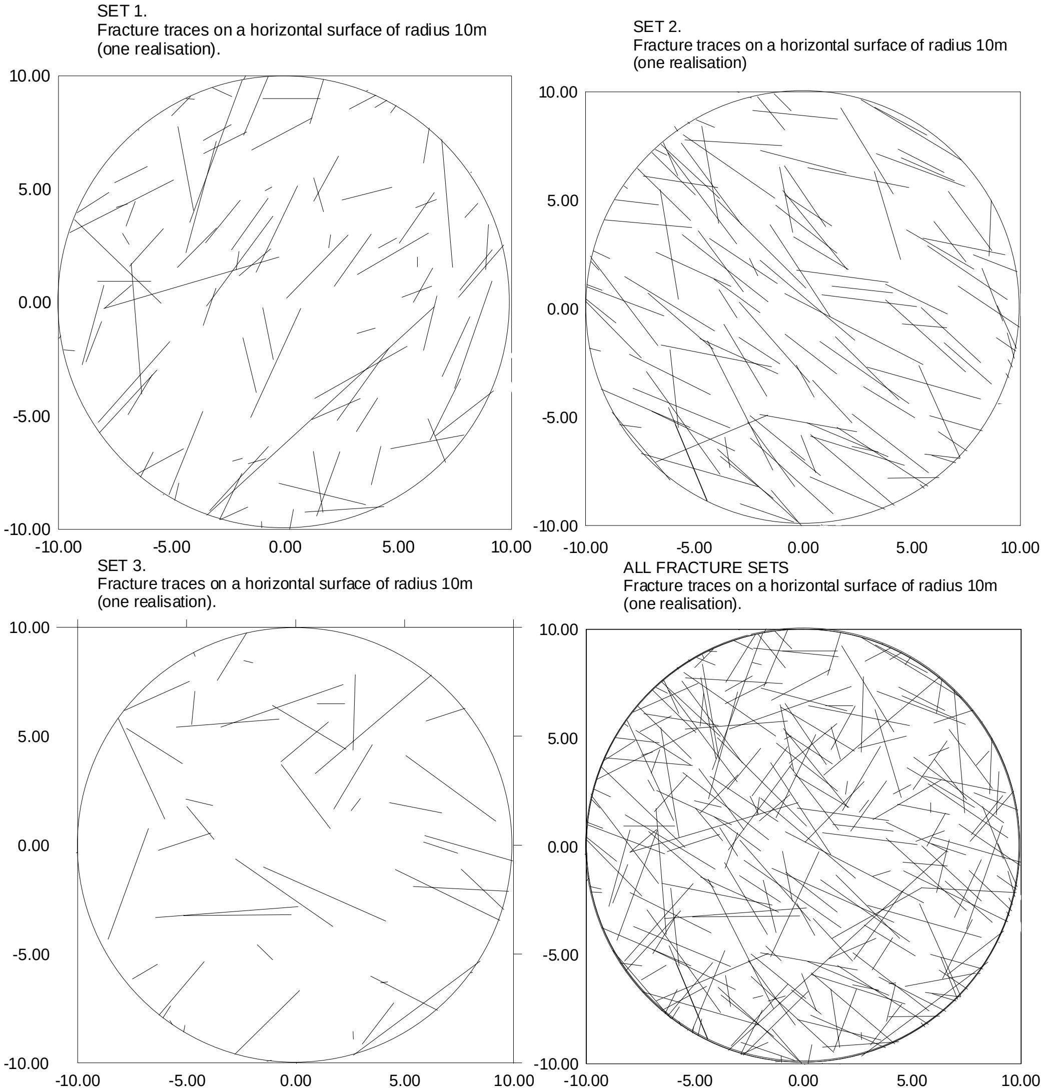
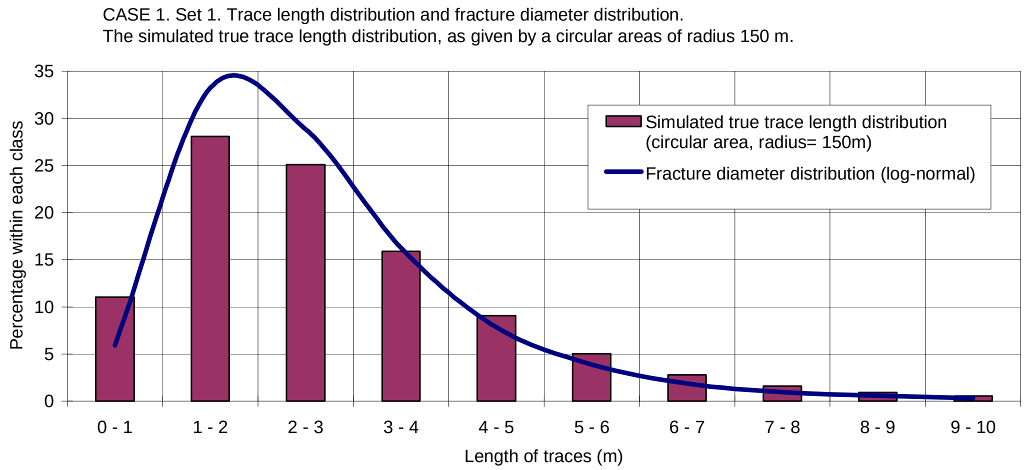
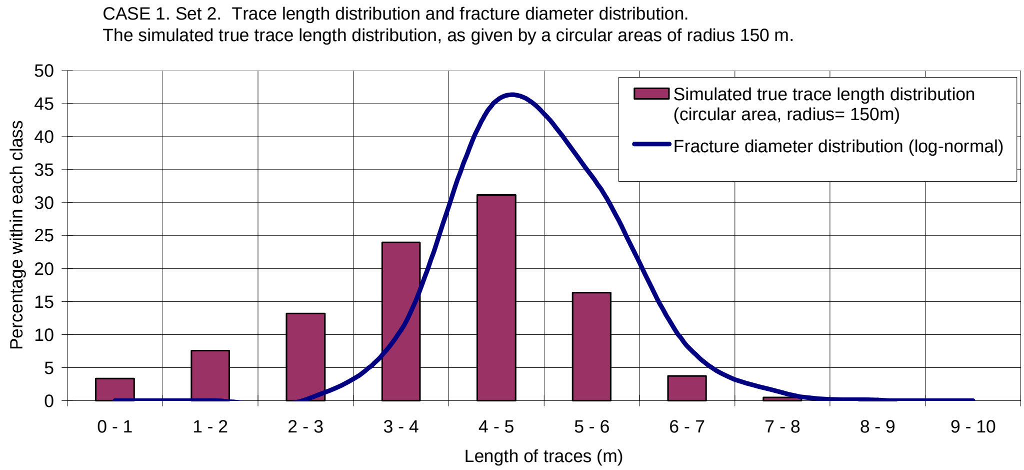


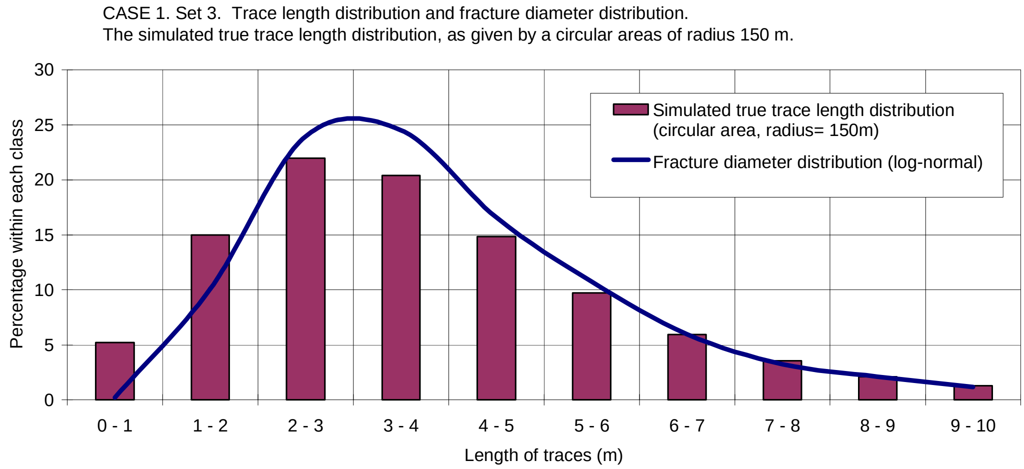
Figure 8-2. *Sensitivity Case 1. Fracture traces on circular horizontal surfaces with radius 10m. The plotted fracture traces represents one realisation of the fracture population studied (the fracture network). Each of the four figures includes a different number of fractures, dependent on the orientation and density of the fracture set studied. The length of the traces divided by the surface area is the P21 parameter. For very large horizontal surfaces, the P21 values are as follows: Set1 P21= 0.77, Set 2 P21= 1.51, Set 3 P21= 0.41; hence 29% of the trace-lengths belongs to Set 1, and 56% belongs to Set 2, and 15% belongs to Set 3. Considering the number of traces on a very large horizontal surface, on the average 35% belongs to Set1, 51% belongs to Set 2 and 14% belongs to Set 3.*



SET 1



SET 2



SET 3

Figure 8-3. *CASE 1: Comparison between: (i) the fracture diameter distributions and (ii) the corresponding trace-length distributions, considering the three fracture sets and a window of radius 150 m. The average trace-length distribution for windows of radius 150 m is set as the simulated true trace-length distribution, of Case 1.*

8.2.2 Results considering fracture orientation and density based on data from boreholes

All the tests and analyses presented in previous chapters have also been carried out for the fractures of Case 1. Considering fracture data as seen in boreholes (fracture orientation and fracture density) the results of Case 1 are **identical** to the results of the base-case presented in previous chapters. (It should perhaps be noted that also the estimates of *P32*-values from borehole data in Case 1 are identical to the estimates of the base-case.)

This can be explained in the following way. The fractures of Case 1 are smaller, hence the probability of seeing a specific fracture in a borehole is smaller than for a specific fracture of the base-case, on the other hand there is a larger number of fractures in Case 1 than in the base-case. In all, the number of fractures seen in the borehole is the same as in the base-case, and the *P10* values are the same. Hence, everything else being equal, the fracture radius may vary, but as long as the *P32* is constant, the number of fractures seen in a borehole will be the same, regardless of the mean and standard deviation of the fracture radius distributions. (Presuming that the rock volume studied is large enough for being statistically homogeneous).

This is important, especially when establishing DFN-models of a fractured rock mass, because it tells us that for a given value of the *P32* of the rock mass, the uncertainty in actual shape of the fracture radius distribution will not influence the estimates of fracture set orientation as seen in boreholes. Note that this conclusion assumes that the geometrical form of fractures are the same (e.g. circular), it is the size of the fractures that may vary.

8.2.3 Results considering fracture orientation, density and trace-lengths based on data from rock surfaces

All the tests and analyses presented in previous chapters have also been carried out for the fractures of Case 1. Considering fracture data as seen on rock surfaces (fracture trace density, fracture trace-length distribution and fracture trace-strike distribution) the results of Case 1 are **not identical** to the results of the base-case presented in previous chapters.

This can be explained in the following way. The fractures of Case 1 are smaller, hence the probability of seeing the trace of a specific fracture on a surface (window) is smaller than for a specific fracture of the base-case; on the other hand there is a larger number of fractures in Case 1 than in the base-case, hence the number of fracture traces seen on a surface will not be the same as in the base-case. On the average more fracture traces are seen on a surface in Case 1 than in the base-case (presuming that the two surfaces are of equal size), and these traces are on the average shorter than the traces of the base-case.

However, to what amount and in which way this will influence the point estimates will depend on the properties of the fracture sets studied and the size and shape of the windows studied. It is difficult to make any detailed or even general conclusions, except the following: A large number of small fractures will on the average produce a more efficient point estimate (considering a given increase in window radius) than a small number of large fractures (considering the same window radii).

Fracture trace density – *P21*

The fracture density is the same in Case 1 as in the base-case, it follows that the true *P21* value for very large windows (the *P21* parameter) will be the same as in the base-case. This follows from Eq. 6-1. But the point estimate of the *P21*-values in Case 1 will not converge towards the *P21* parameter in the same way as in the base-case. The larger number of smaller fractures in Case 1 will produce smaller differences between different windows (realisations) than in the base-case, which will give a more efficient point estimate than in the base-case. The number of boundary truncated fracture traces will be smaller in Case 1 than in the base-case, which will also influence the point estimate. A summary of the results for Case 1 and the base-case is given in Table 8-2 (below).

As can be seen in the table below, considering the estimation of the *P21* parameter, for Set 1 and Set 3 the necessary sample sizes are smaller for Case 1 than for the base-case. The necessary radius of the window studied in Case 1 is half of the necessary radius of the base-case, for the two criterions studied (Deviation <= +/–15% of true value, and Deviation <= +/–10% of true value).

Considering Set 2 and comparing the results of the two cases, as given in Table 8-2, the results are approximately the same for the two cases. That is however for the criterions (confidence intervals) given in Table 8-2, but for a criterion (confidence interval) of plus/minus 5% of the true value, the necessary window size is somewhat smaller for Case 1 than for the base-case.

The different responses demonstrated by the different fracture sets illustrates the difficulty in making any general conclusions regarding the sensitivity of the point estimate of *P21*, to the properties of the fracture radius distribution. It should however be noted that for the two studied cases, both cases produce the same values of *P21*, if the window studied is large enough.

Table 8-2. P21, Case 1 and the base-case. Summary of results.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	FRACTURE SET	SIZE *(Sample size)*
BASE-CASE ***P21*** [*Trace-length per surface area]* Analysis of circular horizontal surfaces	Deviation < = +/–15% of true value (1)	>= 90%	Horizontal circular Surface Set 1 Set 2 Set 3	Radius of surface >= 24 m >= 22 m >= 40 m
	Deviation < = +/–10% of true value (1)	>= 90%	Horizontal circular Surface Set 1 Set 2 Set 3	Radius of surface >= 38 m >= 32 m >= 65 m
CASE 1 ***P21*** [*Trace-length per surface area]* Analysis of circular horizontal surfaces	Deviation < = +/–15% of true value (1)	>= 90%	Horizontal circular Surface Set 1 Set 2 Set 3	Radius of surface >= 12 m >= 22 m >= 20 m
	Deviation < = +/–10% of true value (1)	>= 90%	Horizontal circular Surface Set 1 Set 2 Set 3	Radius of surface >= 19 m >= 32 m >= 32 m

(1) Samples are within a range of plus or minus 5, 10 or 15 percent of the True value, considering a range centred on the true value, i.e. within: 0.85*TV–1.15*TV (TV=TrueValue)

Fracture trace-length distribution

The *P32* parameter is the same in Case 1 as in the base-case, but the fracture radius distributions are different. Consequently the fracture trace-length distributions will be different and the efficiencies of the point estimates of the trace-length distributions will be different as well.

The smaller fractures of Case 1 will produce smaller traces, consequently less boundary truncation will take place in Case 1 and a larger part of the trace-length distribution will be represented within a limited window. In addition, the larger number of smaller fractures in Case 1 will produce smaller differences between different windows than in the base-case, which will give a more efficient point estimates than in the base-case. A summary of the results for Case 1 and the base-case is given Table 8-3 (below).

As can be seen in the table below, considering the estimation of the mean of the trace-length distributions, the necessary sample sizes are much smaller for Case 1 than for the base-case. The necessary radius of the window studied in Case 1, could be half of the necessary radius for the base-case, or even less. Considering Sets 1, 2 and 3, and the mean values of the trace-length distributions, the necessary window radii of Case 1 are 56% (Set 1), 44% (Set 2) and 42% (Set 3) of the corresponding radii of the base case, for the studied criterion (Deviation <= 15% of true value).

For the standard deviation of the trace-length distribution and for Set 1and Set 3, the necessary radii of the windows in Case 1 are smaller than those of the base-case. Considering Set 1 and Set 3, and the standard deviation of the trace-length distributions, the necessary window radii of Case 1 are 35% (Set 1) and 33% (Set 3) of the corresponding radii of the base case, for the studied criterion (Deviation <= 15% of true value).

