EXPERIMENTS AND A PHENOMENOLOGICAL MODEL OF BENTONITE SWELLING IN A NARROW CHANNEL

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Objective:

To construct a hydromechanical model that

- includes finite deformations, plasticity and swelling
- is based on direct experimental data (rather than on micro-scale modelling)
- is experimentally validated

Applications:

- bentonite buffer behaviour
- erosion modelling

- The set of experiments needed to find the necessary material properties should be feasible and minimal.
- The model is first applied in one-dimensional swelling of purified bentonite in a narrow channel => Validation.
- Generalized to an arbitrary three-dimensional case => Comparison with wetting/swelling data from X-ray tomography.

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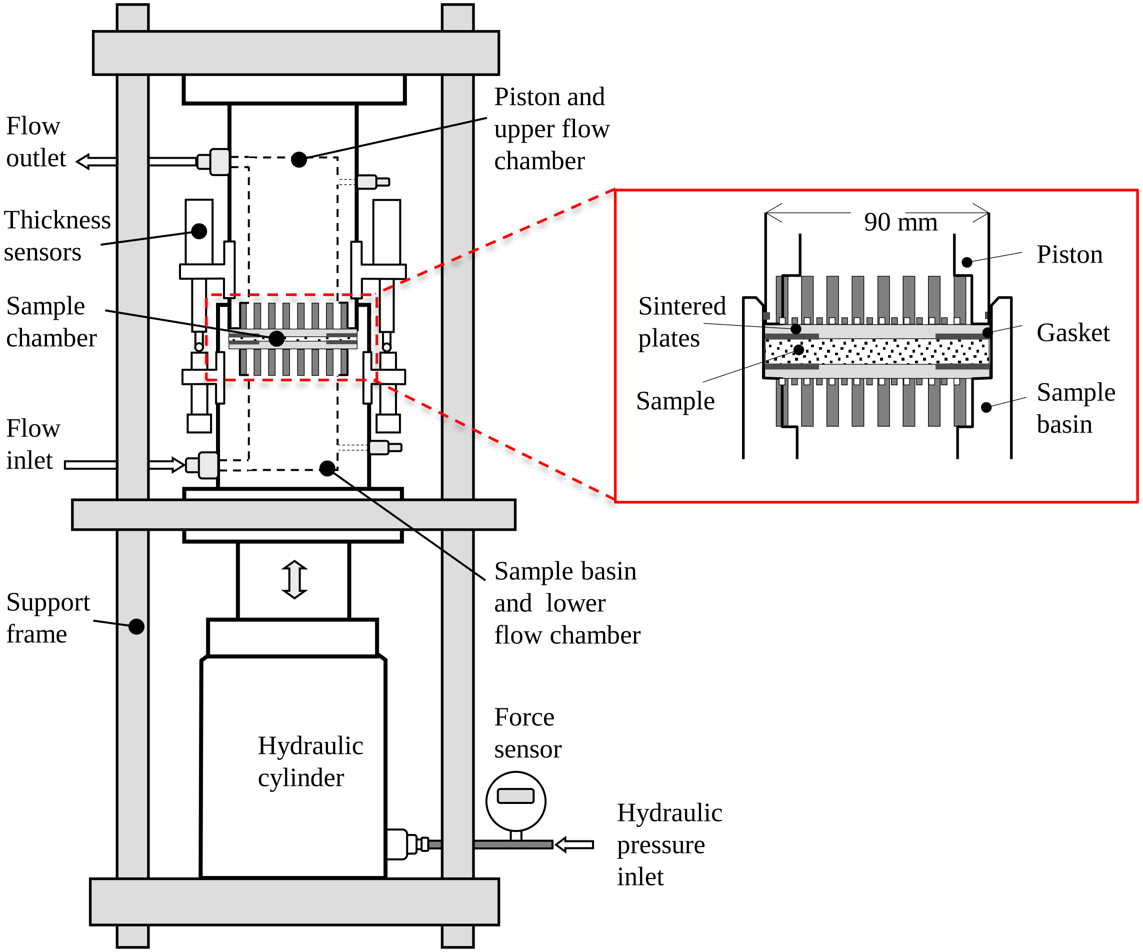
Material properties. 1D compression experiment

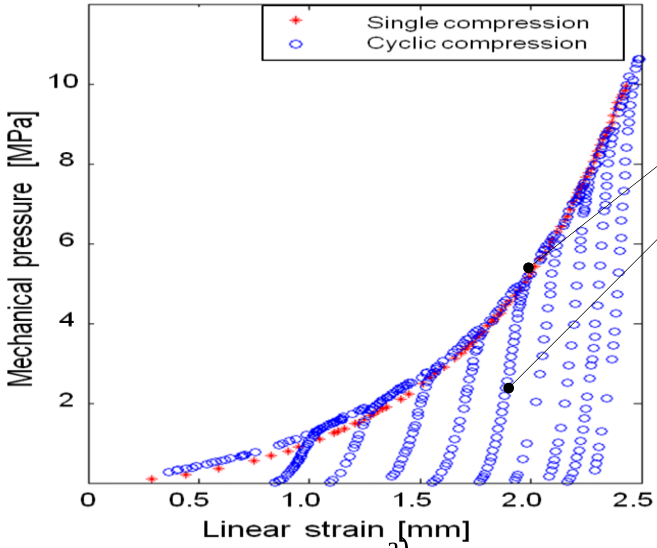
- Wet bentonite is considered as an elasto-plastic material with well defined elastic domain and yield behaviour

