Hydromechanical Model for Bentonite Based on X-ray Tomography Experiments

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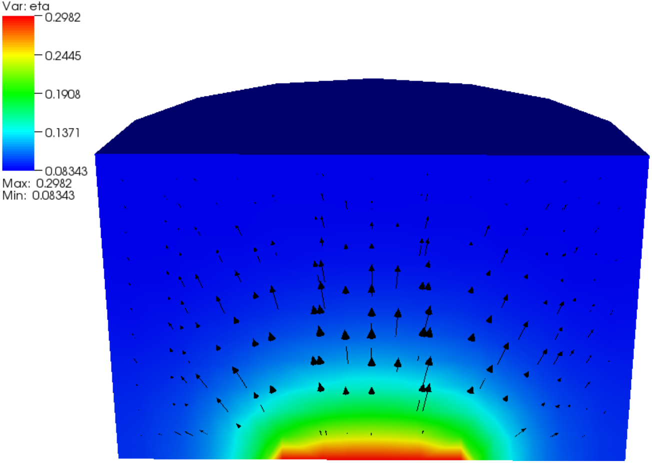
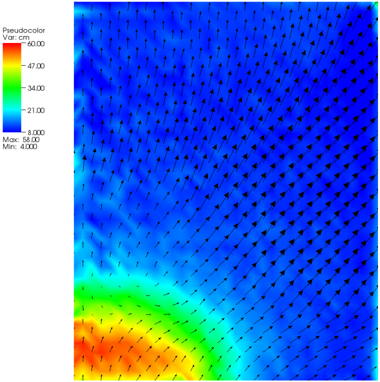
Goal: Validated hydromechanical 3D-solver

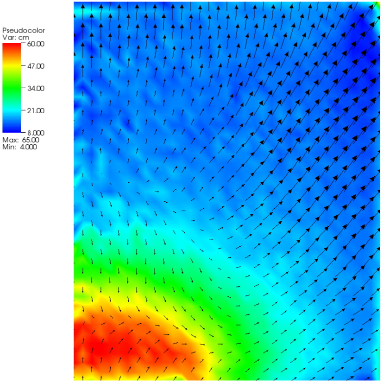


Numerola Oy (Ltd)

- Computational services for wind power developers and technology R & D
- Employs 9 experts in modelling, simulation and software development
- Located in Jyväskylä, Finland
- Participates in Finnish KYT2014 programme, subcontractor for VTT in BELBaR

Measured water content and deformation (JyU)





t = 0.29 days

t = 0.73 days

Modelling assumptions

- Bentonite is multiphase mixture of water and solid
- Mechanical equilibrium is reached instantly compared to time scales of water transport
- Bentonite is elastoplastic material
- Mechanical material properties depend on dry density, water content and possibly history of straining

Solution algorithm

For each time step:

1. Solve new water content
2. Update mechanical material properties depending on water content
3. Increment stress by pressure induced by the change in water content
4. Solve mechanical model
5. Update geometry

Hydrological model

$$\frac{\partial \rho_w}{\partial t} - \nabla \cdot (D_w \nabla \rho_w) = r$$

$$\frac{\partial (\phi \rho_v)}{\partial t} - \nabla \cdot (D_v \nabla \rho_v) = -r$$

with

ρ_w	liquid water density
ρ_v	water vapor density
t	time
ϕ	porosity
D_v, D_w	diffusion coefficients
r	sink/source due to gas–liquid phase change

Hypoelastic mechanical model (Markku Kataja)

$$\nabla \cdot \sigma = 0$$

$$D_t \sigma_{ij} = C^{ijkl}(D_{kl} - D^p_{kl})$$

with

σ	stress tensor
$D_t\sigma$	covariant time derivative of stress tensor
C^{ijkl}	elasticity coefficients
D_{kl}	rate of deformation tensor, computed from solid velocity
D^p_{kl}	rate of plastic deformation tensor

Yield law is needed to derive D^p

Yield law (modified von Mises)

$$f = \sigma_{\text{eff}} - \sigma_Y$$

with

f	yield function
σ_{eff}	effective (scalar) stress
σ_Y	yield stress (experimental)

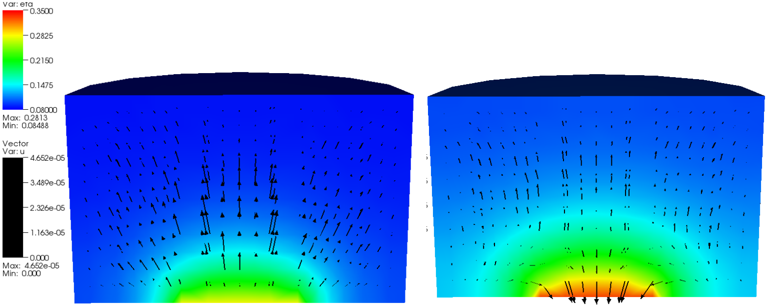
f defines D^p and determines deformation type:

deformation is elastic if $f < 0$
deformation is plastic if $f = 0$

Mechanical model algorithm (return mapping):

1. Solve 'trial' velocity by assuming only elastic deformations
2. For each point in geometry, check if trial velocity leads to plastic stresses: if $f > 0$, solve $f = 0$ to obtain plastic strain and stress
3. Solve final velocity with elastic and plastic deformations
4. Update geometry

Preliminary computed results:



t = 0.3 days   t = 0.6 days