It is a different situation for Set 2. Considering Set 2 and the two cases (bas-case and Case 1) the results for the mean-values of Set 2 are in line with the results of Set 1 and Set 3 (i.e., the necessary window sizes are reduced for Case 1). However, for the standard deviation of the trace-length distribution of Set 2 the opposite takes place, the necessary window sizes are larger for Case 1 than for the base-case. For Set 1 the necessary window radius of Case 1 is 208% of the radius of the base-case for the criterion tested (Deviation <= 15% of true value). The reason for the difficulty to estimate the standard deviation of the trace-lengths of Set 2 in Case 1, is caused by the shape of the fractures radius distribution of Set 2. It is a distribution with a small variance in comparison to its mean. Very few small fractures occur in the distribution. However, there will always be small fracture traces because only a part of a fracture may intersect a surface. This will create a fracture trace distribution that is not symmetric and in comparison to the mode of the distribution, the left part (lower tail) of the distribution contains more traces than the right part (upper tail). This will lead to an overestimation of the standard deviation of the trace-length distribution in Case 1, at small window sizes. The overestimation of the standard deviation will not take place in the same way in the base-case, partly because in the base-case the fractures and the traces are larger, which will make the boundary truncation to a dominant process at small window sizes and will reduce the length of the traces observed. Considering a window with a radius of 10 metre, in the base-case 72% of the traces are boundary-truncated, while in Case 1 the amount of boundary truncated traces are 43% Hence, for the base-case and for Set 2, the boundary truncation may actually help to prevent an overestimation at small window sizes.

Again, the different responses demonstrated by the different fracture sets illustrates the difficulty in making any general conclusions regarding point estimates of trace-length distributions and fracture radius distributions.

The analyses of the shape of the distributions are carried out as a Chi-square test of “goodness-of-fit”. A comparison of the results of these tests (for Case 1 and the base-case) demonstrates the following. Considering Set 2 and Set 3, and the amount of accepted samples, the necessary window radius of Case 1 are 60% (Set 2) and 67% (Set 3) of the radius of the base case, for the studied criterion (Confidence level of test= 99%). For Set 1the reduction in necessary size of window is much smaller than for the other fracture sets, for Set 1 the necessary window radius of Case 1 is 97% of the radius of the base-case for the criterion tested (Confidence level of test= 99%).

Table 8-3. Trace-length distributions, Case 1 and the base-case. Summary of results.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	TYPE & FRACTURE SET	SIZE *(Sample size)*
BASE-CASE **MOMENTS** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Deviation < = +/–15% of true value (1)	>= 90%	**MEAN** Set 1 Set 2 Set 3	Radius of surface >= 32 m >= 45 m >= 52 m
			STANDARD DEVIATION Set 1 Set 2 Set 3	Radius of surface >= 52 m >= 12 m >= 70 m
CASE 1 **MOMENTS** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Deviation < = +/–15% of true value (1)	>= 90%	**MEAN** Set 1 Set 2 Set 3	Radius of surface >= 18 m >= 20 m >= 22 m
			STANDARD DEVIATION Set 1 Set 2 Set 3	Radius of surface >= 18 m >= 25 m >= 23 m
BASE-CASE **SHAPE** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Chi-square test “goodness-of-fit” Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 14 m >= 38 m >= 33 m
CASE 1 **SHAPE** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Chi-square test “goodness-of-fit” Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 13 m >= 23 m >= 22 m

(1) Samples are within a range of plus or minus 15 percent of the true value, considering a range centred on the true value, i.e. within: 0.85*TV–1.15*TV (TV=TrueValue)

(2) Samples are accepted based on the result of a Chi-square goodness-of-fit test, which compares the shape of the sample distribution to the shape of the true distribution. The confidence level of the test was set to 99%

Fracture strike distribution

The *P32* parameter is the same in Case 1 as in the base-case, but the fractures are smaller in Case 1 than in the base-case, it follows that a larger number of fractures takes place in the rock mass of Case 1 than in the base-case, and consequently a larger number of traces are on the average seen on rock surfaces in Case 1. The traces observed in Case 1 are on the average shorter than the traces of the base-case, that is however of no importance when measuring the strike of a trace, presuming that the trace is large enough to make a measurement possible.

The efficiency of the point estimate of the fracture trace strike distribution is proportional to the sample size, and as a larger number of traces are (on the average) seen on surfaces in Case 1 than in the base-case, the point estimate of Case 1 is more efficient than the that of the base-case. A summary of the results for Case 1 and the base-case is given (below).

As can be seen in the table below, considering the estimation of the fracture trace strike distributions, the necessary sample sizes are much smaller for Case 1 than for the base-case. The necessary radius of the windows studied in Case 1, could be half of the necessary radius of the base-case, or even less.

For Set 1, considering mean strike and shape of the strike distribution, the necessary radius of the windows studied in Case 1 are 63% (regarding mean) and 61% (regarding shape) of the necessary radius for the base-case. These results corresponds to the criterions studied, sample mean within +/–15 degrees of true value and distribution shape is tested by a Chi-square test with a confidence level of 99%

For Set 2, considering mean strike and shape of the strike distribution, the necessary radius of the windows studied in Case 1 are 22% (regarding mean) and 54% (regarding shape) of the necessary radius for the base-case. These results corresponds to the criterions studied, sample mean within +/–15 degrees of true value and distribution shape is tested by a Chi-square test with a confidence level of 99%

For Set 3, considering mean strike and shape of the strike distribution, the necessary radius of the windows studied in Case 1 are 57% (regarding mean) and 58% (regarding shape) of the necessary radius for the base-case. These results corresponds to the criterions studied, sample mean within +/–15 degrees of true value and distribution shape is tested by a Chi-square test with a confidence level of 99%

Table 8-4. Fracture trace strike distributions, Case 1 and the base-case. Summary of results.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	FRACTURE SET	RADIUS OF SURFACE *(Sample size)*
BASE-CASE MEAN STRIKE	Deviation < = 15 deg (1)	>= 90%	Set 1 Set 2 Set 3	>= 35 m >= 18 m >= 60 m
CASE 1 MEAN STRIKE	Deviation < = 15 deg (1)	>= 90%	Set 1 Set 2 Set 3	>= 22 m >= 4 m >= 34 m
BASE-CASE STRIKE DISTRIBUTION	Chi-square test “goodness-of-fit” Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	>= 13 m >= 11 m >= 24 m
CASE 1 STRIKE DISTRIBUTION	Chi-square test “goodness-of-fit” Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	>= 8 m >= 6 m >= 14 m

(1)Samples are within a range of plus or minus 15 degrees of the True value, considering a range centred on the true value.
(2) Samples are accepted based on the result of a Chi-square goodness-of-fit test, which compares the shape of the sample distribution to the shape of the true distribution. The confidence level of the test was set to 99%

8.3 Case 2 – Same fracture radii but different *P32*-value

8.3.1 Definition of Sensitivity Case 2

Case 2 is identical to the base-case of this study (see Section 2.3), except that the *P32* parameter of the fracture population is defined as two times that of the base-case. The fracture radius distributions of Case 2 are identical to those of the base-case. It follows that, as the fracture radius distribution is the same and the *P32*-value is two times larger, the number of fractures will be larger in Case 2 than in the base-case.

The *P32*-values of the three fracture sets in Case 2 are defined as follows:

$P32_{\mathrm{CASE\ 2}} = 2 \times P32_{\mathrm{BASE\ CASE}}$

- Case 2. $P32_{\mathrm{SET\ 1}}$ = 2 x 0.85 = 1.70
- Case 2. $P32_{\mathrm{SET\ 2}}$ = 2 x 1.59 = 3.18
- Case 3. $P32_{\mathrm{SET\ 3}}$ = 2 x 0.97 = 1.94

8.3.2 Results considering fracture orientation and density based on data from boreholes

All the tests and analyses presented in previous chapters have also been carried out for the fracture of Case 2. Considering fracture data as seen in boreholes (fracture orientation and fracture density) the results of Case 2 are **not identical** to the results of the base-case.

This can be explained in the following way. As the fractures are of the same sizes in both Case 2 and in the base-case (same fracture radius distribution) and the *P32*-values of Case 2 are two times the values of the base-case, the number of fractures in the rock mass of Case 2 will be two times the number of fractures of the base case. It follows that more fractures will be seen in a borehole. On the average in Case 2, the *P10*-values (number of fracture per metre) in a borehole will be two times the values of the base-case.

Hence, the number of fractures in a sample, for a given length of borehole, is in Case 2 (on the average) two times the number of fractures of the base-case. Consequently, the necessary length of borehole, to reach a certain sample size is on the average for Case 2 half of the lengths necessary in the base-case. The variance in number of fracture traces observed is the same if the mean number of fracture traces is the same; this follows from the prerequisite that the only difference between the cases is the *P32*-values.

Consequently, the necessary length of borehole, to reach a certain confidence level, is for Case 2 half of the lengths necessary in the base-case. (It should perhaps be noted this conclusion is also applicable to the estimate of *P32* from borehole data.)

Thus, considering two identical boreholes in two different fracture networks that are equal, except for the fracture density (*P32*-values), for such a situation the following equation is applicable:

$$\frac{LBH_B}{LBH_A} = \frac{P32_A}{P32_B} = \frac{P21_A}{P21_B} = \frac{P10_A}{P10_B} \qquad \text{8-1}$$

LBH_A = Necessary length of borehole, to reach a confidence level in Case A

LBH_B = Necessary length of borehole, to reach a confidence level in Case B

In the equation above, the index A and B correspond to two different fracture networks.

The equation above demonstrates that, everything else being equal except the fracture density, the necessary length of borehole to reach a confidence level is inversely proportional to the fracture density (*P32*-, *P21*- or *P10*- value) of the studied fracture network (fracture set). The conclusions above opens for a possibility to estimate the necessary borehole lengths to reach a certain confidence level (or to estimate a *P32*-value), based on fractures observed in boreholes or rock surfaces and a comparison to results obtained in a DFN-model.

8.3.3 Results considering fracture orientation, density and trace-lengths based on data from rock surfaces

All the tests and analyses presented in previous chapters have also been carried out for the fractures of Case 2. Considering fracture data as seen on rock surfaces (fracture trace density, fracture trace-length distribution and fracture trace-strike orientation), the results of Case 2 are **not identical** to the results of the base-case. In addition, the results are not inversely proportional to the *P32*-value, as was the case for the borehole data. This can be explained in the following way. There are more fractures in the rock mass of Case 2 than in the base-case, hence in comparison with the base-case on the average more fracture traces are seen on a surface in Case 2 than in the base-case (presuming that the two surfaces are of equal size).

Fracture trace density – *P21*

The *P32* parameter is not the same in Case 2 as in the base-case. Considering the different measures of fracture density, the following equation is applicable for fracture networks with different fracture densities:

$$\frac{P10_A}{P10_B} = \frac{P21_A}{P21_B} = \frac{P32_A}{P32_B} \qquad \text{8-2}$$

In the equation above, the index *A* and *B* correspond to two different fracture networks. It follows from Eq. 9-2 that if the *P32* values of the two networks are different, the *P10* and *P21* values will be different as well. By use of the equation above it is possible to calculate the *P10* and *P21* values of Case 2, as the *P32* value of Case 2 is known, as well as the *P32*, *P21* and *P10* values of the base case. According to the equation above the following values are obtained.

SET 1. $P21_{\text{CASE 2 HORIZONTAL CIRCULAR SURFACE}} = 1.54$

SET 2. $P21_{\text{CASE 2 HORIZONTAL CIRCULAR SURFACE}} = 3.0$

SET 3. $P21_{\text{CASE 2 HORIZONTAL CIRCULAR SURFACE}} = 0.82$

These values are also observed in the Case 2 DFN-model for large window radii.

The point estimates of the *P21*-values in Case 2 will not be the same as in the base-case, as the *P21*-values are different and as the number of observed fractures are different. The larger number of fractures in Case 2 will produce smaller differences between different windows (realisations) than in the base-case, which will give a more efficient point estimate than in the base-case. Expressed as a percentage of all observed fracture traces, the number of boundary truncated fracture traces will be the same in Case 2 as in the base-case (because the fracture radius distribution is the same) A summary of the results for Case 1 and the base-case is given below in Table 8-5.

Considering the estimation of the *P21* parameter, and comparing Case 2 and the base-case, Table 8-5 demonstrates the following:

- For Set 1, the necessary sample sizes are smaller for Case 2 than for the base-case. The necessary radius of the windows studied in Case 2 are 75% , 71% and 60% of the necessary radius for the base-case, considering the three different criterions (Deviation <= 15% of true value, Deviation <= 10% of true value and Deviation <= 5% of true value).

- For Set 2, the necessary sample sizes are smaller for Case 2 than for the base-case. The necessary radius of the windows studied in Case 2 are 77% , 72% and 75% of the necessary radius for the base-case, considering the three different criterions (Deviation <= 15% of true value, Deviation <= 10% of true value and Deviation <= 5% of true value).

- For Set 3, the necessary sample sizes are smaller for Case 2 than for the base-case. The necessary radius of the windows studied in Case 2 are 75% , 72% and 72% of the necessary radius for the base-case, considering the three different criterions (Deviation <= 15% of true value, Deviation <= 10% of true value and Deviation <= 5% of true value).

Table 8-5. P21, Case 2 and the base-case. Summary of results.

PARAMETER	CRITERION (*Confidence interval)*	PROBABILITY *(Confidence level)*	TYPE	SIZE *(Sample size)*
BASE-CASE ***P21*** [*Trace-length per surface area]* Analysis of circular horizontal surfaces	Deviation < = +/–15% of true value (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 24 m >= 22 m >= 40 m
	Deviation < = +/–10% of true value (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 38 m >= 32 m >= 65 m
	Deviation < = +/–5% of true value (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 83 m >= 60 m >= 135 m
CASE 2 ***P21*** [*Trace-length per surface area]* Analysis of circular horizontal surfaces	Deviation < = +/–15% of true value (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 18 m >= 17 m >= 30 m
	Deviation < = +/–10% of true value (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 27 m >= 23 m >= 47 m
	Deviation < = +/–5% of true value (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 50 m >= 45 m >= 97 m

(1) Samples are within a range of plus or minus 5, 10 or 15 percent of the True value, considering a range centred on the true value, i.e. within: 0.85*TV–1.15*TV (TV=TrueValue)

Fracture trace-length distribution

The *P32* parameter of Case 2 is two times that of the base-case, but the fracture radius distributions are the same. Consequently the fracture trace-length distributions will be the same, but the number of fracture traces observed on a surface will be larger in Case 2 than in the base case.

Considering trace-length distributions and point estimates of the moments and shape of such distributions, the larger number of fractures in Case 2 will produce smaller differences between different windows (realisations) than in the base-case, which will theoretically produce more efficient point estimates of the trace-length distributions than in the base-case. However, when estimating the true moments and shape of a trace-length distribution, the most important factor is the size of the window studied in relation to the length of the traces observed (size of fracture radius distribution), and not the number of traces observed. Hence, even if the number of traces observed is larger in Case 2, the efficiency of the point estimate can be close to the point estimate of the base-case, as the efficiency is strongly dependent on the size of the windows studied. A comparison of the point estimates of trace-length distribution of Case 2 and the base-case is given below in Table 8-6.

Considering Set 1, a comparison between Case 2 and the base-case demonstrates that the two point estimates are **close, but not identical.** Set 1 of Case 2 demonstrates a more efficient point estimate than the base case, because of (i) the large number of fractures in Case 2, and (ii) as the fractures of Set 1 are small, the relation between fracture size and window size is favourable (a small amount of boundary truncated fractures). For mean and standard deviation of the trace-length distribution of Set 1, the necessary radius of the windows studied in Case 2 are 91% (regarding mean) and 83% (regarding standard dev.) of the necessary radius for the base-case, considering the criterion studied (sample within +/–15% of true value).

Considering Set 2, a comparison between Case 2 and the base-case demonstrates that the two point estimates are **nearly identical** (Case 2 is somewhat more efficient). For mean and standard deviation of the trace-length distribution of Set 2, the necessary radius of the windows studied in Case 2 are 96% (regarding mean) and 92% (regarding standard dev.) of the necessary radius for the base-case, considering the criterion studied (sample within +/–15% of true value).

Considering Set 3, a comparison between Case 2 and the base-case demonstrates that the two point estimates are **close, but not identical.** Set 3 of Case 2 demonstrates a more efficient point estimate than the base case, because of the larger number of fractures in Case 2 in comparison to the base-case. For mean and standard deviation of the trace-length distribution of Set 3, the necessary radius of the windows studied in Case 2 are 88% (regarding mean) and 83% (regarding standard dev.) of the necessary radius for the base-case, considering the criterion studied (sample within +/–15% of true value).

The analyses of the shape of the distributions are carried out as a Chi-square test of "goodness-of-fit". A comparison of the results of these tests (Case 2 and the base-case) demonstrates the following. Considering Set 1 and Set 3, and the amount of accepted samples; the necessary window radius of Case 2 are 71% (Set 2) and 75% (Set 3) of the radius of the base case, for the studied criterion (Confidence level of test= 99%). For Set 2 the reduction in necessary size of window is much smaller than for the other fracture sets, for Set 2 the necessary window radius of Case 2 is 97% of the radius of the base-case for the criterion tested (Confidence level of test= 99%).