Uniaxial compression device



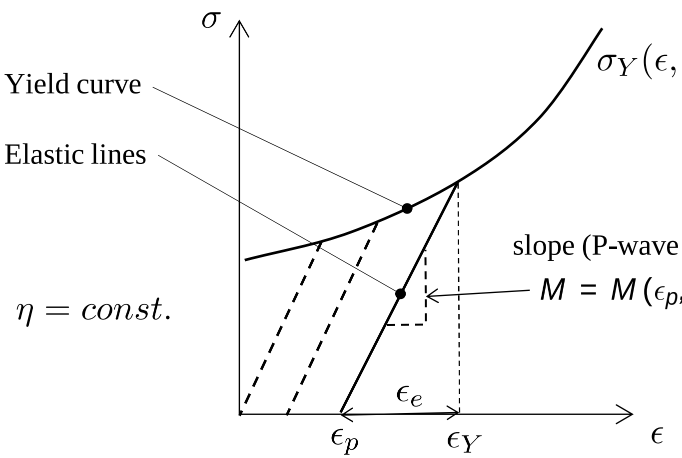
Result: elasto-plastic behaviour in uniaxial compression:

Exp. result (for a constant water content)



a)

Simplified behaviour in terms of effective strain



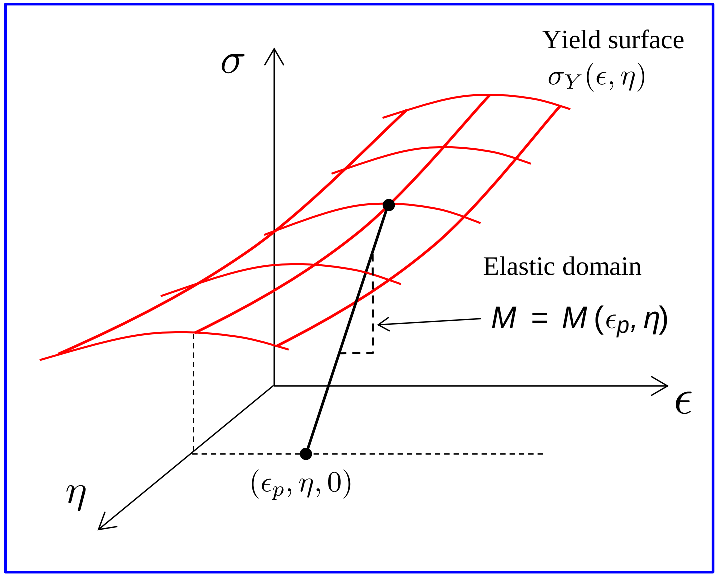
The experiments give

- Yield stress $\sigma_Y = \sigma_Y(\epsilon,\eta)$
- P –wave modulus $M = M(\epsilon_p,\eta)$
- Max. elastic strain $\epsilon_e = \epsilon_e(\epsilon_p,\eta)$

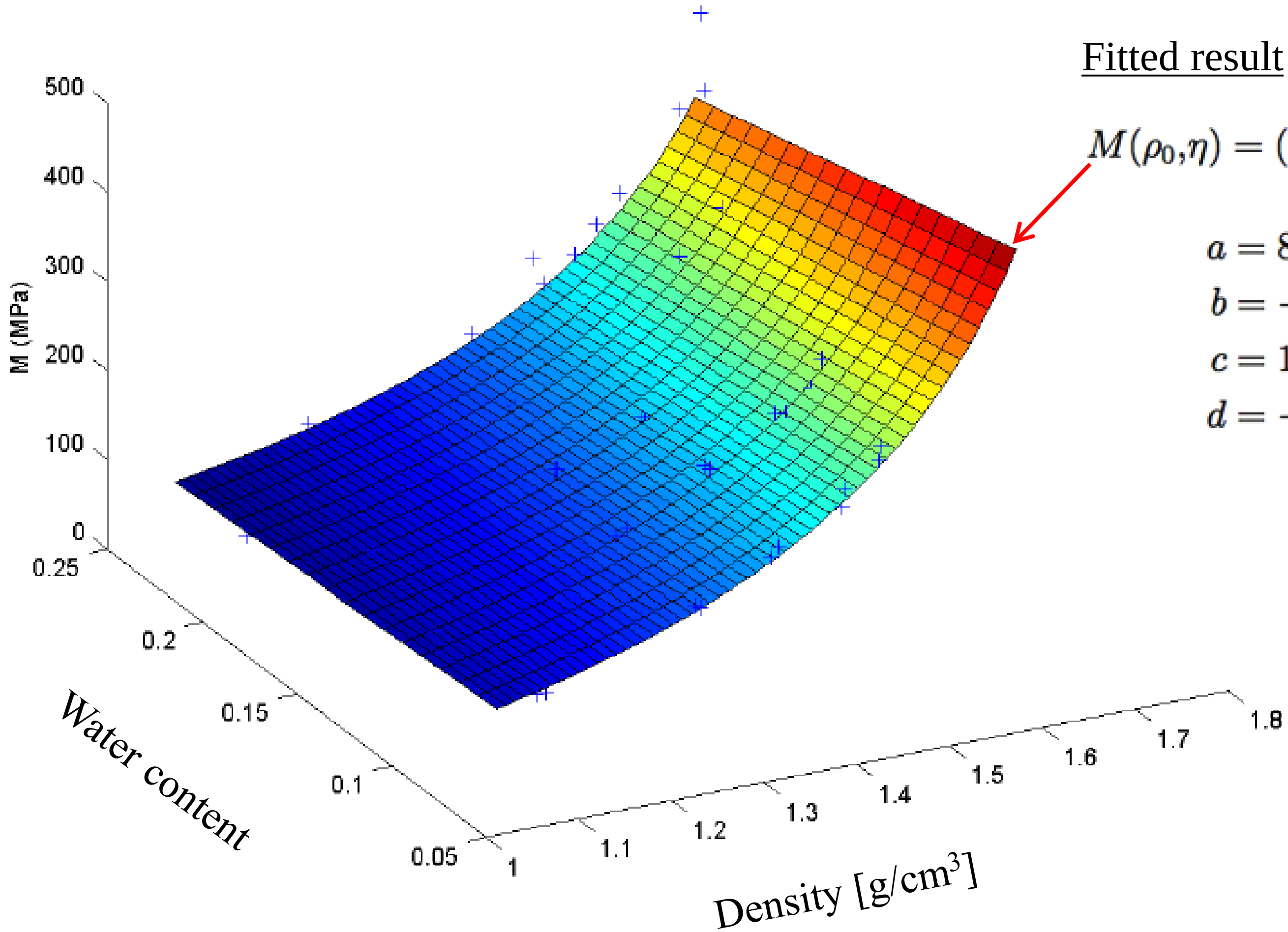
where $\epsilon = \Delta h/h_0$ = linear strain,

ϵ_p = plastic strain

η = water to solid mass ratio



P-wave modulus of purified bentonite as a function of density and water content (1D compression at constant moisture).



<u>Fitted result</u>

$$M(\rho_0,\eta) = (a + b\eta)\left(\frac{\rho_0}{c-\rho_0}\right) + d,$$

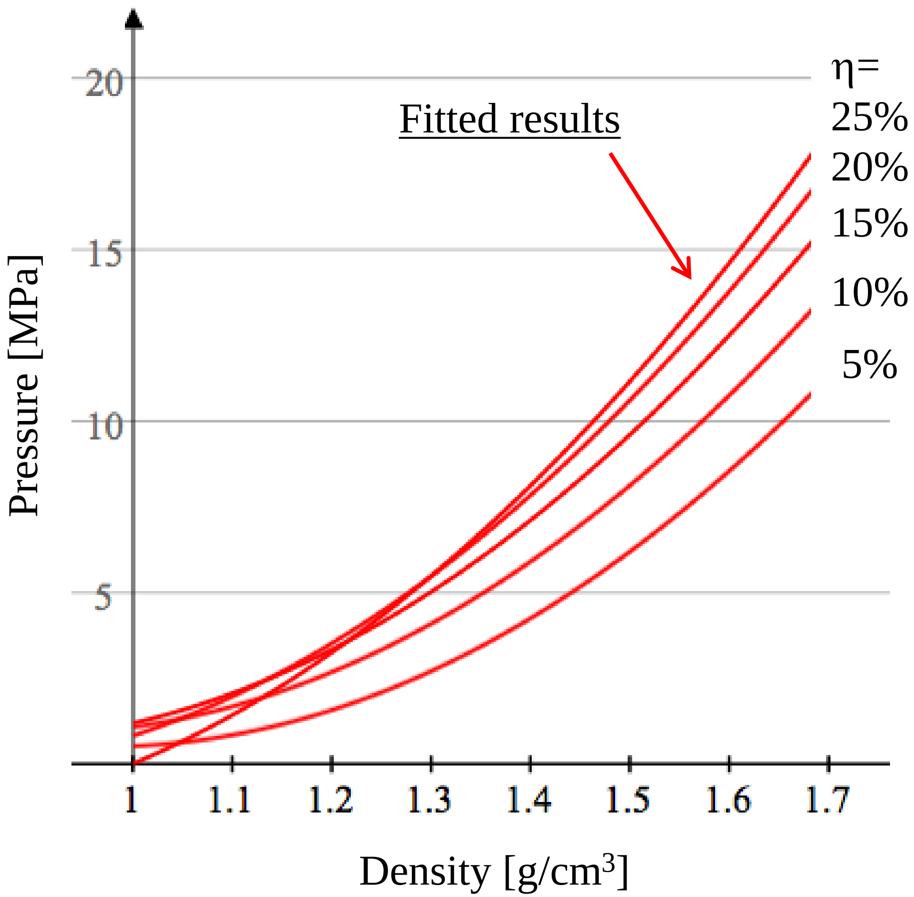
$a = 86.4381\,\mathrm{MPa}$

$b = -97.2583\,\mathrm{MPa}$

$c = 1.8779\,\mathrm{g\,cm^{-3}}$

$d = -12.3159\,\mathrm{MPa},$

Yield stress of purified bentonite as a function of density and water content (1D compression at constant moisture).

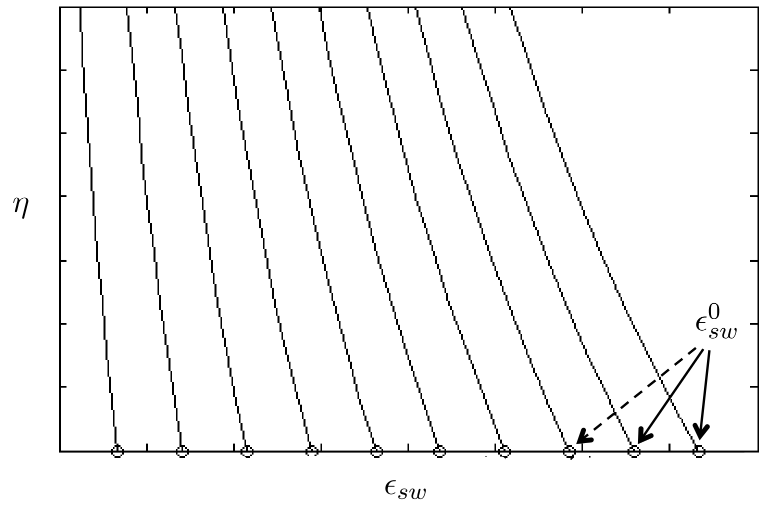


$$p_{\text{yield}}(\rho_0,\eta) = a_1 + a_2\rho_0 + a_3\eta + a_4\rho_0^2 + a_5\rho_0\eta + a_6\eta^2,$$