Table 8-6. Trace-length distributions, Case 2 and the base-case. Summary of results.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	TYPE & FRACTURE SET	SIZE *(Sample size)*
BASE-CASE **MOMENTS** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Deviation < = +/–15% of true value (1)	>= 90%	**MEAN** Set 1 Set 2 Set 3	Radius of surface >= 32 m >= 45 m >= 52 m
			STANDARD DEVIATION Set 1 Set 2 Set 3	Radius of surface >= 52 m >= 12 m >= 70 m
CASE 2 **MOMENTS** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Deviation < = +/–15% of true value (1)	>= 90%	**MEAN** Set 1 Set 2 Set 3	Radius of surface >= 29 m >= 43 m >= 46 m
			STANDARD DEVIATION Set 1 Set 2 Set 3	Radius of surface >= 43 m >= 11 m >= 58 m
BASE-CASE **SHAPE** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Chi-square test “goodness-of-fit” Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 14 m >= 38 m >= 33 m
CASE 2 **SHAPE** OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Chi-square test “goodness-of-fit” Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 10 m >= 37 m >= 25 m

(1) Samples are within a range of plus or minus 15 percent of the true value, considering a range centred on the true value, i.e. within: 0.85*TV–1.15*TV (TV=TrueValue)

(2) Samples are accepted based on the result of a Chi-square goodness-of-fit test, which compares the shape of the sample distribution to the shape of the true distribution. The confidence level of the test was set to 99%

Fracture strike distribution

The *P32* value of Case 2 is two times that of the base-case and the fracture radius distributions are the same as in the base-case, it follows that a larger number of fractures takes place in the rock mass of Case 2 than in the base-case, and consequently a larger number of traces are on the average seen on rock surfaces in Case 2. The traces observed in Case 2 are on the average of the same length as in the base-case, however the length of a trace is of no importance when measuring the strike of a trace, presuming that the trace is large enough to make a measurement possible.

The efficiency of the point estimate of the fracture trace strike distribution is proportional to the sample size, and as a larger number of traces are (on the average) seen on surfaces in Case 2 than in the base-case, the point estimate of Case 2 is more efficient than the that of the base-case.

As can be seen in the Table 8-7 below, considering the estimation of the fracture trace strike distributions, the necessary sample sizes are much smaller for Case 2 than for the base-case. The necessary radius of the windows studied in Case 2 are close to 66% of the necessary radius of the base-case.

For Set 1, considering mean strike and shape of the strike distribution, the necessary radius of the windows studied in Case 2 are 66% (regarding mean) and 69% (regarding shape) of the necessary radius for the base-case. These results corresponds to the criterions studied, sample mean within +/–15 degrees of true value and distribution shape is tested by a Chi-square test with a confidence level of 99%

For Set 2, considering mean strike and shape of the strike distribution, the necessary radius of the windows studied in Case 2 are 67% (regarding mean) and 64% (regarding shape) of the necessary radius for the base-case. These results corresponds to the criterions studied, sample mean within +/–15 degrees of true value and distribution shape is tested by a Chi-square test with a confidence level of 99%

For Set 3, considering mean strike and shape of the strike distribution, the necessary radius of the windows studied in Case 2 are 67% (regarding mean) and 67% (regarding shape) of the necessary radius for the base-case. These results corresponds to the criterions studied, sample mean within +/–15 degrees of true value and distribution shape is tested by a Chi-square test with a confidence level of 99%

Table 8-7. Fracture trace strike distributions, Case 2 and the base-case. Summary of results.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	FRACTURE SET	RADIUS OF SURFACE *(Sample size)*
BASE-CASE MEAN STRIKE	Deviation < = 15 deg (1)	>= 90%	Set 1 Set 2 Set 3	>= 35 m >= 18 m >= 60 m
CASE 2 MEAN STRIKE	Deviation < = 15 deg (1)	>= 90%	Set 1 Set 2 Set 3	>= 23 m >= 12 m >= 40 m
BASE-CASE STRIKE DISTRIBUTION	Chi-square test "goodness-of-fit" Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	>= 13 m >= 11 m >= 24 m
CASE 2 STRIKE DISTRIBUTION	Chi-square test "goodness-of-fit" Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	>= 9 m >= 7 m >= 16 m

(1)Samples are within a range of plus or minus 15 degrees of the true value, considering a range centred on the true value.
(2) Samples are accepted based on the result of a Chi-square goodness-of-fit test, which compares the shape of the sample distribution to the shape of the true distribution. The confidence level of the test was set to 99%

The number of fracture traces observed on a window (surface) depends on size of window studied. The number of fracture traces on a window is proportional to the three-dimensional fracture density (the *P32*-parameter). It follows that if we have knowledge of the average number of fracture traces on a window, produced by a fracture network with a known value of *P32*; it is possible to analytically estimate the average number of fracture traces that will take place on a window of equal size, but for another fracture network that is equal to the first network except for its value of *P32*. The average number of fracture traces observed and the variance in the number of fracture traces observed will together produce the necessary size of window, for reaching a certain confidence level in an estimate. As it is possible to analytically estimate the average number of fracture traces on a window, for fracture networks that are equal except for the *P32*-values, it is also possible to analytically estimate the necessary sizes of windows for reaching a certain confidence level, for the same networks.

However, as the number of fracture traces (or fractures) per unit area is scale dependent, there is no simple relationship (such as Eq. 9-1), when comparing, (i) different fracture networks (e.g. Cases) with different *P32*-values and (ii) the necessary sizes of windows to reach a certain sample size or confidence level.

Nevertheless, as the number of fracture traces on a window is proportional to the three-dimensional fracture density (the *P32*-parameter), the following equation is applicable when comparing average numbers of fracture traces (for windows of a given size), produced by two different fracture networks that are equal except for the *P32*-values. (The only difference between Case A and B is the *P32*-values, everything else being equal).

$$\frac{N_{A,r}}{N_{B,r}} = \frac{P32_A}{P32_B} \qquad 8\text{-}3$$

$N_{A,r}$ = Case A, Average number of fracture traces on a surface with radius, r.

$N_{B,r}$ = Case B, Average number of fracture traces on a surface with radius, r.

$P32_A$ = Case A, fracture area per unit volume.

$P32_B$ = Case B, fracture area per unit volume.

It follows that for fracture networks, which are equal to the base-case except for the *P32*-value, the average number of fractures on a surface with a given radius could be easily estimated by use of the following equation.

$$N_{C,r} = \frac{N_{0,r}\ P32_C}{P32_0} \qquad 8\text{-}4$$

$N_{0,r}$ = Base-Case, Average number of fracture traces on a surface with radius, r.

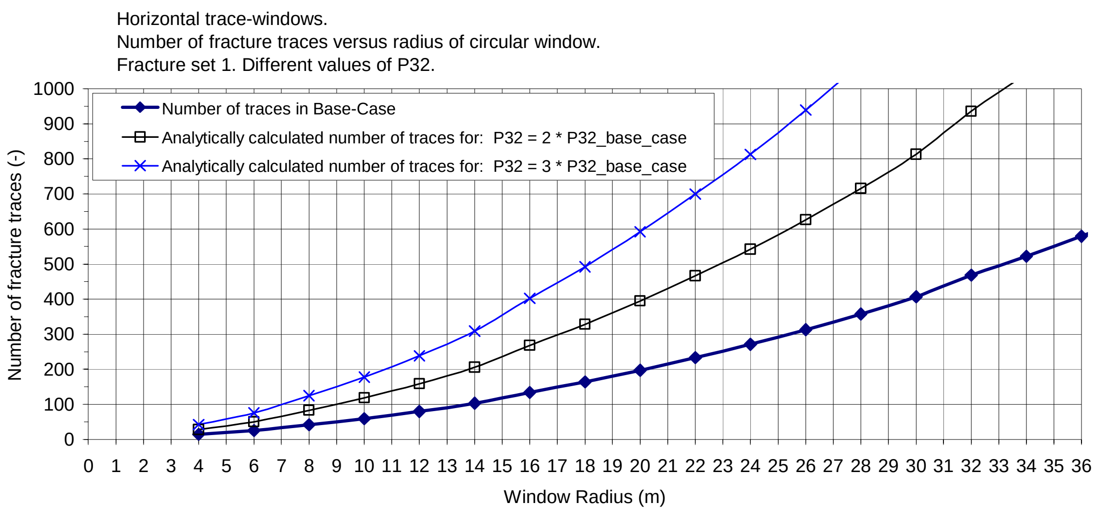
$N_{C,r}$ = Case C, Average number of fracture traces on a surface with radius, r.

$P32_0$ = Base-Case, fracture area per unit volume.

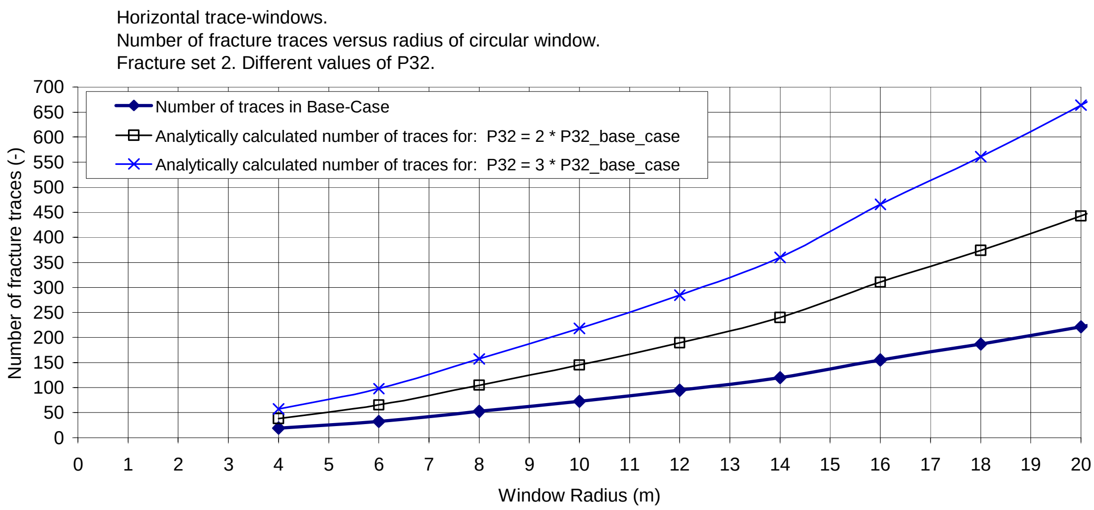
$P32_C$ = Case C, fracture area per unit volume.

Based on the equation above, we have estimated the average number of fracture traces on windows of different sizes, and for different cases that are equal to the base case, except for their *P32*-values. The results are given in Figure 8-4, below.

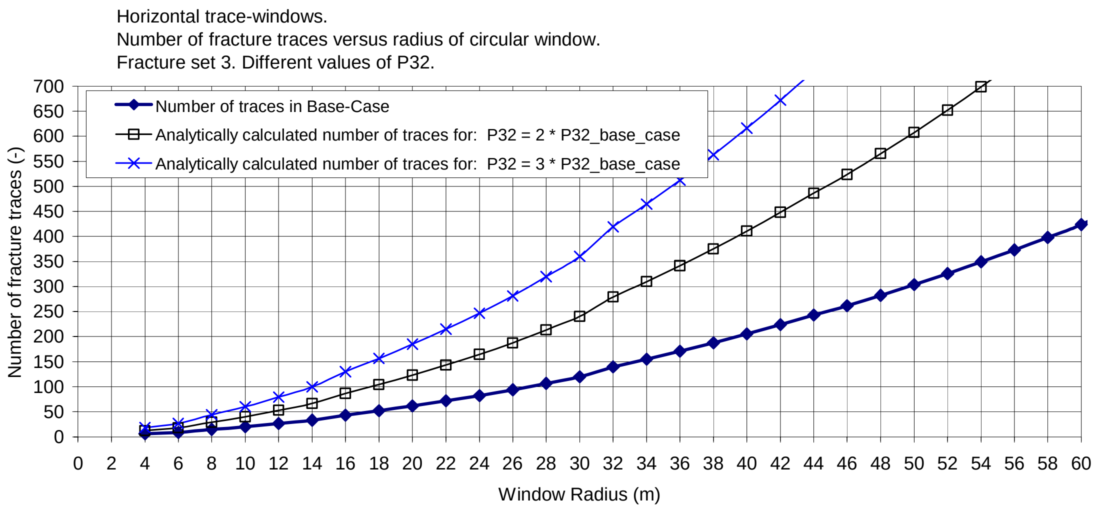
The confidence in an estimate, based on the properties of fracture traces, depends on number of fracture traces observed and on the variance in number of fractures-traces observed. The variance in number of fracture traces observed is the same, if the mean number of fracture traces is the same; this follows from the prerequisite that the only difference between the cases is the *P32*-values. It follows that the confidence is the same, if the average number of fracture traces is the same; hence we can use Figure 8-4 for estimation of the necessary size of windows for reaching a certain confidence level, for the different fracture-networks studied. The following results are obtained.



SET 1



SET 2



SET 3

Figure 8-4. *Number of fracture traces on circular windows of different sizes, for fracture networks that are equal except for their P32-values. Analytical estimate based on the base-case.*

Set 1
For prediction of mean strike, and for a confidence interval given by a deviation less than plus/minus 15 degrees, and for a confidence level of 90%, the base-case predicts a window-radius of 35 m (less than or equal to this radius); this corresponds to an average of approximately 550 fracture traces.

- For a fracture network with a *P32* that is two times the *P32* of the base case, the same number of fractures is obtained for a window-radius of approximately 24 m. Hence, the necessary window-radius is 24 m. The case with a *P32* that is two times the *P32* of the base case, is the same case as the previously presented Case 2. For Case 2, the numerical analysis predicted a necessary window-radius of 23 m.

- For a fracture network with a *P32* that is three times the *P32* of the base case, the same number of fractures is obtained for a window-radius of approximately 19 m. Hence, the necessary window-radius is 19 m.

Set 2
For prediction of mean strike, and for a confidence interval given by a deviation less than plus/minus 15 degrees, and for a confidence level of 90%, the base-case predicts a window-radius of 18 m (less than or equal to this radius); this corresponds to an average of approximately 190 fracture traces.

- For a fracture network with a *P32* that is two times the *P32* of the base case, the same number of fractures is obtained for a window-radius of approximately 12 m. Hence, the necessary window-radius is 12 m. The case with a *P32* that is two times the *P32* of the base case, is the same case as the previously presented Case 2. For Case 2, the numerical analysis predicted the same window-radius (12 m).

- For a fracture network with a *P32* that is three times the *P32* of the base case, the same number of fractures is obtained for a window-radius of approximately 9 m. Hence, the necessary window-radius is 9 m.

Set 3
For prediction of mean strike, and for a confidence interval given by a deviation less than plus/minus 15 degrees, and for a confidence level of 90%, the base-case predicts a window-radius of 60 m (less than or equal to this radius); this corresponds to an average of approximately 425 fracture traces.

- For a fracture network with a *P32* that is two times the *P32* of the base case, the same number of fractures is obtained for a window-radius of approximately 40 m. Hence, the necessary window-radius is 40 m. The case with a *P32* that is two times the *P32* of the base case, is the same case as the previously presented Case 2. For Case 2, the numerical analysis predicted the same window-radius (40 m).

- For a fracture network with a *P32* that is three times the *P32* of the base case, the same number of fractures is obtained for a window-radius of approximately 32 m. Hence, the necessary window-radius is 32 m.

9 Conclusions

9.1.1 Applicability and limitations of the presented results

Results and conclusions given in this study are only directly applicable to the fracture networks studied; however, rock masses having similar fracture networks will produce similar results. Nevertheless, great care should be taken when generalising results and conclusions given in this study. It is important to note the following:

- Considering the studied parameters of the rock mass, the results correspond to a rock unit having statistically homogeneous properties. When analysing real data from field investigations, the applicability of this assumption needs to be statistically evaluated.

- The rock unit studied is of a certain size and is assigned statistically homogeneous properties. Sample sizes have been calculated—sample sizes that are necessary to reach a certain confidence level when predicting the properties of the rock unit. When applying the results of this study to an actual rock unit it is not a prerequisite that the actual rock unit must be of the same size and form as the unit used in this study when the necessary sample sizes were calculated. However, the actual rock unit needs to be larger than the calculated necessary sample size, and it should carry statistically properties that are close to homogeneous within the volume considered. For example, it is a result of this study that for a certain rock mass the mean direction of a certain fracture set could be estimated (within a certain acceptable deviation) using a vertical borehole with a length of 20 m. Such a result is applicable to a rock unit that is larger than 20 m and carries statistically homogeneous properties within that scale.

- The fracture network studied does not contain any spatial correlation of the fractures. For a fracture network that has such a correlation, the necessary length of boreholes and size of rock outcrops, for producing an estimate with a certain confidence level, is larger than for the network of this study.