$$a_1 = 4.1510\,\text{MPa}$$
$$a_2 = -25.669\,\text{MPa}\cdot\text{cm}^3/\text{g}$$
$$a_3 = 84.7691\,\text{MPa}$$
$$a_4 = 20.2826\,\text{MPa}\cdot(\text{cm}^3/\text{g})^2$$
$$a_5 = -54.8058\,\text{MPa}\cdot\text{cm}^3/\text{g}$$
$$a_6 = -91.4177\,\text{MPa}.$$

Free swelling experiment/model (?)

Dry density vs. water content for various initial densities (expected qualitative behaviour)



Problem: How to ensure homogeneous wetting/swelling.

The experiment yields

- Free swelling strain $\epsilon_{sw} = \epsilon_{sw}(\epsilon_{sw}^0, \eta)$.

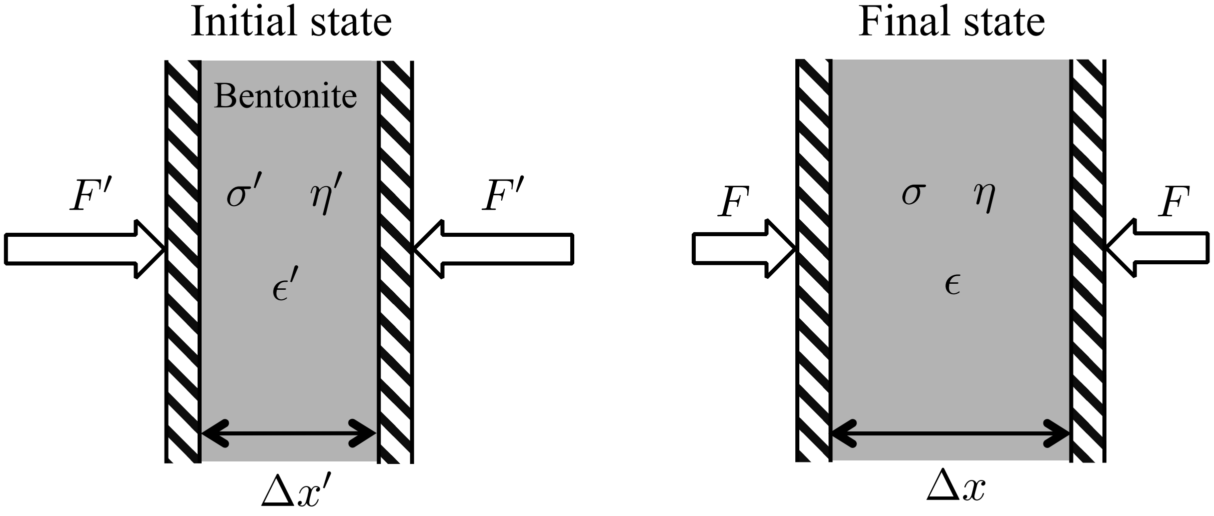
where ϵ_{sw}^0 = initial strain (given by the initial dry density) of the swelling experiment.

1D swelling model

Assumptions:

- The state of the material system is prescribed by three variables: strain, water content and stress: (ϵ, η, σ)
- The stress-strain relationship at a given water content η is independent of the path taken by the system to reach that value of water content

Consider a change of state of a material element from a known initial state $(\epsilon', \eta', \sigma')$ to a final state (ϵ, η, σ)

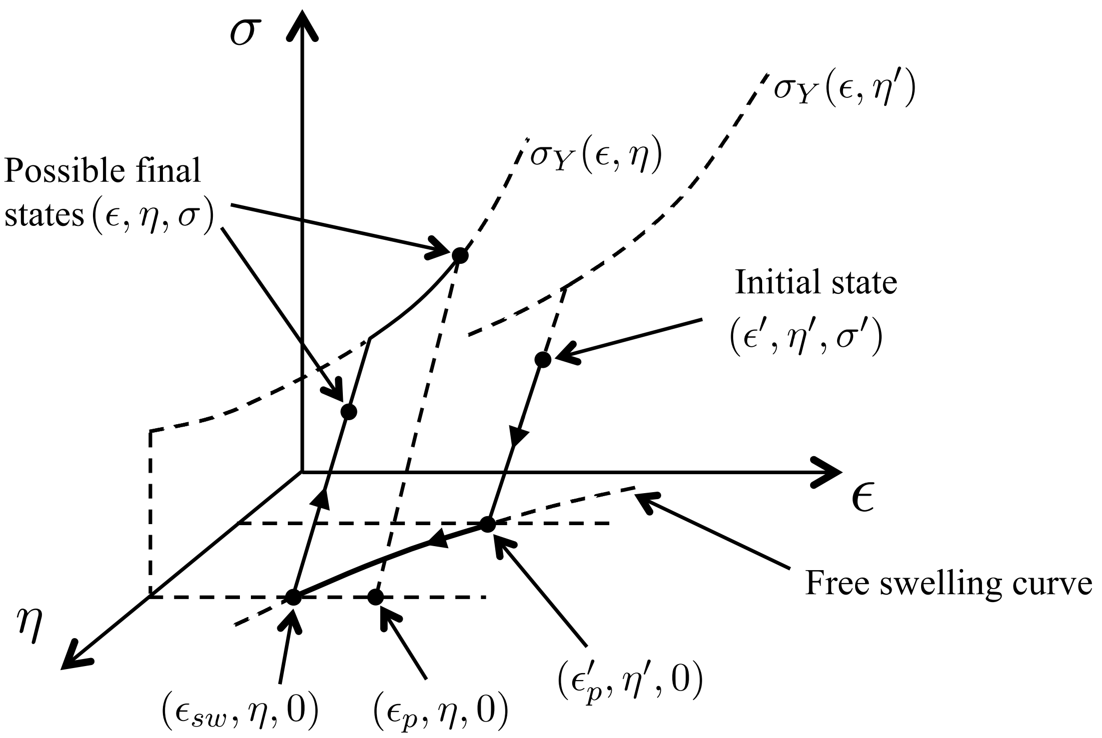


Assume that the new water content η and strain ϵ are known.
The task is to calculate the new stress value σ

Calcuation of stress

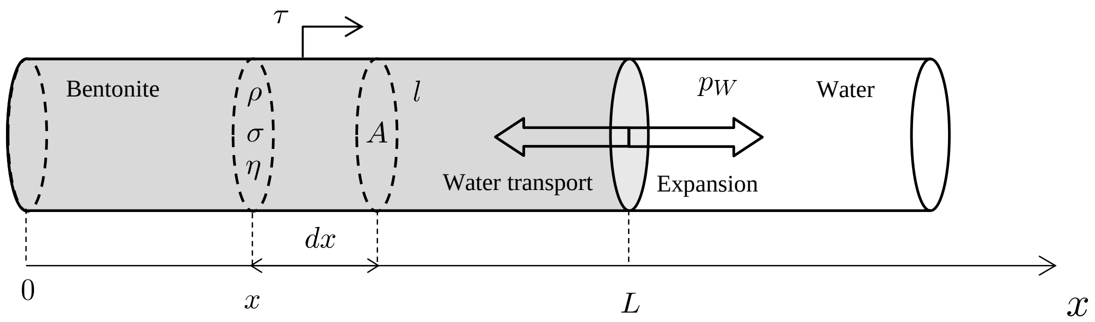
Assume that the process $(\epsilon', \eta', \sigma') \to (\epsilon, \eta, \sigma)$ can be divided in three steps:

- Elastic expansion to zero stress at constant water content η': $(\epsilon', \eta', \sigma') \to (\epsilon'_p, \eta', 0)$.
- Wetting and free swelling to new water content η: $(\epsilon'_p, \eta', 0) \to (\epsilon_{sw}, \eta, 0)$.
- Compression to final stress at constant water content η: $(\epsilon_{sw}, \eta, 0) \to (\epsilon, \eta, \sigma)$



Hydromechanical model of swelling in a narrow channel

Consider a long straight channel with cross-section *A* and circumference *l* partially filled with compacted bentonite and water at known pressure p_W .



Assumptions:

- Wet bentonite treated as locally homogeneous mixture of solid and water (gas phase taken into account implicitely through material properties of unsaturated bentonite)
- Bentonite is an elasto-plastic material (as specified earlier)
- Water transport mechanism in bentonite is assumed known (here, simple diffusion).
- The swelling process is assumed quasistationary such that
  - Time evolution determined by water transport
  - At each instant of time, bentonite is assumed to be in a mechanical equilibrium with respect to the internal stress and wall friction, at a given moisture distribution and boundary conditions.

1D model equations

1. Water transport (Here: diffusion in the material frame)

$$\frac{d}{dt}\rho_w(t,x) = \frac{d}{dx}\left(D\frac{d}{dx}\rho_w(t,x)\right) \Longrightarrow \eta(t,x) = \frac{\rho_w}{\rho} = \frac{\rho_w}{\rho_r}(1-\epsilon)$$

2. Deformation. A strain field found such that the mechanical balance equation

$$\frac{d}{dx}\sigma = \frac{l}{A}\tau$$

and the given boundary conditions are fulfilled.

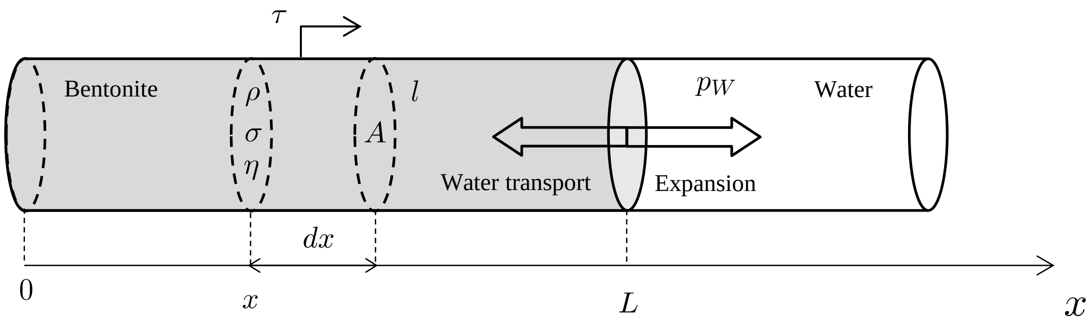
Here: $\tau = \pm\min\left\{\frac{A}{l}\left|\frac{d\sigma}{dx}\right|, \tau_c\right\}$ Wall friction