- The effects of different methods for identification of fracture sets are not included in this study.

- The fracture orientations observed in boreholes were corrected for sampling bias by use of Terzaghi correction; such a correction is essential and should always be included when analysing fracture orientations data from boreholes.

- This study is a theoretical study, all data form the boreholes and rock-surfaces are numerically collected from a numerical fracture-network. No measurement errors occur in this study and all data is collected with the same high precision and quality.

Below are a few important observations that should be considered when generalising the results and conclusions given in this study (more details are given in the sensitivity analysis presented in Chapter 8).

- The results depend on the properties of the fracture network, i.e. fracture orientation and fracture density (intensity) and fracture size. Generally, for a fracture network with a higher fracture density than that of the network studied, the lengths of boreholes and sizes of rock outcrops, necessary for deriving an estimate within a certain confidence interval and at a certain confidence level, is less than for the network studied. It follows that for a rock mass with a lower fracture density, the necessary length of boreholes and size of rock outcrops is larger than for the network of this study.

- The dispersion of the orientations of the fractures of a fracture set will influence the length of a borehole and the size of a rock outcrop (window), necessary for deriving an estimate with a certain confidence. In general, when analysing a fracture set with a large dispersion, the necessary length of borehole and size of rock outcrop is larger than for a fracture set with a smaller dispersion (everything else being equal). Also the type of distribution of orientations within a fracture set (e.g. Fisher distribution) will influence the necessary lengths and areas. However, the efficiency of a point estimate, based on data gathered by boreholes and rock surfaces, will also depend on the orientation of boreholes and surfaces studied. In theory it is possible that for a fracture set with a very small dispersion, sampled with a borehole or a surface that is approximately at right angel to the mean direction (trend and plunge) of the fracture set, the necessary length of borehole or size of surfaces could be very large. (If the dispersion is zero (or negligible) and the sampling borehole or plane is at right angelto the fracture set, the necessary borehole length and window size could be infinite.)

- When analysing a fracture set with a sampling structure, i.e. a borehole (a scan-line) or a rock-outcrop (a window). The length or size of the sampling structure, necessary for deriving an estimate with a certain confidence, depends on the orientation of the sampling structure in relation to the mean orientation of the fracture set studied. In general the most favourable orientation of a sampling structure is an orientation parallel to the mean direction (defined by trend and plunge) of the fracture set studied (i.e. on the average the fracture planes should be at right angles to the structure). For boreholes, the use of Terzaghi-correction will compensate for the systematic bias caused by sampling a three-dimensional fracture system with a one-dimensional scan-line. Therefore, also a borehole that is approximately at right angles to the to the mean direction (defined by trend and plunge) can be used for sampling (i.e. the borehole is along the fracture planes). The Terzaghi-correction is not perfect and for very small confidence intervals (acceptable deviations), the remaining bias may come to dominate the derived estimates.

- Consider estimations based on observations in boreholes. Everything else being equal, the necessary length of borehole for producing an estimate with a certain confidence level is linearly proportional to the fracture density of the population studied (the *P32*-value or the *P21*-value or the *P10*-value).

- For a rock mass with a given fracture density, the mean and the variance of the fracture radius distributions (fractures defined as circular planar discs), will not influence the number of fractures that intersects a borehole. Hence, on the average a small number of large fractures will produce the same number of fracture observations in a borehole as a large number of small fractures, presuming that the fracture density of the rock mass is the same (*P32* is constant). It follows that for estimations based on observations in boreholes, the necessary length of borehole for producing an estimate with a certain confidence level, is independent on mean and variance of the fracture radius distributions, presuming that the fracture density of the rock mass is the same (*P32* is constant).

- Consider estimations based on observations on surfaces. Everything else being equal, the necessary size of rock-outcrop (window) for producing an estimate with a certain confidence level is not linearly proportional to the fracture density of the population studied (the *P32*-value or the *P21*-value or the *P10*-value). Estimation of the trace-length distributions is difficult, the necessary size of window for producing an estimate with a certain confidence level depends on (i) the orientations of window studied in relation to that of the fracture set studied, (ii) the size of the window studied in relation to the properties of the fracture-radius distribution that created the fracture traces, as well as on (iii) the fracture density (the *P32*-value) and the dispersion of the fracture set studied. It follows that it is difficult to make any general conclusions regarding the necessary window size for estimating the properties of a trace-length distribution (with a certain confidence). For estimation of the mean of a strike distribution (derived from direction of fracture traces), the necessary window-radius for deriving an estimate with a certain confidence level, is related to the fracture density (*P32*-value) in a non-linear way. However, for fracture networks that are equal, except for the *P32*-value, this relationship can be analytically estimated.

9.1.2 Summary of detailed results

The analysed fracture network consists of three fracture sets (see Section 2.3).

- Set 1 is sub-vertical, it has a very large dispersion and the smallest value of fracture density (*P32*), the fractures of Set 1 is on the average small.

- Set 2 is sub-vertical, its dispersion is much less than that of Set 1, it has the largest value of fracture density (approximately two times that of Set 1) and on the average it contains the largest fractures.

- Set 3 is sub-horizontal, it has the same dispersion as Set 2, its fracture density is smaller that that of Set 2, but somewhat larger than that of Set 1. On the average it contains smaller fractures than Set 2, but larger fractures than Set 1.

Below we will present some detailed results; these results are also summarised in tables at the end of this chapter. Different aspects of the applied statistical tests are given in Section 2.7. The results given below can be interpreted as confidence levels and confidence intervals, even if they are not presented that way. The presented maximum acceptable deviation in estimation corresponds to a confidence interval. The probability for an estimate within the given maximum deviation corresponds to a confidence level.

The length of borehole or radius of studied window corresponds to a sample size – the size that is necessary to reach the confidence level.

The results given below are only examples of results that can be deduced from the figures of the main report.

Fracture orientation – mean direction

The mean direction of the fracture sets studied, were calculated based on sampling of fracture orientations, in both the vertical and the inclined borehole. The acceptable deviation (confidence interval) in degrees, discussed below, corresponds to the acute angle between the true mean direction and that of a sample.

Considering a vertical borehole.

With a probability larger than 90 percent (confidence level), the deviation in estimation is less than 15 degrees, if the borehole length is larger than:

For Set 1: 140 metres. For Set 2: 50 metres. For Set 3: 20 metres.

Considering an inclined borehole (45 degrees form vertical).

With a probability larger than 90 percent (confidence level), the deviation in estimation is less than 15 degrees, if the borehole length is larger than:

For Set 1: 90 metres. For Set 2: 35 metres. For Set 3: 35 metres.

Fracture orientation – dispersion

The dispersion of a fracture set is a measure of the concentration (or spread) of the fracture orientations about some mean direction. In this study we have analysed two dispersion parameters, the Kappa parameter which corresponds to a Fisher distribution, and the SR1 parameter which is a general dispersion parameter. The dispersion of the fracture sets studied, were calculated based on sampling of fracture orientations, in both the vertical and the inclined borehole.

Analysis of the Kappa parameter by use of a Vertical borehole. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true Kappa value, if the borehole length is larger than or equal to:

For Set 1: 500 metres. For Set 2: 420 metres. For Set 3: 200 metres.

Analysis of the Kappa parameter by use of an Inclined (45 deg.) borehole. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true Kappa value, if the borehole length is larger than or equal to:

For Set 1: 420 metres. For Set 2: 360 metres. For Set 3: 500 metres.

Analysis of the SR1 parameter by use of a Vertical borehole. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true SR1 values if the borehole lengths are larger than or equal to:

For Set 1: 1100 metres. For Set 2: 250 metres. For Set 3: 100 metres.

Analysis of the SR1 parameter by use of an Inclined (45 deg.) borehole. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true SR1 values, if the borehole lengths are larger than or equal to:

For Set 1: 750 metres. For Set 2: 170 metres. For Set 3: 250 metres.

Fracture density – the *P10* parameter (fracture frequency)

The one-dimensional fracture density is the *P10* parameter; it is equal to *number of fractures per unit length*, taken along a straight line (a scan-line). The value of the *P10* parameter varies with direction of scan-line. The *P10*-parameter of the fracture sets studied, were calculated based on sampling of fractures, in both the vertical and the inclined borehole.

Analysis of the *P10* parameter by use of a Vertical borehole. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true *P10* values, if the borehole lengths are larger than or equal to:

For Set 1: 400 metres. For Set 2: 300 metres. For Set 3: 150 metres.

Analysis of the *P10* parameter by use of An Inclined borehole (45 deg.). With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true *P10* values, if the borehole lengths are larger than or equal to:

For Set 1: 350 metres. For Set 2: 150 metres. For Set 3: 210 metres.

Fracture density – the *P21* parameter

The two-dimensional fracture density is the *P21* parameter; it is equal to *fracture trace-length per unit surface area*, taken over a two-dimensional plane. The *P21*-parameter of the fracture sets studied, were calculated based on sampling of fracture traces on circular horizontal windows (surfaces). With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true *P21* value, if the circular window has a radius larger than or equal to:

For Set 1: 24 metres. For Set 2: 22 metres. For Set 3: 40 metres.

Fracture density – the *P32* parameter

The three-dimensional fracture density is the *P32* parameter; it is equal to *fracture surface area per unit rock volume*, taken over a volume. It is possible to derive a *P32*-value based on a *P21*-value and/or a *P10*-value, by use of a trial and error procedure in a DFN-model. Hence, the results for the *P21* and *P10* parameters are also applicable to the *P32* parameter. However, also the uncertainty and errors stemming from the trial and error procedure, will influence the estimation of the *P32*-parameter. In addition the *P32*-parameter can be estimated based on the fracture-surface area as seen in a borehole and the corresponding borehole volume. The *P32*-parameter of a fracture set is well estimated by sampling in both the vertical and the inclined borehole, presuming that the borehole length is large.

Analysis of the *P32* parameter by use of a Vertical borehole. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true *P32*-value, if the borehole length is larger than or equal to:

For Set 1: 850 metres. For Set 2: 650 metres. For Set 3: 150 metres.

Analysis of the *P32* parameter by use of an Inclined borehole (45 deg.). With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true *P32*-value, if the borehole length is larger than or equal to:

For Set 1: 480 metres. For Set 2: 320 metres. For Set 3: 380 metres.

Fracture trace-length distribution

Trace-length distributions were derived by analysing fracture traces on horizontal circular windows (surfaces). Sample trace-length distributions were derived for each fracture set separately, for different window-radii.

Moments of sample distributions

The results are given for two different moments, mean and standard deviation. We conclude the following results.

Mean of trace-length distribution. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true mean value, if the window radius is larger than or equal to:

For Set 1: 32 metres. For Set 2: 45 metres. For Set 3: 52 metres.

Standard deviation of trace-length distribution. With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true standard deviation value, if the window radius is larger than or equal to:

For Set 1: 52 metres. For Set 2: 12 metres. For Set 3: 70 metres.

Moments of log-normal distributions fitted to an sample distributions

The sample trace-length distributions were also analysed by fitting a log-normal distribution to the sample distribution. The moments of these log-normal distributions were compared to the moments of a log normal distribution fitted to the simulated true trace-length distribution (a distribution obtained from very large windows).

Mean of log-normal distribution (mean in normal-space). With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true mean value, if the window radius is larger than or equal to:

For Set 1: 25 metres. For Set 2: *Not well represented*. For Set 3: 45 metres.

Standard deviation of log-normal distribution (values in normal-space). With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15% of the true standard deviation value, if the window radius is larger than or equal to:

For Set 1: 42 metres. For Set 2: *Not well represented*. For Set 3: 65 metres.

Shape of sample distributions

The sample trace-length distributions was also analysed by comparing the shape of the sample distribution to the shape of the simulated true distribution, by use of a chi-square "goodness-of-fit" test. With a probability larger than 90 percent, the shape of a trace-length distribution derived from a sample is a good representation of the simulated true distribution (the confidence level of the test was set to 99 percent), if the window radius is larger than or equal to:

For Set 1: 13 metres. For Set 2: 38 metres. For Set 3: 32 metres.

Fracture Trace strike distribution

The trace strike distributions of the fracture sets studied were calculated based on sampling of the strike of fracture traces on horizontal circular windows (surfaces).

Mean of sample distributions

With a probability larger than 90 percent, the deviation in estimation is less than plus/minus 15 degrees of the true mean value, if the window radius is larger than or equal to:

For Set 1: 35 metres. For Set 2: 18 metres. For Set 3: 60 metres.

Shape of sample distributions

The sample fracture trace strike distributions were also analysed by comparing the shape of the sample distributions to the shape of the simulated true distribution, by use of a chi-square "goodness-of-fit" test. With a probability larger than 90 percent, the shape of a trace strike distribution derived from a sample is a good representation of the simulated true distribution (the confidence level of the test was set to 99 percent), if the window radius is larger than or equal to:

For Set 1: 13 metres. For Set 2: 11 metres. For Set 3: 24 metres.

Concluding remarks

When comparing the results for the different fracture sets, it is demonstrated that the most difficult fracture set to analyse is Set 1, because this set has a large dispersion and the smallest value of *P32* (fracture density) of the three sets studied.

When comparing the results of a specific fracture set considering different borehole directions, the variation in results is in line with the variation in number of fractures observed in boreholes with different orientations.

Considering the orientation of the fractures of a fracture set, it is more difficult to estimate the dispersion of the fracture-orientations than the mean of the fracture-orientations.

Considering fracture set 3 and horizontal windows, the large radius necessary for good estimates of the parameters of Set 3 is caused by the sub-horizontal orientation of Set 3, because the fractures of a sub-horizontal fracture set only rarely intersects a sub-horizontal surface. A fracture set with such an orientation is not well analysed by use of sub-horizontal surfaces, unless a correction for sampling bias is applied and in this study such a correction was not used when the surface data were analysed. (Correction for orientation sampling bias was only applied to borehole data.)

Estimation of the trace-length distributions is difficult, as such estimations (among other things) depend on the size of the window studied in relation to the properties of the fracture-radius distribution that created the fracture traces. Therefore the results for different fracture sets could be very different, for the same size of window.

9.1.3 On parametric tests and calculated confidence intervals

Parametric statistical tests were carried out regarding mean direction and dispersion of the three fracture sets of the population, considering observations of fracture orientation in theoretical boreholes (see Sections 3.5 and 4.4). As the population (the fracture network) is created by use of Fisher distributions, the tests were based on the assumption that samples were drawn from (represent) Fisher distributions. Even if we had not known that the fracture sets were generated that way, the circular shape of the fracture clusters as revealed by the SR2 parameter (see Section 4.2.1), indicates that parametric tests against Fisher distributions are appropriate. The tested hypothesis was that the mean direction and the dispersion of the population, as estimated by the samples, are equal to the known true properties of the population. We know that this is a correct hypothesis; but due to sampling bias, remaining in the samples after application of Terzaghi correction, the hypothesis will not necessarily be confirmed by the samples. The results of the tests demonstrate a larger amount of rejected samples, than the amount prescribed by the confidence level of the tests; for some of the fracture sets and especially when analysing the samples from the inclined borehole. This is a consequence of a systematic bias in the point estimates of the properties of the population. This bias follows from the fact that a borehole is a one-dimensional line that samples a three-dimensional fracture network. The applied Terzaghi correction, which removes most of this bias, is not perfect and some aspects of the bias remain in the samples. The following conclusion can be made: If we assume that (i) samples are drawn from perfect Fisher distributions and that (ii) the systematic sampling bias is fully corrected by use of Terzaghi correction; we may derive confidence intervals, based on parametrical statistical analysis, that are to small and which do not reflect the actual uncertainties. This is especially the case if the sample size is large (a sample that contains a large number of fracture observations) as the confidence intervals, derived through parametric statistical analyses, are small for such samples.

9.1.4 On optimal orientation of a borehole

Based on observations in theoretical boreholes, we have estimated fracture set orientation, mean direction and dispersion, as well as the fracture density parameters *P10* and *P32*. Two different boreholes have been used, a vertical and an inclined borehole. By comparing the efficiency of the point estimates, as produced by the two boreholes, we can make conclusions regarding the optimal orientation of a borehole.

Let us first consider the *P10*-parameter (fracture frequency in a borehole); it is a direction-dependent parameter and as such it is calculated without Terzaghi correction. The point estimate of the *P10* parameter relates to borehole length and not to number of fractures in a sample. However, the efficiency of the point estimate increases with number of fractures observed in a sample; hence for a given borehole length, the borehole that intersects most fractures will produce the most efficient point estimate as regards the *P10*-parameter. Considering the two borehole directions studied, the inclined borehole (45 deg.) produces on the average, when adding together all three fracture sets, the largest samples (number of fractures per metre of borehole), and consequently as regards *P10* the point estimate is most effective for the inclined borehole.