$\tau_c = \mu\sigma_\perp$ Critical wall friction

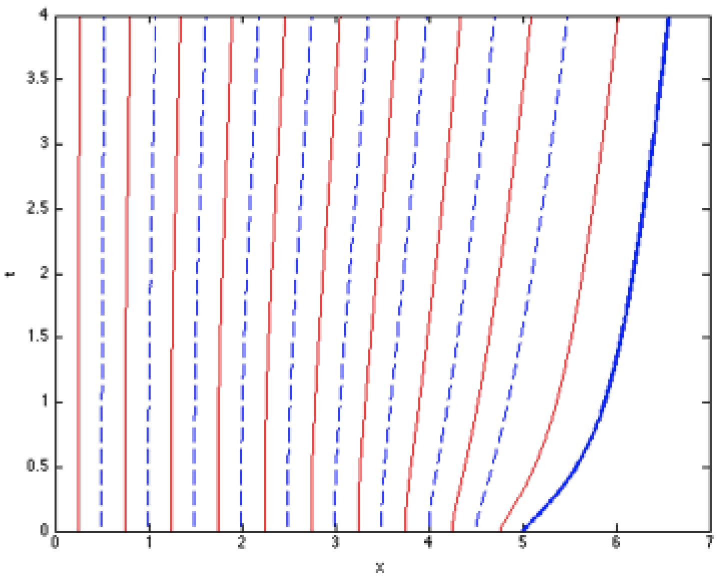
$\sigma_\perp = \frac{\nu}{1-\nu}\sigma$ Transverse stress

$\nu = \nu(\epsilon_p, \eta)$ Poisson ratio

Numerical solution



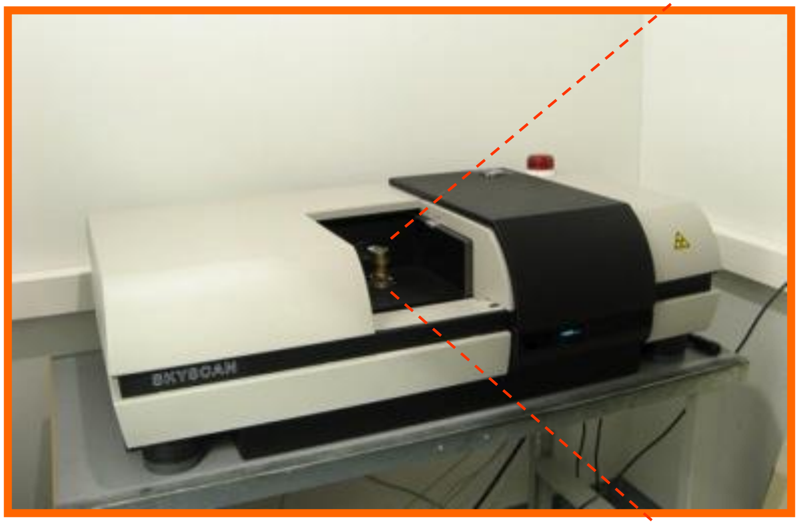
Solid phase pathlines in the channel
according to the 1D expansion model



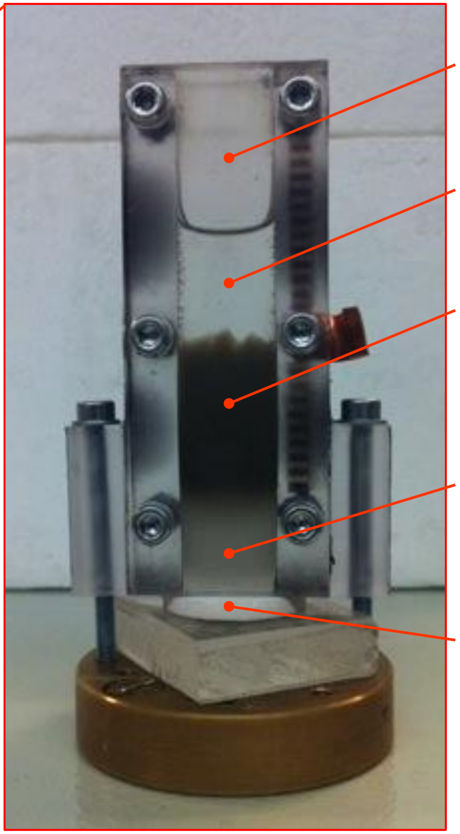
Experiments using X-ray imaging

X-ray tomographic scanner used in making an x-ray image movie of the wetting process

Scanner (SkyScan 1072)



Sample holder and sample after 24 h wetting



Acrylic square channel (13 mm x 5 mm)