For all parameters analysed by use of boreholes, on the average the most efficient point estimate takes place for the borehole direction for which most fractures are intersected. Hence, in order to reach the largest efficiency when analysing a single fracture set, the borehole should not necessarily be an inclined borehole, but directed so that the mean direction (defined by trend and plunge) of the fracture set studied is parallel to the borehole (i.e. on the average the fracture planes are at right angles to the borehole), because on the average this is the borehole direction that produces the largest samples (for a given borehole length). Consequently, different borehole directions are optimal for different fracture sets.

The borehole length necessary for deriving acceptable estimates of all properties studied of all fracture sets studied is determined by the length necessary for deriving an acceptable estimate of the property and fracture set that is the most difficult to estimate. The properties that are easier to estimate will be derived within the borehole length necessary for the most difficult estimation. For example, if we want to estimate the mean orientation and dispersion (Kappa) of the three fracture sets studied, by use of a vertical borehole, the necessary length is 500 m (confidence level=90%; confidence interval =+/– 10 degrees (orientation) and +/–15% (Kappa)). By use of an inclined borehole, the necessary length is 500m as well. For the vertical borehole the most difficult parameter to estimate is the dispersion of Set 1, consequently this is the parameter that determines the borehole length for the vertical borehole. For the inclined borehole the most difficult parameter to estimate is the dispersion of Set 3, and consequently this is the parameter that determines the borehole length for the inclined borehole. For both boreholes the necessary borehole length is 500m.

Even if the necessary length of borehole was the same for the two borehole orientations, as this length was determined by the most difficult estimation, the necessary lengths for estimating the other parameters were not the same. As a measure of the average efficiency of a borehole orientation we have calculated the average necessary length for estimating certain parameters in the same borehole.

$$A = \frac{\sum_{i=1}^{n} L_i}{n} \qquad \text{9-1}$$

A = Average necessary length.

Li = Necessary borehole length to derive an estimate of a parameter of a fracture set, within a given confidence interval and confidence level.

n = Number of estimates studied.

The results for the *P10* and *P32* parameters are given in Table 9-1 (below).

Table 9-1. Average necessary borehole lengths for deriving estimates of all three fracture sets, in boreholes of different directions, considering P10 and P32.

Parameter	Confidence interval	Confidence level	BH-type	Average
P10	Deviation<=+/–15%	90%	Vertical	283m
P10	Deviation<=+/–15%	90%	Inclined(45deg)	236m
P32	Deviation<=+/–15%	90%	Vertical	550m
P32	Deviation<=+/–15%	90%	Inclined(45deg)	393m

Considering fracture frequency *P10* and a vertical borehole, the necessary lengths are 400 m (Set 1), 300 m (Set 2) and 150 m (Set 3), producing an average necessary length of 283 m (confidence interval= +/–15% of true value and confidence level= 90%). For an inclined borehole the average necessary length is 236 m. The average necessary length of the inclined borehole is 84% of that of the vertical borehole. Considering fracture density *P32* (based on borehole data), the average necessary length of the inclined borehole (393 m) is 71% of that of the vertical borehole (550 m). Hence, the inclined borehole produces on the average the best estimates, especially for the *P32* parameter. On the other hand, if the acceptable deviation (confidence interval) is *not* set as very small and the available borehole lengths are large, the direction of the borehole is not very important, as acceptable estimates could be derived for any direction.

Estimates of fracture set orientation should, as little as possible, be dependent on the orientation of the investigation borehole. Therefore all orientation data from boreholes should be corrected by use of Terzaghi correction (see Appendix B). The Terzaghi correction will compensate for most of the systematic sampling bias. After application of Terzaghi correction, the sample sizes necessary for deriving an estimate with a certain confidence, should only be weakly dependent on the orientation of the borehole, however the necessary lengths will still be dependent on dispersion and fracture density; and as the Terzaghi correction is not perfect and some systematic bias will remain in the samples, it follows that some borehole orientations are better than other orientations. The number of fractures observed and the efficiency (completeness) of the Terzaghi correction depends on the acute angle between the borehole and the mean orientation of the fracture set studied. When considering the efficiency (completeness) of the Terzaghi correction, different directions of borehole are optimal for different fracture sets (as they occur in a rock unit). The remaining bias will have the least influence if the bias is distributed in a symmetric way around the predicted mean orientation, which is achieved for boreholes that are at right angles or parallel to the mean direction of the fracture set.

Hence, for best efficiency of the Terzaghi correction, the borehole should be directed in a way that the mean direction (trend and plunge) of the fracture set studied is parallel to the borehole (i.e. fracture planes at right angles to the borehole), as most fractures are intersected for this direction, and because the remaining bias will be symmetric for such a direction. A borehole direction that is at right angle to the mean direction (trend and plunge) of a fracture set (i.e. borehole direction along fracture planes) could (theoretically) be an efficient investigation borehole, assuming that it is has a large length. Because for very large lengths of such a borehole direction, the derived estimate will be close to the true value, as the remaining bias is symmetrically distributed for such a borehole direction.

For a borehole that is not parallel and not at right angles to the mean direction of the fracture set studied, and if the acceptable deviation (confidence interval) is set as very small, for such a situation the necessary borehole lengths could be infinite (especially for large values of the confidence level). Because the estimates might converge not towards the true value but towards a value that is slightly off the true value, due to the remaining sampling bias (see Figure 2-7 and Appendix B). (If the acceptable deviation (confidence interval) is set as very small, the estimate may converge towards a value outside of the confidence interval.)

The necessary average lengths, considering mean directions of a fracture set, are given in Table 9-2 (below).

Table 9-2. Average necessary borehole lengths for deriving estimates of all three fracture sets, in boreholes of different directions, considering mean direction of fracture sets.

Parameter	Confidence interval	Confidence level	BH-type	Average
Mean direction	Deviat.<=15deg	90%	Vertical	70m
Mean direction	Deviat.<=15deg	90%	Inclined(45deg)	53m
Mean direction	Deviat.<=10deg	90%	Vertical	133m
Mean direction	Deviat.<=10deg	90%	Inclined(45deg)	113m
Mean direction	Deviat.<= 5deg	90%	Vertical	550m
Mean direction	Deviat.<= 5deg	90%	Inclined(45deg)	Not possible

The average necessary length of the inclined borehole (53 m) is 76% of that of the vertical borehole (70 m), for an acceptable deviation (confidence interval) of 15 degrees and a confidence level of 90%. For an acceptable deviation of 10 degrees, the average necessary length of the inclined borehole (113 m) is 85% of that of the vertical borehole (133 m). And finally, for an acceptable deviation of 5 degrees, the average necessary length of the inclined borehole is undefined. Because by use of an inclined (45 deg) borehole it is not possible to estimate the mean direction of Set 1 at such a small acceptable deviation (confidence interval) together with a confidence level of 90%. Hence, the inclined borehole is better than the vertical borehole, except if the confidence interval (acceptable deviation) and confidence level is set as very small, for such a situation the direction of the borehole has to be optimised for each fracture set. On the other hand, if the acceptable deviation (confidence interval) is not very small, the direction of the borehole is not very important, as acceptable estimates could be derived for any direction, and the difference in total lengths for different borehole directions is not very large.

The necessary total lengths, considering dispersion of a fracture set, are given in Table 9-3 (below).

Table 9-3. Average necessary borehole lengths for deriving estimates of all three fracture sets, in boreholes of different directions, considering dispersion of fracture sets.

Parameter	Confidence interval	Confidence level	BH-type	Average
SR1	Deviat.<=+/–15%	90%	Vertical	483m
SR1	Deviat.<=+/–15%	90%	Inclined(45deg)	390m
Kappa	Deviat.<=+/–15%	90%	Vertical	373m
Kappa	Deviat.<=+/–15%	90%	Inclined(45deg)	427m

Considering dispersion in fracture orientation, as represented by the SR1 dispersion parameter, the average necessary length of the inclined borehole (390 m) is 81% of that of the vertical borehole (483 m), for an acceptable deviation (confidence interval) of +/– 15% of the true values and a confidence level of 90%. This is in line with the results for the mean direction (above). It should however be noted that the different necessary lengths for each individual fracture set, considering the SR1 parameter (see Section 4.2, page 64), are very large (e.g. vertical borehole, Set 1=1100 m, Set 2=250 m and Set 3=100 m). Considering dispersion in fracture orientation, as represented by the Kappa dispersion parameter (see Section 4.3, page 75), the average necessary length of the inclined borehole (427 m) is 114% of that of the vertical borehole (373 m). This is different from the results regarding mean direction, and it follows from the remaining sampling bias of the inclined borehole.

Thus, it is more difficult to predict dispersion than mean value (which is the way it should be, as dispersion is a measure of variance), it follows that the borehole direction is more important when estimating dispersion than when estimating mean direction of a fracture set.

The borehole direction is also more important when estimating *P32* than when estimating *P10*. In general, the necessary lengths of boreholes are larger when estimating *P32* than for estimation of *P10*. However, if the borehole direction and mean direction (trend and plunge) of the fracture set is parallel, the *P10*-value in the borehole is equal to the *P32*-value of the fracture set; this conclusion underlines the importance of borehole direction.

If the acceptable deviation (confidence interval) is not very small, and large borehole lengths are available, any borehole direction will do, but if the acceptable deviation (confidence interval) has to be very small and/or only short borehole lengths are available, for such a situation the borehole direction is important and needs to be optimised considering each fracture set. In general it is better to have three somewhat shorter boreholes, with different optimised directions, than one borehole with a large length

9.1.5 On number of investigation boreholes and rock surfaces

In this study the analysed fracture network is statistically homogeneous, it follows that the results are only applicable to a rock unit with statistically homogeneous properties. Considering the use of boreholes for investigation of fracture sets orientation (mean direction and dispersion) and the *P10* fracture density parameter, the necessary size of samples for the estimation of the parameter does not have to come from a single borehole. If the rock mass has statistically homogeneous properties, the analysed sample can come from several different boreholes that together produce the necessary size of sample. For example, three boreholes of length 50 metres can together form a sample representing approximately the same size of sample as observations in a single borehole of length 150 metres (presuming that they all are in the same rock unit with statistically homogeneous properties). Hence, in practise when analysing a real rock mass, it is very important to know which observations belong to which rock unit, especially if several boreholes are used; that is however also a concern when analysing observations from a single borehole with a large length.

It is however a different situation when considering the mapping of fracture trace-length distributions on rock surfaces. There are several biases that come from sampling a three dimensional system with a two-dimensional surface of a given form (e.g. circular), this is discussed in Section 6.2; but regarding the topic of this section, the most important bias is the boundary truncation of the large fracture traces. This is stated in Section 6.3.1 in the following way “The efficiency of a point estimate increases with sample size, however for the sampling of traces also the size of the studied window is important. The observations are made on windows that have a limited size, and the upper tail of the trace-length distribution (traces with a large length) can only be directly observed on windows of a size (radius) comparable to length of the large traces. Hence, for small windows there will be a systematic bias in the estimate of the trace-length distribution, due to boundary truncation, even if the sample size is large. (Small window sizes could be sufficient if it is possible to fit a mathematical distribution to the observed truncated trace-length distributions, even if such a curve fitting procedure will introduce uncertainty regarding the ability of such a distribution to represent the part of the true distribution that is unknown at small window sizes.) ”

It follows from the statement above that regarding the trace-length distribution it is not possible to replace observations on one large window with observations on several smaller windows, even if all windows are from the same rock unit with statistically homogeneous properties. However, as stated above, by fitting a mathematical function to observations made on small windows, an approximate estimation can be derived of the upper tail of the trace-length distribution. We will in this study not discuss the best method for such a curve fitting; an example of curve fitting is however given in Chapter 6. The given example is the fitting of a Log-Normal distribution to the observed trace-length distribution.

It is again a different situation when considering observations of fracture strike distributions, derived from directions of fracture traces, as observed on rock surfaces. As for the trace-length distribution there are several biases that come from sampling a three dimensional system with a two-dimensional surface of a given form (see Section 6.2). However, there is no systematic bias in the estimate of the strike distribution, due to boundary truncation of large fracture traces. Hence, when estimating the strike distribution it is possible to replace observations on one large window with observations on several smaller windows and thereby gather one large sample, presuming that all windows are from the same rock unit with statistically homogeneous properties.

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	SAMPLE TYPE FRACTURE SET	BOREHOLE LENGTH (1) *(Sample size)*
ORIENTATION **MEAN DIRECTION** The deviation in degrees corresponds to the acute angle between the true mean direction and that of a sample.	Deviation < = 15 deg	>= 90%	Vertical BH	
			Set 1	>= 140 m
			Set 2	>= 50 m
			Set 3	>= 20 m
			Inclined BH	
			Set 1	>= 90 m
			Set 2	>= 35 m
			Set 3	>= 35 m
	Deviation < = 10 deg	>= 90%	Vertical BH	
			Set 1	>= 270 m
			Set 2	>= 90 m
			Set 3	>= 40 m
			Inclined BH	
			Set 1	>= 200 m
			Set 2	>= 70 m
			Set 3	>= 70 m
	Deviation < = 5 deg	>= 90%	Vertical BH	
			Set 1	>= 1200 m
			Set 2	>= 340 m
			Set 3	>= 110 m
			Inclined BH	
			Set 1	Not possible
			Set 2	>= 260 m
			Set 3	>= 280 m
ORIENTATION Dispersion **SR1**	Deviation < = +/– 15% of true value (2)	>= 90%	Vertical BH	
			Set 1	>= 1100 m
			Set 2	>= 250 m
			Set 3	>= 100 m
			Inclined BH	
			Set 1	>= 750 m
			Set 2	>= 170 m
			Set 3	>= 250 m
ORIENTATION Dispersion **Kappa**	Deviation < = +/– 15% of true value (2)	>= 90%	Vertical BH	
			Set 1	>= 500 m
			Set 2	>= 420 m
			Set 3	>= 200 m
			Inclined BH	
			Set 1	>= 420 m
			Set 2	>= 360 m
			Set 3	>= 500 m

(1). Results and conclusions given in this study are only directly applicable to the fracture network studied, see Section 2.3.

(2). Samples are accepted if the deviation from the true value is within a range of plus or minus 15% of the True value, considering a range centred on the true value, i.e. within: 0.85*TV–1.15*TV (TV=TrueValue)

Figure 9-1. *CONCLUSIONS: FRACTURE ORIENTATION FROM BOREHOLES. (Presuming that trend and plunge is measured without any errors and that Set ID is known for all fractures studied.)*

<table>
<tr><th>PARAMETER</th><th>CRITERION
(Confidence interval)</th><th>PROBABILITY
(Confidence level)</th><th>SAMPLE TYPE
FRACTURE SET</th><th>SIZE (1)
(Sample size)</th></tr>
<tr><td rowspan="2">***P10***
Fracture frequency
[fractures/metre]
Analyses of boreholes</td><td rowspan="2">Deviation
< = +/–15%
of true value
(2)</td><td rowspan="2">>= 90%</td><td>Vertical BH
Set 1
Set 2
Set 3</td><td>BH length
>= 400 m
>= 300 m
>= 150 m</td></tr>
<tr><td>Inclined BH
Set 1
Set 2
Set 3</td><td>BH length
>= 350 m
>= 150 m
>= 210 m</td></tr>
<tr><td rowspan="2">***P21***
[Trace-length per surface area]
Analysis of circular horizontal surfaces</td><td>Deviation
< = +/–15%
of true value
(2)</td><td>>= 90%</td><td>Horizontal circular Surface
Set 1
Set 2
Set 3</td><td>Radius of surface
>= 24 m
>= 22 m
>= 40 m</td></tr>
<tr><td>Deviation
< = +/–10%
of true value
(3)</td><td>>= 90%</td><td>Horizontal circular Surface
Set 1
Set 2
Set 3</td><td>Radius of surface
>= 37 m
>= 32 m
>= 60 m</td></tr>
<tr><td>***P32***
[fracture surface area per volume]
INDIRECT
Analysis of surfaces or boreholes</td><td colspan="4">By use of a trial and error procedure in a DFN-model, the *P32* parameter can be Estimated based on the *P10* or the *P21* values. Therefore the results are the same as for the *P10* and *P21* parameters, with the addition of convergence deviations of the trial and error procedure.
$\text{Deviation}_{P32} = \text{Deviation}_{P21}$
$\text{Deviation}_{P32} = \text{Deviation}_{P10}$</td></tr>
<tr><td rowspan="2">***P32***
[fracture surface area per volume]
DIRECT
From boreholes. Considering fracture area inside borehole and borehole volume.</td><td rowspan="2">Deviation
< = +/–15%
of true value
(2)</td><td rowspan="2">>= 90%</td><td>Vertical BH
Set 1
Set 2
Set 3</td><td>BH length
>= 850 m
>= 650 m
>= 150 m</td></tr>
<tr><td>Inclined BH
Set 1
Set 2
Set 3</td><td>BH length
>= 480 m
>= 320 m
>= 380 m</td></tr>
<tr><td colspan="5">(1). Results and conclusions given in this study are only directly applicable to the fracture network studied, see Section 2.3.
(2). Samples are accepted if the deviation from the true value is within a range of plus or minus 10% or 15% of the true value (TV), considering a range centred on the true value</td></tr>
</table>

Figure 9-2. *CONCLUSIONS: FRACTURE DENSITY.*

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	SAMPLE TYPE FRACTURE SET	SIZE (1) *(Sample size)*
MOMENTS OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface.	Deviation < = + /– 15% of true value (2)	>= 90%	**MEAN** Set 1 Set 2 Set 3	Radius of surface >= 32 m >= 45 m >= 52 m
			STANDARD DEVIATION Set 1 Set 2 Set 3	Radius of surface >= 52 m >= 12 m >= 70 m
MOMENTS OF LOG-NORMAL DIST. FITTED TO THE SAMPLE DISTRIBUTION Analysis of circular horizontal surface. All Sets together.	Deviation < = + /– 15% of true value (2)	>= 90%	**MEAN** 10Log(Trace L.) Set 1 Set 2 Set 3	Radius of surface >= 25 m Not well represented >= 45 m
			STANDARD DEVIATION 10Log(Trace L.) Set 1 Set 2 Set 3	Radius of surface >= 42 m Not well represented >= 65 m
SHAPE OF SAMPLE DISTRIBUTION Analysis of circular horizontal surface. All Sets together.	Chi-square test “goodness-of-fit” Confidence level 99% (3)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 13 m >= 38 m >= 32 m

(1). Results and conclusions given in this study are only directly applicable to the fracture network studied, see Section 2.3.
(2). Samples are accepted if the deviation from the true value is within a range of plus or minus 15% of the true value, considering a range centred on the true value, i.e. within: 0.85*TV– 1.15*TV (TV=TrueValue)
(3). Samples are accepted based on the result of a Chi-square goodness-of-fit test, which compares the shape of the sample distribution to the shape of the true distribution. The confidence level of the test was set to 99%

Figure 9-3. *CONCLUSIONS: FRACTURE TRACE-LENGTH DISTRIBUTION.*

PARAMETER	CRITERION *(Confidence interval)*	PROBABILITY *(Confidence level)*	FRACTURE SET	RADIUS OF SURFACE *(Sample size)*
MEAN OF SAMPLE STRIKE DISTRIBUTION Analysis of circular horizontal surface.	Deviation < = 15 deg (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 35 m >= 18 m >= 60 m
	Deviation < = 10 deg (1)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 50 m >= 27 m >= 68 m
SHAPE OF SAMPLE STRIKE DISTRIBUTION Analysis of circular horizontal surface.	Chi-square test "goodness-of-fit" Confidence level 99% (2)	>= 90%	Set 1 Set 2 Set 3	Radius of surface >= 13 m >= 11 m >= 24 m
(1)Samples are within a range of plus or minus 15 or 10 degrees of the true value, considering a range centred on the true value. (2) Samples are accepted based on the result of a Chi-square goodness-of-fit test, which compares the shape of the sample distribution to the shape of the true distribution. The confidence level of the test was set to 99%				

Figure 9-4. *CONCLUSIONS: FRACTURE TRACE STRIKE DISTRIBUTION.*

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The eigenvector and resultant vector methods for calculation of mean direction of a group of fractures

1.1 Introduction

The mean orientation of a sample of fractures (or a cluster of fracture poles) and the dispersion of this sample (cluster) can be evaluated with different methods. Two of these methods are presented below. They are the eigenvector method and the resultant vector method.

1.2 Eigenvectors and eigenvalues, the eigenvalues method

For a given sample of fractures, every fracture *i* can be characterised by its normal vector $\vec{n}_i$. This vector is defined by three co-ordinates so that $[n_i] = \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \end{bmatrix}$. The length of the vector $\vec{n}_i$ is set by definition to one. In the following, fracture poles, also referred to as normal vectors, will simply be called vectors.

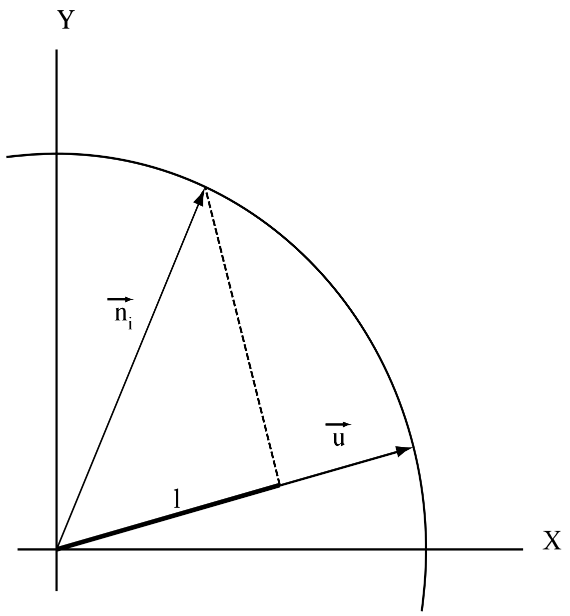

Figure A-1. *Two-dimensional illustration of the projection of vector $\vec{n}_i$ on vector $\vec{u}$. The length of projection of the vector n on u is l. It is defined as a positive scalar.*

Let θ_i be the acute angle between the direction of a exploration borehole and a normal to a fracture plane *i* hit by the exploration borehole. According to the Terzaghi correction theory /Terzaghi, 1965/, the fracture is assigned a weight equal to:

$w_i = 1/\cos\theta_i \qquad \theta < 90$ *deg.*

When θ approaches 90 deg. *w* becomes very large, to the extent that a single data point could dominate the distribution of orientaion values, to avoid this a maximum weighting value is introduced, e.g. $w_{maximum} = 7$.

This means that: $w_i = \min(1/\cos\theta_i, 7)$.

A matrix T is defined as follow:

$$T = \sum_{i=1}^{N} w_i \cdot \left(\vec{n_i} \times \vec{n_i}^T \right)$$

Note that T is a symmetric matrix.

The length of projection of the vector $\vec{n_i}$ on $\vec{u}$ is l_i (see Figure A-1). It is defined as a positive scalar. Let us define a scalar called M as:

$$M = \sum_{i=1}^{N} l_i^2$$

For an arbitrary vector $\vec{u}$ with co-ordinates [*u*], one has

$$M = [u]^T [T] [u]$$

Since we are looking for an unknown representative vector that reflects the orientation of a sample of fractures (or a fracture pole cluster*,) the representative vector is the vector for which the following condition applies. *The representative vector is the vector for which the sum of the projections of the fracture vectors is the largest*. It is also true for the sum of the squared value of the projections of the poles. In other words, M should take its largest value (maximum value) for the representative vector.

Linear algebra provides a solution to our problem, the maximum possible value of M is the largest eigenvalue λ_1 of the symmetric matrix T. The associated eigen vector $\vec{g}_1$ of T gives the orientation of the pole cluster. Hence:

$$\lambda_1 = [g_1]^T [T] [g_1]$$

Since the matrix T is symmetric, the eigen vectors define an orthogonal co-ordinate system. The eigen vectors $\vec{g}_2$ and $\vec{g}_3$ are thus located on a representative (average) plane of the fracture cluster. The ratio between the eigen values provides information on the degree of clustering (or dispersion) of the sample studied, and also on the shape of the cluster studied.

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* By cluster we mean a concentration of fracture poles on a spherical projection

The eigen values associated to eigen-vectors $\vec{g}_2$ and $\vec{g}_3$ are λ_2 and λ_3. The clustering or dispersion of the fractures is given by the ratios LN(λ_1/λ_2) and LN(λ_2/λ_3), see /Woodcock, 1977/.

1.3 Resultant vectors

For a given sample of fractures, every fracture *i* can be characterised by its normal vector $\vec{n}_i$. This vector is defined by three co-ordinates so that $[n_i] = \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \end{bmatrix}$.

The length of the vector $\vec{n}_i$ is set by definition to one. In the following, fracture poles, also referred to as normal-vectors, will simply be called vectors.

Let θ_i be the acute angle between the direction of a exploration borehole and a normal to a fracture plane *i* hit by the exploration borehole. According to the Terzaghi correction theory, the fracture is assigned a weight equal to:

$w_i = 1/\cos\theta_i \qquad \theta < 90$ *deg.*

When θ approaches 90 deg. *w* becomes very large, to the extent that a single data point could dominate the distribution of orientaion values, to avoid this a maximum weighting value is introduced, e.g. $w_{maximum} = 7$.

This means that: $w_i = \min(1/\cos\theta_i, 7)$.

The resultant vector $\vec{V}$ of a group of fractures (a sample) is defined as the average orientation-vector of the cluster, hence:

$$\vec{V} = \frac{1}{\sum_{i=1}^{N} w_i} \sum_{i=1}^{N} w_i \vec{n}_i$$

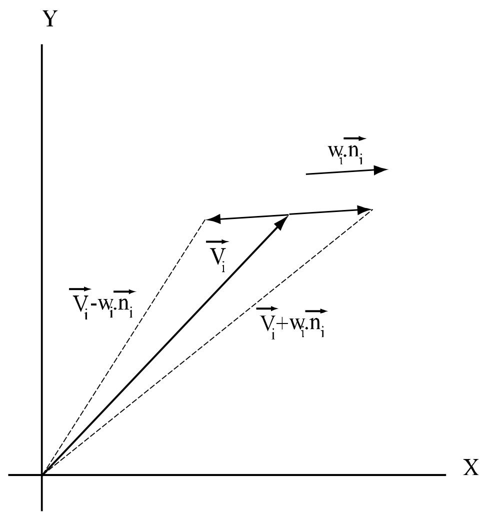


Figure A-2. *Aspects of the resultant vector method. When adding a fracture-normal to a resultant vector, two directions are possible; this is illustrated by the figure. Only one direction is correct. Hence, it is important to check that the selected direction is the correct one.*

This definition is not rigorous. In fact, the normal vector $\vec{n_i}$ can point in two opposite directions since a fracture has two faces. When calculating the resultant vector $\vec{V}$, one should be cautious and check that: *all vectors ($\vec{n_i}$) point towards the same general direction i.e. all directions are inside the same half sphere*. Otherwise, the resultant vector will not reflect the overall orientation of the fracture group (see Figure A-2). The condition that all vectors should point towards the same half sphere can be achieved by different techniques as part of iterative numerical algorithms. However, the best approach is to first apply the eigenvalues method on the sample studied, for deriving a good estimate of the mean direction, i.e. deriving a representative vector. As a second step the resultant vector method is applied on the sample, when the resultant vector method is applied it is checked that all normal-vectors of the sample point towards a half sphere centred about the representative vector. If the resultant vector method is constrained by the results of the eigenvalues method, it will produce correct results. The condition that all normal-vectors should point in the same general direction (same half sphere) limits the theoretical dispersion of a studied sample. This limitation is however correct, because fracture-normals are axes and not true vectors, and a distribution of axes (fracture-normals) can only occur in one half sphere (this is discussed in Chap.2).

The length the resultant vector $\vec{V}$ reflects the dispersion of the sample (or group of fractures, or cluster of fracture poles). For a given number of fractures, the longer the resultant vector $\vec{V}$, the smaller the dispersion of the cluster (and the larger the degree of clustering). The resultant vector method will not provide information on the circularity of a fracture cluster. Assuming that the cluster of concern is “circular” and that the orientation of the fractures follows a Fisher distribution /see Fisher, 1953/, it is possible to calculate the dispersion by use of the Fisher kappa parameter. /Fisher, 1953/ showed that an estimate *k* of the population kappa κ can be found from a sample of *M* unit vectors, for which the resultant vector is $|r_n|$.

This estimate is expressed by:

$$\frac{e^k + e^{-k}}{e^k - e^{-k}} - \frac{1}{k} = \frac{|r_n|}{M}$$

For sufficiently large values of k, approximately $k > 5$, the variable e^{-k} is negligible and the equation reduces to:

$$k \approx \frac{M}{M - |r_n|}$$

With the introduction of weighting factors, this equation can be expressed as:

$$k \approx \frac{\sum_{i=1}^{N} w_i}{\sum_{i=1}^{N} w_i - |\vec{V}|}$$

1.4 Relationship between SR1 and K for the Fisher distribution

1.4.1 Generation of a Fisher distributed variable θ

/Priest, 1993/ suggests a Fisher distributed random deviation θ from a reference vector $\vec{u}$ can be generated by:

$$\theta_\kappa = Arc\cos\left(\frac{\ln(1 - R_U)}{\kappa} + 1\right)$$

where R_U is uniformly random distributed value. $\mathbf{R_U} \in [0;1]$

For $\vec{u}$ defined as the reference vector (see Figure A-1),

$$M = \sum_{i=1}^{N} l_i^2$$

$$\therefore M = \sum_{i=1}^{N} \left(\cos\theta_{i,\kappa}\right)^2$$

We saw that the maximum possible value of M is the largest eigen-value λ_1 solution of our problem. The associated eigen-vector $\vec{g}_1$ gives the orientation of the pole cluster.

The Fisher distributed random deviation θ from vector $\vec{g}_1$ gives:

$$\lambda_1 = \sum_{i=1}^{N} \cos(\theta_{i,\kappa})^2$$
$$\therefore \lambda_1 = \sum_{i=1}^{N} \left(\frac{\ln(1-R_U)}{\kappa} + 1 \right)^2$$
$$\therefore \frac{\lambda_1}{N} = \frac{1}{\kappa^2} \frac{1}{N} \sum_{i=1}^{N} \left(\ln(1-R_U)\right)^2 + \frac{2}{\kappa} \frac{1}{N} \sum_{i=1}^{N} \ln(1-R_U) + 1$$

Let's define A and B such as:

$$A = \frac{1}{N} \sum_{i=1}^{N} \left(\ln(1-R_U)\right)^2$$
$$and$$
$$B = \frac{1}{N} \sum_{i=1}^{N} \ln(1-R_U)$$

Hence

$$\frac{\lambda_1}{N} = \frac{A}{\kappa^2} + \frac{2B}{\kappa} + 1 \qquad \text{Equ A.4.1}$$

1.4.2 Calculus of SR1 as a function of K.

Let's define the measure of the cluster dispersion SR1 as SR1= $\ln(\lambda_1/\lambda_2)$ /Woodcock, 1977/.

Assuming that the fracture orientations are Fisher distributed, i.e. the fracture cluster is circular. We have thus the relationship $\lambda_2=\lambda_3$.

Linear algebra claims that for a sample of N fractures, the trace of the matrix T=$\lambda_1+\lambda_2+\lambda_3=\lambda_1+2\lambda_2=N$

Hence the definition of SR1:

$$SR1 = \ln\left(\frac{2\lambda_1}{N-\lambda_1} \right)$$
$$\therefore SR1 = \ln\left(\frac{2\frac{\lambda_1}{N}}{1-\frac{\lambda_1}{N}} \right) \qquad \text{Equ. A.4.2}$$

Replacing Equ A.4.1 in Eq A.4.2 gives

$$SR1 = \ln\left(\frac{\frac{2A}{\kappa^2} + \frac{4B}{\kappa} + 2}{-\frac{A}{\kappa^2} - \frac{2B}{\kappa}}\right)$$

One has:

$$A \xrightarrow[N \to \infty]{} 2$$
$$B \xrightarrow[N \to \infty]{} -1$$

Hence for a sufficiently large N,

$$\mathbf{SR}1 = \ln\left(\frac{(\boldsymbol{\kappa} - 1)^2 + 1}{\boldsymbol{\kappa} - 1}\right); \boldsymbol{\kappa} \in [1; +\infty]$$

Note that this function is not defined for values of K<1.

This function is illustrated in the figure below (Figure A-3.).