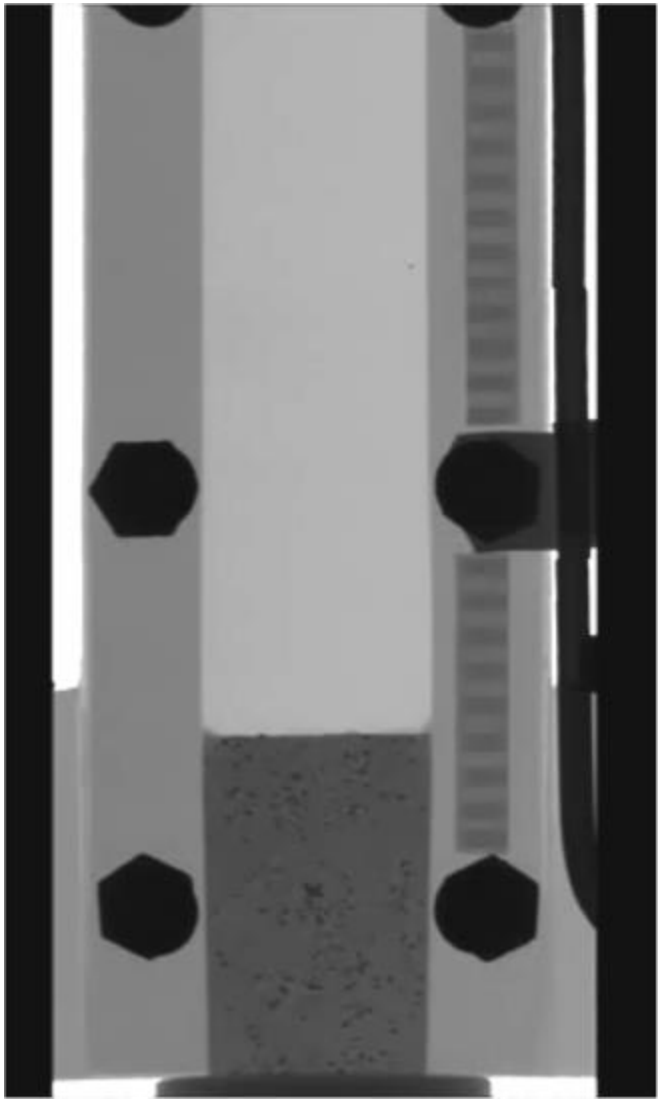
Water

Wetted zone

Dry bentonite

Sintered plate bottom

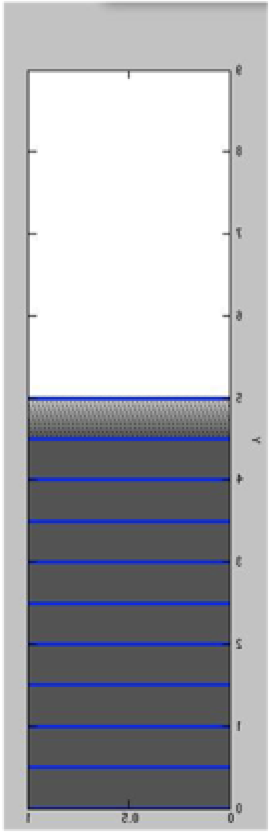
X-ray movie of swelling bentonite with metallic marker particles



Motion tracking by image correlation

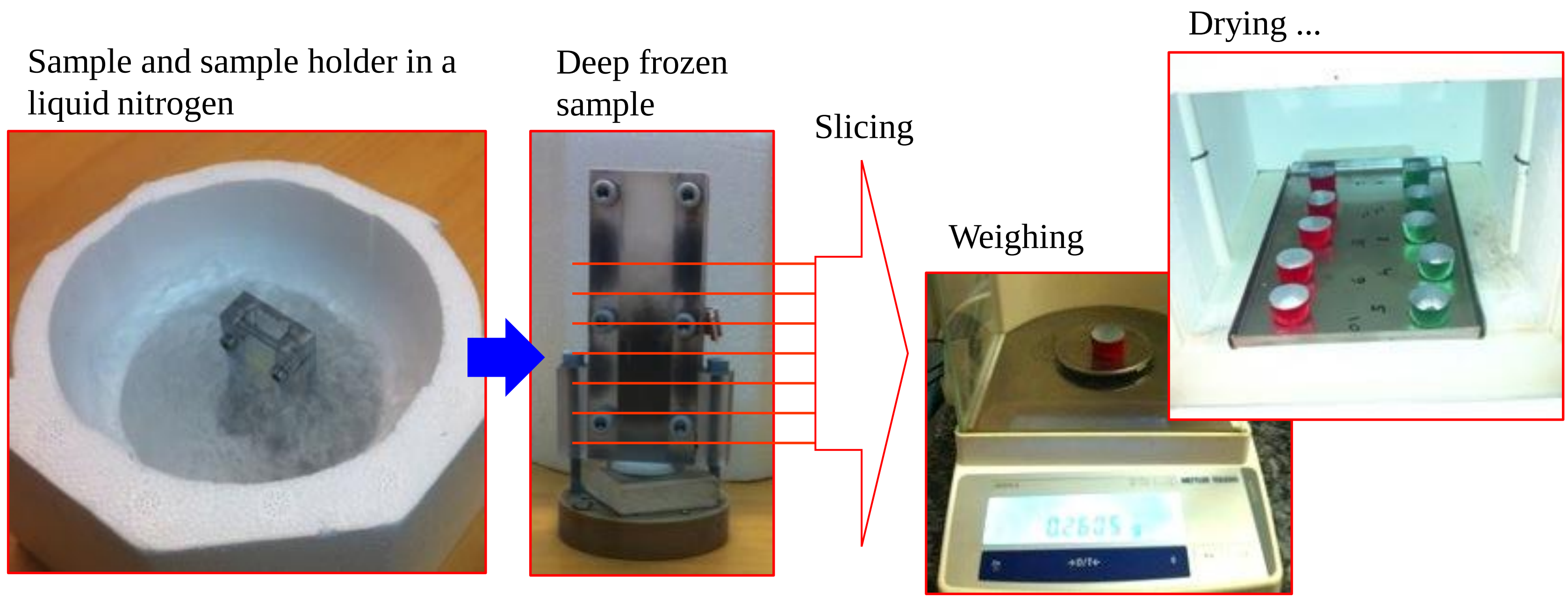
Experiment

Model



Calibration of X-ray image grey scale (plan)

Measurement of local water content distribution in the final state (partially wetted sample)



Prospective: Combined information from motion tracking and post mortem water content measurement allows calibration of x-ray image grey scale to local solid and water content during swelling.

→ Direct experimental measurement of solid density $\rho(t,x)$ and water content $\eta(t,x)$ → Model validation etc... ?