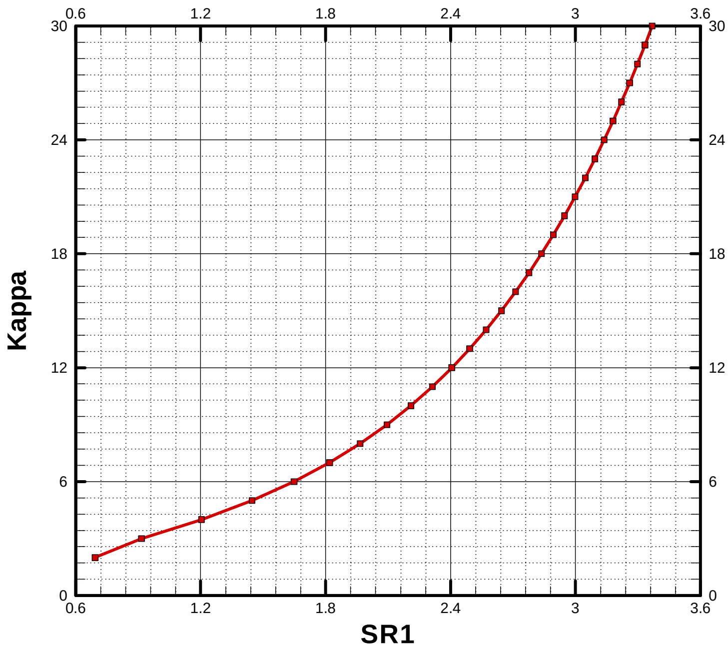


Figure A-3. *Relationship between the dispersion parameter K of a circular fracture cluster /Fisher, 1953/ and SR1 /Woodcock, 1977/.*

1.4.3 Calculus of K as a function of SR1.

The function SR1 above can be easily inverted and provide the dispersion parameter K as a function of SR1. This function is valid under the same assumptions as SR1=f(K), except the domain of validity. This inverse function is given by:

$$\mathbf{K} = \frac{1}{2}\mathbf{e}^{\mathbf{SR}1} + 1 + \sqrt{\frac{\mathbf{e}^{2\mathbf{SR}1}}{4} - 1} \text{ ; } \mathbf{SR}1 \in [\ln(2);+\infty]$$

It should be noted that the domain of validity represents eigen-values so that $\lambda_1 \geq 2 \cdot \lambda_2$.

SR1=ln(2) represents $\lambda_1 = 2 \cdot \lambda_2$. This gives a minimum value of K=2.

Relationship between SR1 and kappa considering a Fisher distribution

1.1 Terzaghi correction; methodology and examples

One-dimensional sampling is sampling along a straight line (a scanline). Such sampling of fracture orientation in a three-dimensional fracture system will introduce an orientation sampling bias. The bias follows from the fact that the probability for intersecting a fracture depends on the angle between the sampling line and the fracture, as well as on the area of the fracture. For a more thorough discussion of this we refer to /Terzaghi, 1965/ or /Priest, 1993/. The discussion below is based on /Priest, 1993/.

For compensation of this bias /Terzaghi, 1965/ proposed the application of a geometrical correction factor based on the observed angle between the sampling line and the normal to a particular fracture. In this study, such a correction is called "Terzaghi correction".

The acute angle between a normal to the fracture plane and the borehole is called "δ". The highest probability for intersection occurs when $\delta = 0$ deg. The lowest probability of intersection is zero; this occurs when $\delta = 90$ deg. Any direction of sampling line will therefore produce a sample that is biased to contain a lower amount of fractures than the actual amount. The reduced sample size at the higher values of δ can be compensated for by assigning a higer weighting to those fractures that are sampled. A fracture sampled by a sampling line is assigned a weighting W given by:

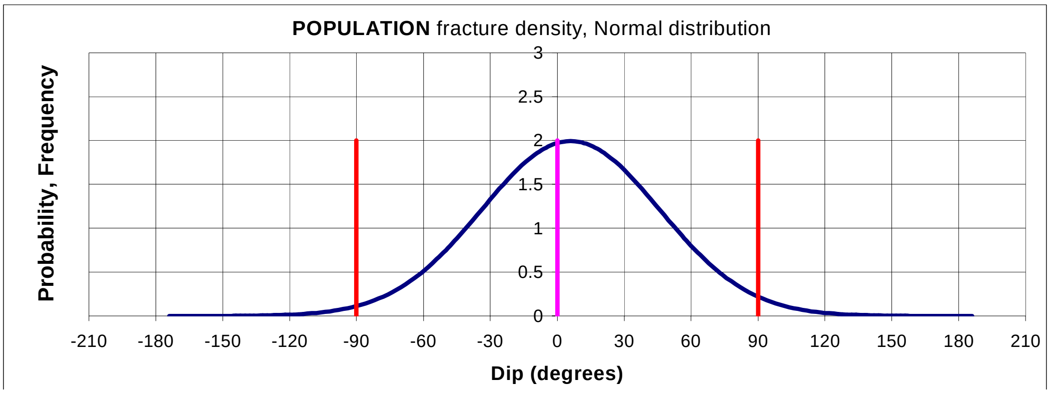
$$W = \frac{1}{\cos \delta} \quad \delta < 90 \text{ deg}.$$

For a large sample size this weighting will serve to balance the orientation sampling bias introduced by linear sampling. When δ approaches 90 deg. W becomes very large, to the extent that a single data point could dominate the distribution of orientaion values, to avoid this a maximum weighting value is introduced, e.g. $W_{maximum} = 7$.

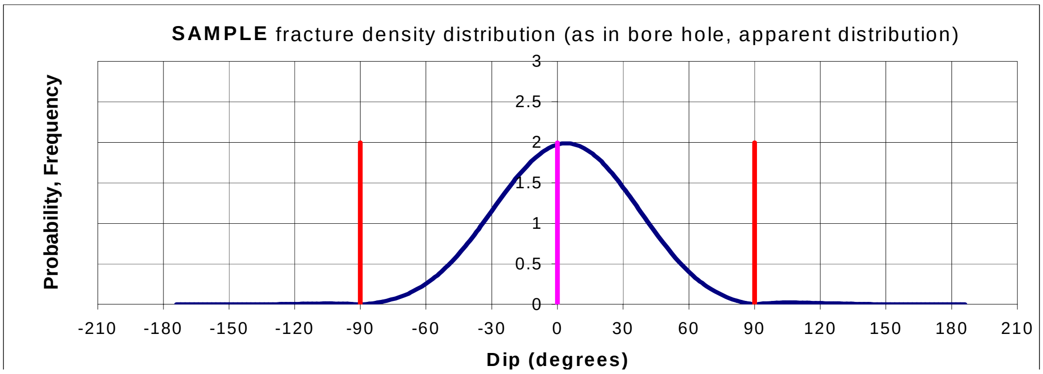
Below we will present three theoretical examples demonstrating the way the Terzaghi correction works (Figures B-1, B-2 and B-3). These three examples are based two-dimensional data, and demonstrate only the principle of the Terzaghi correction. For each example we will present: (i) the true distribution as it occurs in the rock mass, (ii) the apparent distribution as it occurs in a borehole and (iii) the corrected distribution as it will look after application of Terzaghi correction.

These figures (Figures B-1, B-2 and B-3) demonstrate that even after application of Terzaghi correction, due to the maximum weighting value of the correction, the distributions carries a minor distortion. For a vertical borehole sampling sub-vertical and sub-horizontal fracture sets, the distortion of the distributions are symmetric around the mean value of the distribution. Considering an inclined borehole (e.g. 45 deg), used for sampling sub-vertical and sub-horizontal fracture sets, the distortion of the distributions are not symmetric around the mean value of the distributions.

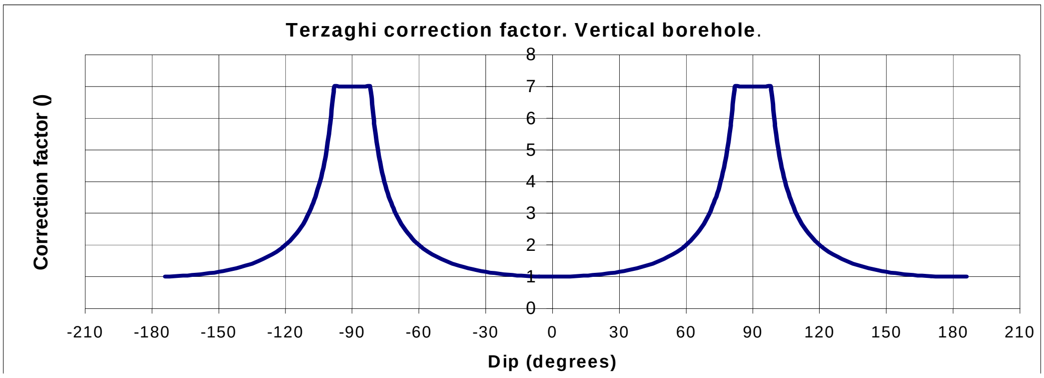
THEORETICAL EXAMPLE:
SUB-HORIZONTAL FRACTURE SET AND VERTICAL BOREHOLE.



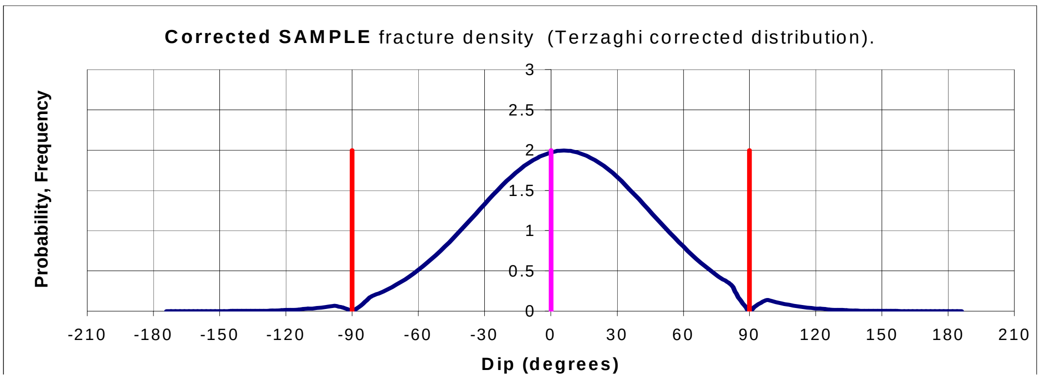
DIP: Normal distribution, Mean 6 deg. STD= 40 deg, Number of fractures: 100%



DIP: Mean 3.7 deg. STD= 31.6 deg, Number of fractures: 79.2%



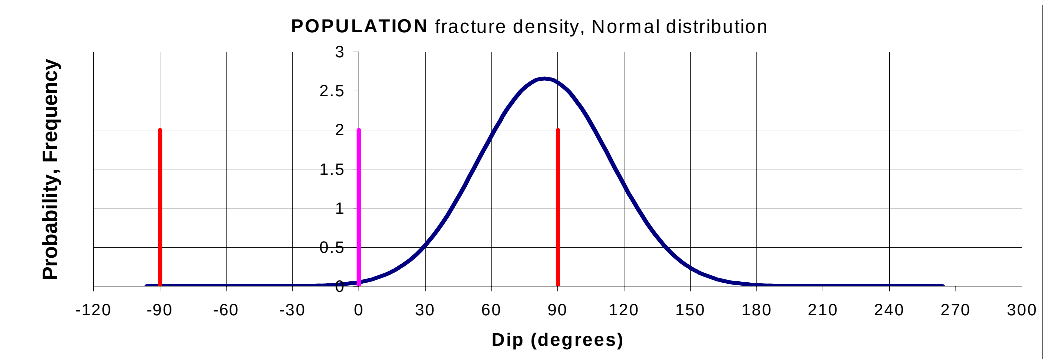
Terzaghi correction, weight factor, max weight= 7.0



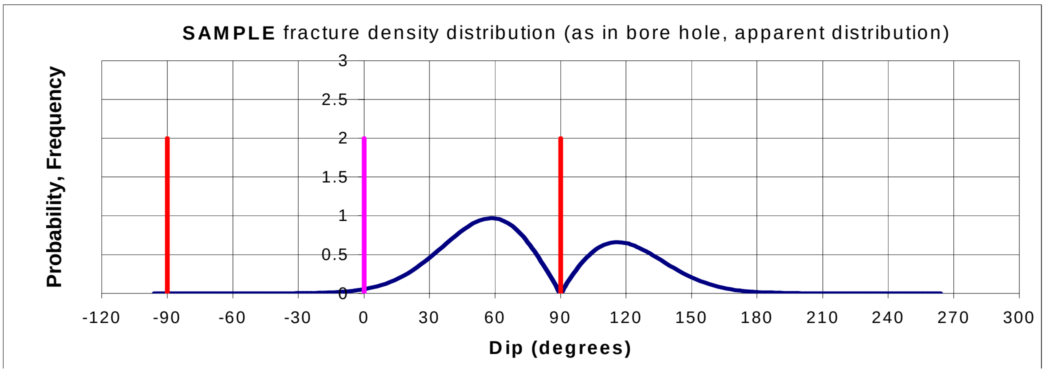
DIP: Mean 5.7 deg. STD= 38.9 deg, Number of fractures: 98.6%

Figure B-1. *Theoretical example: sub-horizontal fracture set and vertical borehole.*

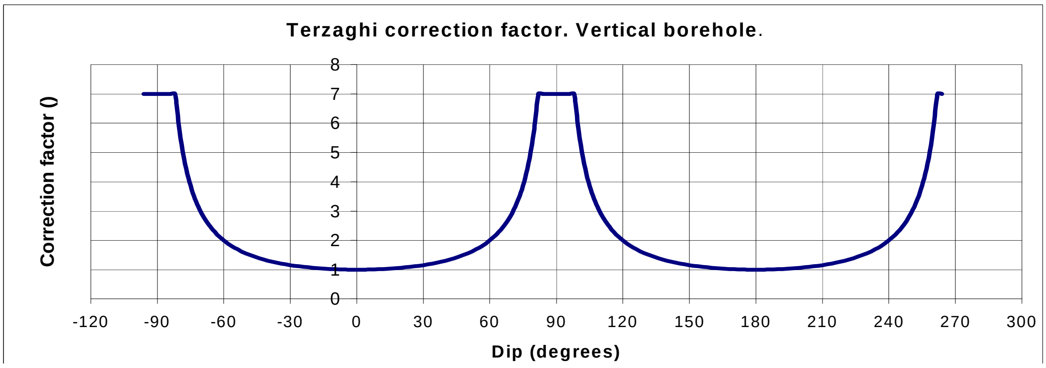
THEORETICAL EXAMPLE:
SUB-VERTICAL FRACTURE SET AND VERTICAL BOREHOLE.



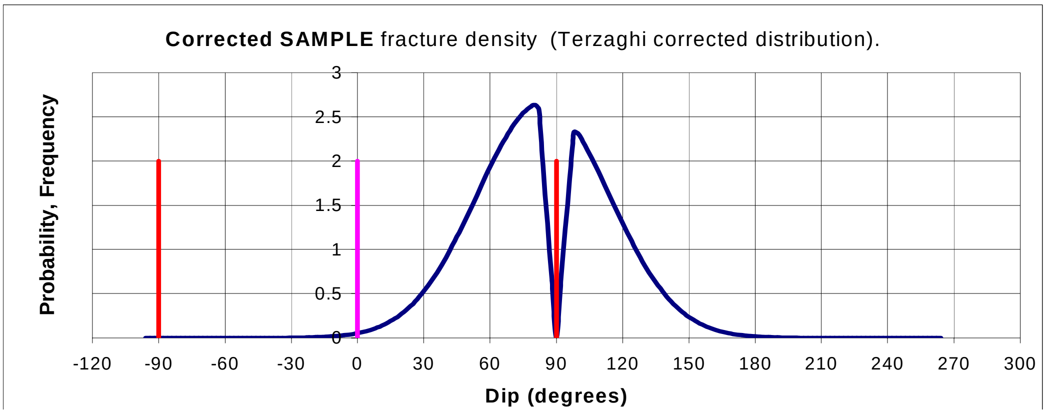
DIP: Normal distribution, Mean 84 deg. STD= 30 deg, Number of fractures: 100%



DIP: Mean 79.2 deg. STD= 39.8 deg, Number of fractures: 38.8%



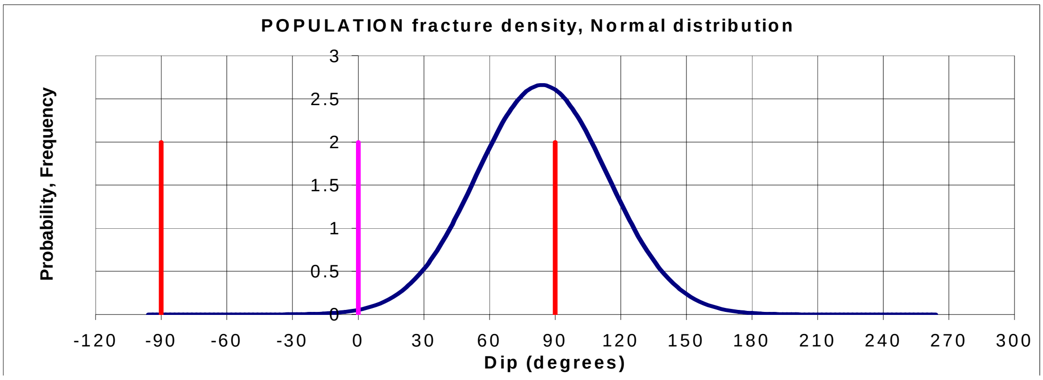
Terzaghi correction, weight factor, max weight= 7.0