General 3D model. Governing equations

(Based on hypoelastic formulation generalized to include wetting)

Rate equations for stress tensor, effective strain and effective plastic strain

$$\dot{\sigma}^{ij} = 2G\left[\left(D^{ij} + \frac{\mu}{3}\mathrm{Tr}(D)g^{ij}\right) - I_Y\left(D_p^{ij} + \frac{\mu}{3}\mathrm{Tr}(D_p)g^{ij}\right)\right]$$

$$\dot{\bar{\epsilon}}_p = \bar{D}_p,$$

where

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x^j} + \frac{\partial v_j}{\partial x^i}\right) \quad \text{(Rate of strain tensor)}$$

$$D_p^{ij} = \dot{\lambda}F_p^{ij} \quad \text{(Plastic rate of strain tensor)}$$

$$\bar{D}_p = \sqrt{b\,\mathrm{Tr}(D_p \cdot D_p)} \quad \text{(Effective plastic rate of strain)}$$

$$I_Y = \text{The plastic domain indicator function}$$

$$F_p^{ij} = \frac{\partial f_Y}{\partial \sigma_{ij}} = \frac{a_Y}{\bar{\sigma}_Y}(\sigma^{ij} + k_Y p g^{ij})$$

$$f_Y = \left[a_Y(\sigma_{ij}\sigma^{ij} - 3k_Y p^2)\right]^{1/2} - \bar{\sigma}_Y(\bar{\epsilon}_p, \eta) \quad \text{(Yield surface)}$$

$$\dot{\lambda} = \Gamma(p)\left(\mathrm{Tr}(S \cdot D) - C_e\dot{\eta}\right) \quad \text{(Plastic multiplier)}$$

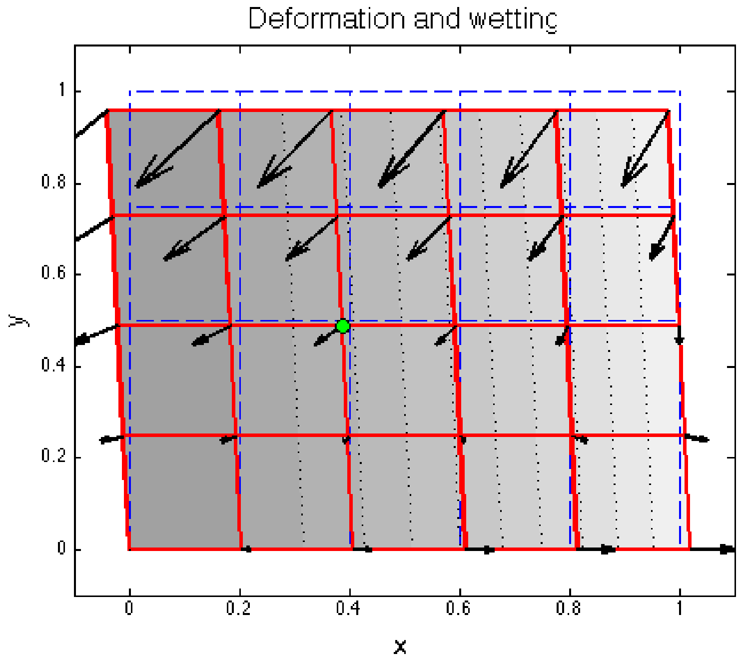
$$\Gamma(p) = \frac{\bar{\sigma}_Y}{(\sigma_Y^2 + \beta p^2) + C_p\bar{\sigma}_Y\sqrt{\bar{\sigma}_Y^2 + \gamma p^2}}$$

$$S^{ij} = \sigma^{ij} + \alpha p g^{ij}$$

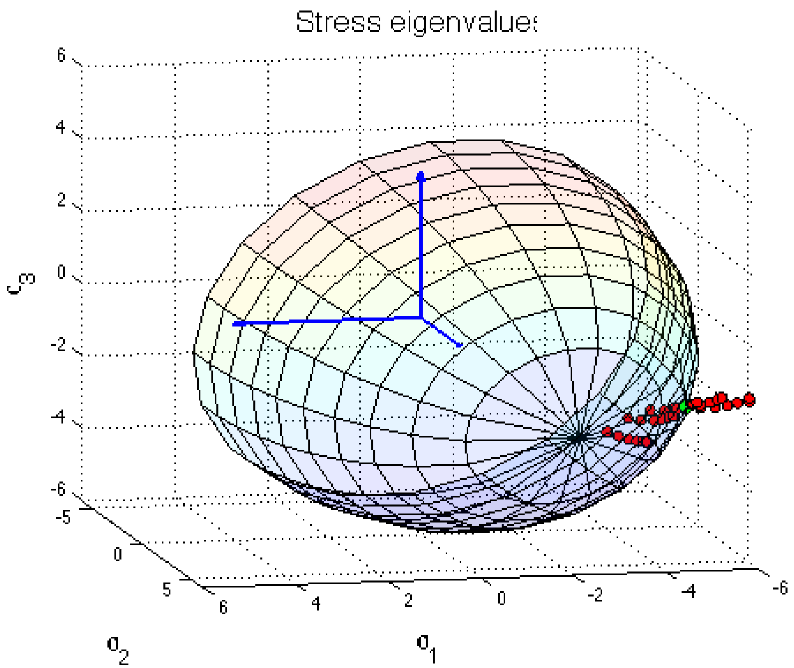
and more ...

General 3D stress model with finite deformations, plasticity and wetting/swelling

Test solutions for predetermined deformation and wetting



Original (dashed lines) and deformed grid (solid lines) with scaled displacement vectors (arrows) and water content in the final state (gray scale color) for a test calculation for the 3D stress model



Stress eigenvalues of the material points of test grid in the final deformed state state (red dots) shown in the 3D stress eigenspace. The surface is the final yield surface of a selected point indicated as green dot in fig. a)