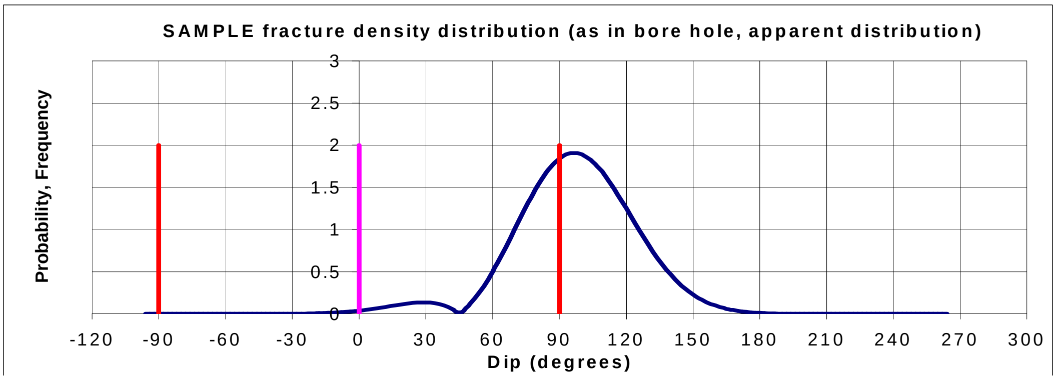
DIP: Mean 83.3 deg. STD= 31.6 deg, Number of fractures: 89.3%

Figure B-2. *Theoretical example: sub-vertical fracture set and vertical borehole.*

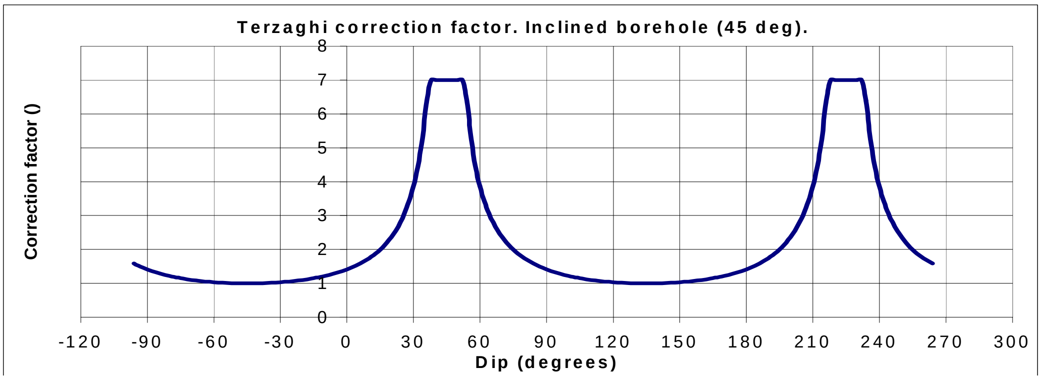
THEORETICAL EXAMPLE:
SUB-VERTICAL FRACTURE SET AND INCLINED BOREHOLE, 45 deg.



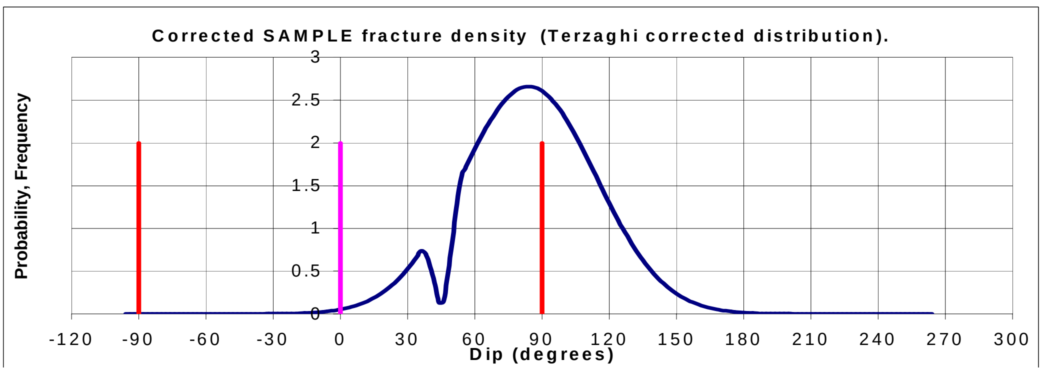
DIP: Normal distribution, Mean 84 deg. STD= 30 deg, Number of fractures: 100%



DIP: Mean 97.0 deg. STD= 27.5 deg, Number of fractures: 59.4%



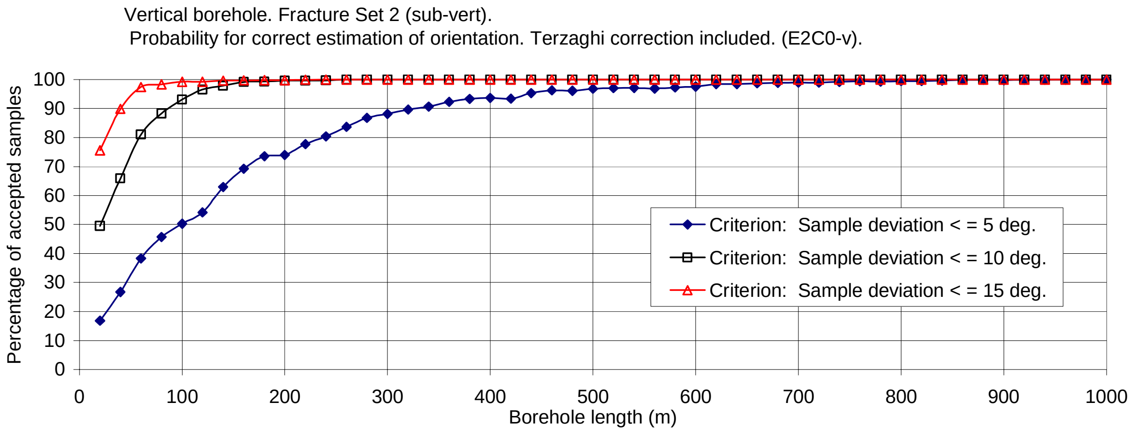
Terzaghi correction, weight factor, max weight= 7.0



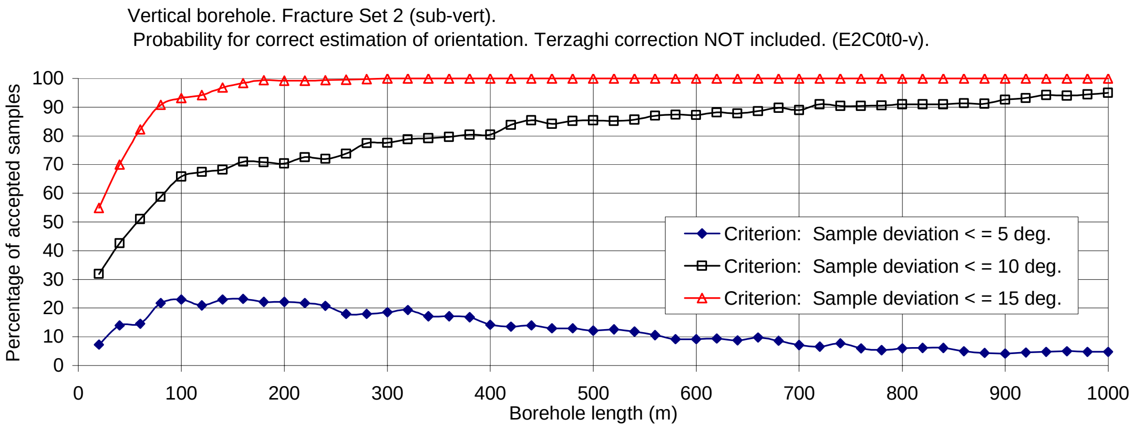
DIP: Mean 85.9 deg. STD= 29.4 deg, Number of fractures: 95.3%

Figure B-3. *Theoretical example: sub-vertical fracture set and inclined borehole (45 degrees).*

Below we will, by use of three examples, demonstrate the consequences of not including the Terzaghi correction in the calculation of mean direction and dispersion of a fracture set. The calculations are based on samples of fracture orientation as seen in a vertical borehole. These examples are based on the same DFN-network as the one presented in Chapter 2. (Tables 2.1 through 2.3). The fracture set studied is Set 2. The only difference compared to the results presented in Chapters 3 and 4, is that the estimates presented in those chapters were calculated with the inclusion of Terzaghi correction (maximum correction factor equal to 30), while the estimates presented below are calculated without Terzaghi correction.

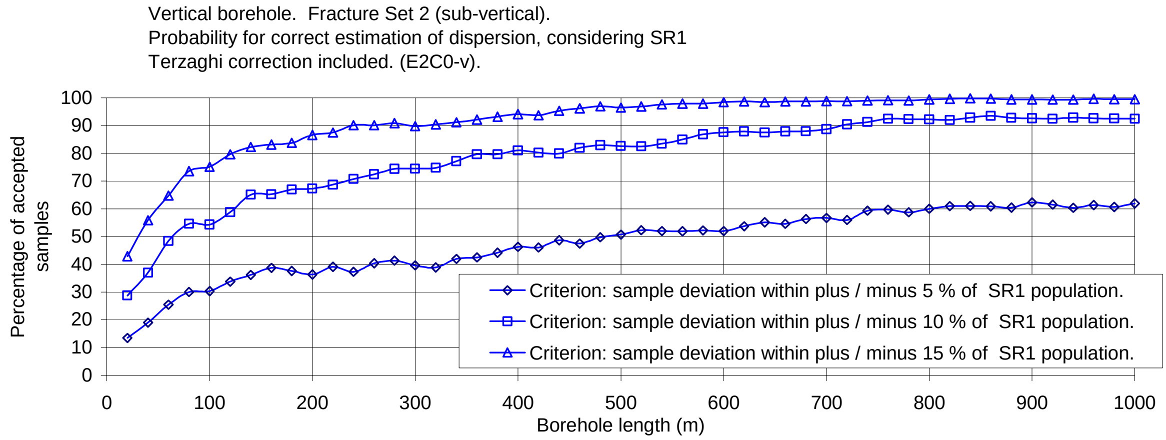


MEAN DIRECTION WITH TERZAGHI CORRECTION (AS IN CHAPTER 3).

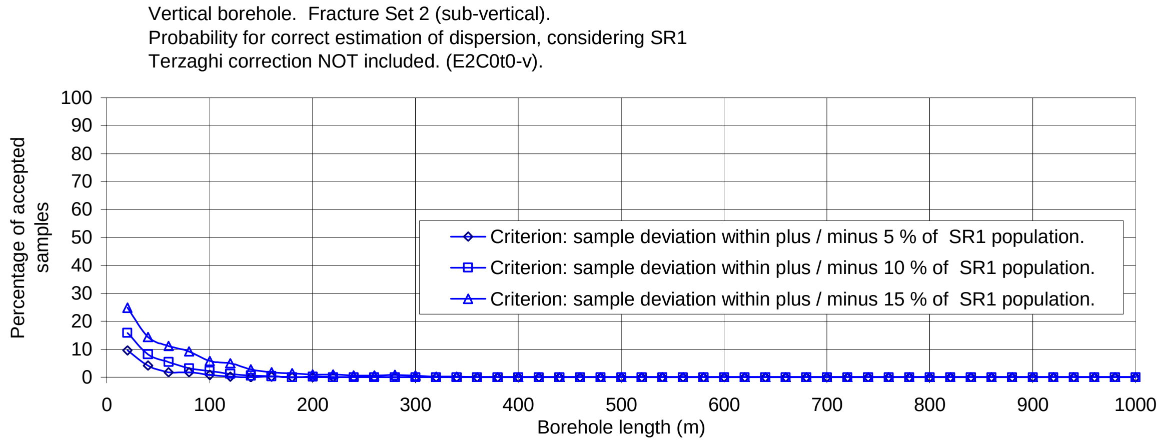


MEAN DIRECTION WITHOUT TERZAGHI CORRECTION.

Figure B-4. *Probability for correct estimation of fracture set orientation. A comparison between estimates of mean direction of a fracture set, calculated with and without Terzaghi correction. The calculations represent a sub-vertical fracture set (Set 2), investigated by use of a vertical borehole.*

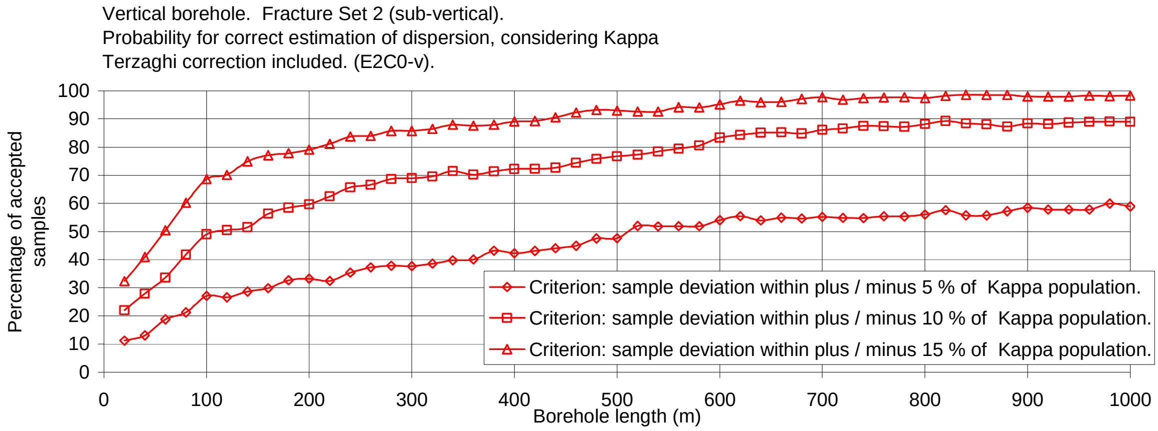


SR1 DISPERSION PARAMETER WITH TERZAGHI CORRECTION. (AS IN CHAPTER 4).

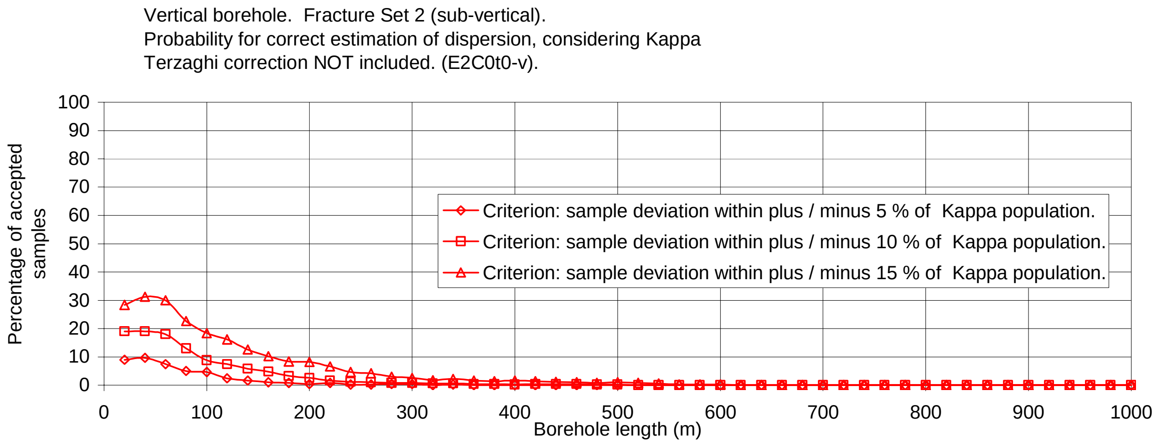


SR1 DISPERSION PARAMETER WITHOUT TERZAGHI CORRECTION.

***Figure B-5.** Probability for correct estimation of fracture set orientation. A comparison between estimates of the SR1 dispersion parameter of a fracture set, calculated with and without Terzaghi correction. The calculations represent a sub-vertical fracture set (Set 2), investigated by use of a vertical borehole.*



KAPPA DISPERSION PARAMETER WITH TERZAGHI CORRECTION (AS IN CHAPTER 4).



KAPPA DISPERSION PARAMETER WITHOUT TERZAGHI CORRECTION.

Figure B-6. *Probability for correct estimation of fracture set orientation. A comparison between estimates of the KAPPA dispersion parameter of a fracture set, calculated with and without Terzaghi correction. The calculations represent a sub-vertical fracture set (Set 2), investigated by use of a vertical borehole.*

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